

(Reminder: Test 9/16)

Today: Present value of payment stream (3-4)

Last time: $FV = A = PMT \frac{(1+i)^n - 1}{i}$
Future value

Example: buy a new car with 5% APR financing (and monthly payments)

$\$20,000 = PV$ (cost of buying car with cash)
present value

$t=0$: loan account balance $A = PV$ $r = 5\%$

$t = \frac{1}{12}$: ① Multiply A by $(1 + \frac{r}{12})$
 $(1+i) = (1 + \frac{0.05}{12})$

② Subtract PMT from A

$$A = PV(1+i) - PMT$$

$$t = \frac{2}{12} : A = (PV(1+i) - PMT)(1+i) - PMT$$

$$A = PV(1+i)^2 - PMT(1+i) - PMT$$

$$t = \frac{3}{12} : A = (PV(1+i)^2 - PMT(1+i) - PMT)(1+i) - PMT$$

$$A = PV(1+i)^3 - PMT(1+i)^2 - PMT(1+i) - PMT$$

$$t = \frac{n}{12} \quad 0 = A = PV(1+i)^n - PMT(1+i)^{n-1} - PMT(1+i)^{n-2}$$

↑
Last payment
n months
later

$$- PMT(1+i)^{n-2}$$

$$\vdots$$

$$- PMT(1+i)^2$$

$$- PMT(1+i)$$

$$- PMT$$

$$FV = PMT \frac{(1+i)^n - 1}{i}$$

$$0 = PV(1+i)^n - FV$$

$$FV = PV(1+i)^n$$

Say $n = 72$ (6 years = 72 months)

$$PV = 20,000 \quad i = \frac{0.05}{12} = \frac{r}{12} \quad n = 72$$

$$FV = PV(1+i)^n = 26,980.36$$

$$PMT \left(\frac{(1+i)^n - 1}{i} \right) = FV$$

$$PMT = FV \left(\frac{i}{(1+i)^n - 1} \right) = 322.10$$

monthly
car
loan
payment

Related Formulas:

$$\rightarrow FV(1+i)^{-n} = PV$$

$$PMT \frac{1 - (1+i)^{-n}}{i} = PV \quad \leftarrow$$

$$PMT = PV \frac{i}{1 - (1+i)^{-n}}$$

Annuity: How much would it cost to buy a 20-year annuity that pay \$1,000 per month, with $r = 4\%$?

$$PV = ? \quad PMT = 1,000 \quad r = 0.04$$

$$n = 12 \cdot 20 = 240$$

$$i = \frac{r}{12} = 0.00\overline{333}\dots$$

$$PMT \left(\frac{1 - (1+i)^{-n}}{i} \right) = PV = \$165,021.86$$

$$PMT * (1 - (1+r/12)^{-n}) / (r/12)$$

$$r = 2\% \Rightarrow PV = \$197,674.03$$

$$r = 8\% \Rightarrow PV = \$119,554.29$$

HW: ① To buy a \$200,000 house with a 30-year, 4.8% APR, mortgage, and a \$40,000 down payment, what is the monthly mortgage payment?

② How much does a 10-year annuity that pays \$700 per month cost if $r = 3.5\%$?

③ What is the monthly car payment for a \$23,000 car at 6% APR over 12 years?

Get more practice from exercises in 3-4.