9/2 Compound interest (3-2)

Monthly compounded interest:

Pay P now. ($t = 0$ years)

Receive $Pr/12$ ($t = \frac{1}{12}$ year = 1 month later)

→ account balance $A = P + Pr/12$

$A = P(1 + r/12)$

Same as simple interest if you only are paid once. → $A = P(1 + rt)$

Every month: multiply by $(1 + r/12)$

initial deposit

start $P = A$

1 month later $P(1 + r/12) = A$

2 months later $P(1 + r/12)^2 = A$

3 months later $P(1 + r/12)^3 = A$

1 year later $P(1 + r/12)^{12} = A$

Never depositing or withdrawing after start

$P = 500$ $r = 2.1\% = 0.021$

compounded monthly

t (years)	A
0	500
1/12	$500(1+0.021/12) = 500.88$
2/12	$500(1+0.021/12)^2$ ~~$= 502.62$~~ 501,75
3/12	$\hookrightarrow = 500.88(1+0.021/12)$ ~~$= 502.62$~~
$1 = 12/12$	$501.75(1+0.021/12) = 502.63$
	$500(1+0.021/12)^3 =$ same
	$500(1+0.021/12)^{12} = 510.60$

Comparte to $\underbrace{500(1+0.021)}_{\text{annually compounded } (t=1)} = 510.50$

$r = 2.1\%$ $P = 500$ $t = 20$ years

$A = P(1+r)^{20}$ (annually compounded)

$A = P(1+r/12)^{240}$ (monthly compounded)

$A = 757.68$ (annually compounded)

$A = 760.70$ (monthly compounded)

If we change r to 12%:

$A = 4,823.15$ (annually comp.)

$A = 5,446.28$ (monthly comp.)

HW#1

m = # times compounded per year

m = 12 means monthly comp.

m = 1 means annually comp.

Fill in this table:

P	m	r	t (years)	A
500	12	2.5%	7	?
5000	1	8%	10	?
1000	4	1.3%	9/12	?
2000	2	0.7%	1	?

At 5% interest compounded monthly, how does it take to double your money?

$$A = P(1 + r/12)^{12t} \qquad (t \text{ in years})$$

$$r = 5\% = 0.05$$

Solve $A = 2P$ for t.

$$2P = P(1 + r/12)^{12t}$$

$$2 = (1 + r/12)^{12t}$$

$$\ln 2 = \ln\left(\left(1 + \frac{0.05}{12}\right)^{12t}\right)$$

$$\ln 2 = 12t \ln\left(1 + \frac{0.05}{12}\right)$$

$$\frac{\ln 2}{12 \ln\left(1 + \frac{0.05}{12}\right)} = t$$

$t = 13.86$ years

$t = 166.4$ months

Compare to $\frac{70}{5} = 14$.

HW #2

At 3% interest compounded quarterly, given an initial deposit of 10,000.00, how much will be in the account in 5 years? How long until there is ≥25,000.00 in the account?