8/29 Logarithms (2.6)

$\ln(a \pm b) = \ln(a \pm b)$

$\ln(ab) = \ln a + \ln b$

$\ln(a/b) = \ln a - \ln b$

$\ln(a^b) = b \ln a$

$\ln(e^x) = x$

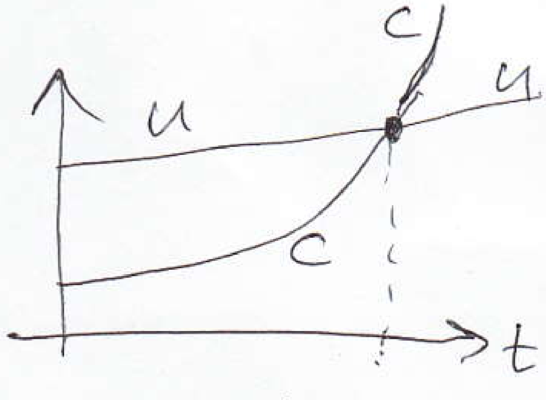
$e^{(x^y)} = e^{(x^y}$

$e^{x+y} = e^x e^y$

$e^{x-y} = e^x / e^y$

$(e^x)^y = e^{xy}$

$e^{\ln x} = x$



$t \approx 14$

$U(t) = 15 (1.01)^t$

$C(t) = 5.3 (1.09)^t$

~~U(t)~~ Solve $U(t) = C(t)$ for t.

$15 (1.01)^t = 5.3 (1.09)^t$

$$\frac{15}{5.3}\left((1.01)^t\right) = (1.09)^t$$

Try: $\sqrt[t]{\frac{15}{5.3}(1.01)^t} = 1.09$

$$\sqrt[t]{\frac{15}{5.3}}\, 1.01 = 1.09$$

Let's try something else.

$$\ln\left(\frac{15}{5.3}\cdot\left((1.01)^t\right)\right) = \ln\left((1.09)^t\right)$$

$$\ln\frac{15}{5.3} + \ln\left((1.01)^t\right) = t \ln 1.09$$

$$\ln\frac{15}{5.3} + t\ln 1.01 = t\ln 1.09$$

$$\ln\frac{15}{5.3} = t\ln 1.09 - t\ln 1.01$$

$$\ln(15/5.3) = t(\ln 1.09 - \ln 1.01)$$

$$\ln(15/5.3) = t\ln(1.09/1.01)$$

$$\frac{\ln(15/5.3)}{\ln(1.09/1.01)} = t \Rightarrow t = 13.65\ldots$$

A bacteria culture has 100 cells initially. An hour later, there are 130 cells.

Assuming exponential growth, how long will it take to grow from 130 to 260 cells?

$$\frac{130}{100} = 1.3 \qquad B(t) = 100(1.3)^t$$

ratio of populations over 1 hour's growth

$$B(0) = 100(1.3)^0 = 100(1) = 100$$

$$B(1) = 100(1.3)^1 = 100(1.3) = 130$$

$$260 = B(t) \qquad t = ?$$

$$260 = 100(1.3)^t$$

$$2.6 = (1.3)^t$$

$$\ln 2.6 = \ln(1.3^t) = t \ln 1.3$$

$$\frac{\ln 2.6}{\ln 1.3} = t$$

Time from $B = 130$ to $B = 260$

is $t - 1 = \dfrac{\ln 2.6}{\ln 1.3} - 1 = 2.64$ hours

HW #1 A bacteria culture starts with 1000 cells. An hour later it has 1450 cells. How long will take to grow from 2000 to 5000 cells?

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^{14}C (carbon-14) has a half-life of 5700 years. (For buried, dead stuff).

years

$X(t)$ = amount of ^{14}C at time t.

$$X(t) = \underbrace{(X(0))}_{\text{initial amount}}\left(\frac{1}{2}\right)^{(t/5700)}$$

$$X(5700)\ \cancel{X(t)} = (X(0))\left(\frac{1}{2}\right)^{(5700/5700)}$$

$$X(5700) = (X(0))\left(\frac{1}{2}\right)^{1}$$

$$X(5700) = \underbrace{(X(0))}_{\text{initial amount}}\left(\frac{1}{2}\right)$$

HW #2 If a skeleton has 1.2% of its initial ^{14}C, how old is it?