

# More examples of applications of present value (3-4).

## Retirement planning:

You save for 40 years;  
you withdraw the savings over 20 years.

Saving stage: monthly deposit of same amount ???

Withdrawing stage: monthly withdrawal of same amount \$2500.

$$r = 5\%$$

① How much money is needed in 40 years?

$$PV = \underbrace{PMT}_{\substack{\text{amount} \\ \text{needed} \\ \text{in 40 years}}} \frac{1 - (1+i)^{-n}}{i}$$

2500

$$i = 0.05/12$$

$$n = 12 \cdot 20 = 240$$

↑  
20 years  
of withdrawals

$$378,813.28$$

② How much should we deposit each month ~~at~~ over 40 years to accumulate 378,813.28?

$$378,813.28 = FV = \underbrace{PMT}_{\substack{\uparrow \\ \text{monthly deposit}}} \frac{(1+i)^n - 1}{i}$$

$$i = 0.05/12$$

$$n = 12 \cdot 40 = 480$$

$$FV \frac{i}{(1+i)^n - 1} = PMT = \boxed{248.24}$$

$$(FV * (0.05/12)) / (((1 + (0.05/12))^{480}) - 1) = PMT$$

HW: Repeat with  $r = 1\%$  &  $r = 9\%$ .

Home equity = value of home -  $\underbrace{\text{unpaid loan balance}}_{\text{present value}}$

Example: ① Buy a house in 2012 for 150,000; ~~make~~ make down-payment of 25,000; ~~to~~ to pay the remaining 125,000, take out 20-year, 5% APR mortgage.

② In 2022, how much equity do you have if you made each monthly mortgage payment and the house is now worth 180,000?

$$\text{Equity} = 180,000 - PV_{\text{unpaid loan}}$$

$$\textcircled{1} \quad \text{PMT} = \frac{125,000}{\text{PV}_{2012}} \cdot \frac{i}{1 - (1+i)^{-n}}$$

$$n = 12 \cdot 20 = 240$$

$$i = .09/12$$

~~8295.54~~

824.94

$$\textcircled{2} \quad \text{PV}_{\text{unpaid loan } 2022} = \text{PMT} \cdot \frac{1 - (1+i)^{-n}}{i}$$

10 years left =

$$n = 12 \cdot 10 = 120$$

$$\text{PV}_{\text{unpaid}} = 77,776.46$$

$$\text{Equity} = 180,000 - \text{PV}_{\text{unpaid}} = 102,223.35$$

HW: 3-4 #49, 42, 48