Compound interest (part II) (3-2)

- continuously compounded interest
- APR & APY

$r = 5\%$ (annual interest rate)

$t = 1$ (year) $P = 1,000.$ (start)

$m = \#$ times interested compounded
          per year

1   year later: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

$$P = 1000 \qquad r = 0.05 \qquad t = 1$$

m	A
annually	1	1,050
quarterly	4	1,050.9453369...
monthly	12	1,050.11618978...
daily	360	1,050.12674464...
continuously	∞	1,050.12710963...

$A = P\left(1 + \frac{r}{m}\right)^{mt}$

$A = Pe^{rt}$

$$A = Pe^{rt} = P\left(e^{(rt)}\right)$$

As $m \to \infty$, $\left(1 + \frac{r}{m}\right)^{m} \to e^{r}$

How long does it take to double
your money at 1.3%
annual interest, compounded continuously?

$$A = Pe^{rt}$$ Solve $A = 2P$

$$2P = Pe^{rt} \Rightarrow 2 = e^{rt} \Rightarrow \ln 2 = \ln(e^{rt})$$

$$\ln 2 = rt \underbrace{\ln(e)}_{1} = rt \Rightarrow t = \frac{\ln 2}{r}$$

$$t = \frac{0.693147\ldots}{0.013} = 53.319\ldots \text{ years}$$

$$t \approx \frac{0.70}{0.013} = \frac{70}{1.3}$$ (rule of thumb for doubling time)

Which is better, 1.2% compounded
continuously, or 1.25% compounded
monthly?

Compare $P\left(1 + \frac{0.0125}{12}\right)^{12t}$ with

$$P e^{0.012t}$$

Use $P = 1$ & $t = 1$ for the comparison.

Compare $\left(1 + \dfrac{0.0125}{12}\right)^{12}$ with

$$e^{0.012}$$

$$\left(1 + \frac{0.0125}{12}\right)^{12} > e^{0.012}$$

These are called APY's $+ 1$.

APY = annual percentage yield

APR = annual percentage rate $= r$

$$\left(1 + \frac{0.012005}{12}\right)^{12} \text{ is a tiny}$$

bit less than $e^{0.012}$.

1.25% compounded monthly ⊗ pays more interest than 1.2% compounded continuously, but 1.2% compounded continuously pays more interest than 1.2005% compounded monthly.

$$APY = \left(1 + \frac{r}{m}\right)^m - 1$$

$$\text{or} \quad APY = e^r - 1 \quad (\text{case } m = \infty)$$

r = APR	m	APY
0.05 = 5%	1	5% = 0.05
5%	2	5.0625% = 0.050625
5%	4	5.0945% = 0.050945
5%	12	5.1162% = 0.051162
5%	360	5.1267% = 0.051267
5%	∞	5.1271% = 0.051271

HW: Which is ~~a better~~ the best

savings account?

A) 2% compounded quarterly

B) 1.99% compounded monthly

C) 1.98% compounded continuously

Compute the APY's and
doubling times for all 3 options.