More Gauss-Jordan

Section 4-3

$$2x + 3y = 7$$
$$-4x - 6y = -14$$

$$\begin{bmatrix} 2 & 3 & 7 \\ -4 & -6 & -14 \end{bmatrix} \quad \frac{1}{2}R_1 \to R_1$$

Starting with left most column, try to get a 1 in the diagonal entry & 0s elsewhere in that column.

$$\begin{bmatrix} 1 & 3/2 & 7/2 \\ -4 & -6 & -14 \end{bmatrix} \quad R_2 + 4R_1 \to R_2$$

$$\begin{bmatrix} 1 & 3/2 & 7/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Goal: $\begin{bmatrix} 1 & 0 & - \\ 0 & 1 & - \end{bmatrix}$

← We can't get a 1 here without changing the left column.

Stop here: This is the best we'll get.

$$\begin{bmatrix} 1 & 3/2 & 7/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x + \frac{3}{2}y = \frac{7}{2}$$
$$0 = 0$$

$$x = \frac{7}{2} - \frac{3}{2}y$$
$$y \text{ can be anything}$$

$\Longleftrightarrow$

$$x = \frac{7}{2} - \frac{3}{2}t$$
$$y = t \text{ arbitrary}$$

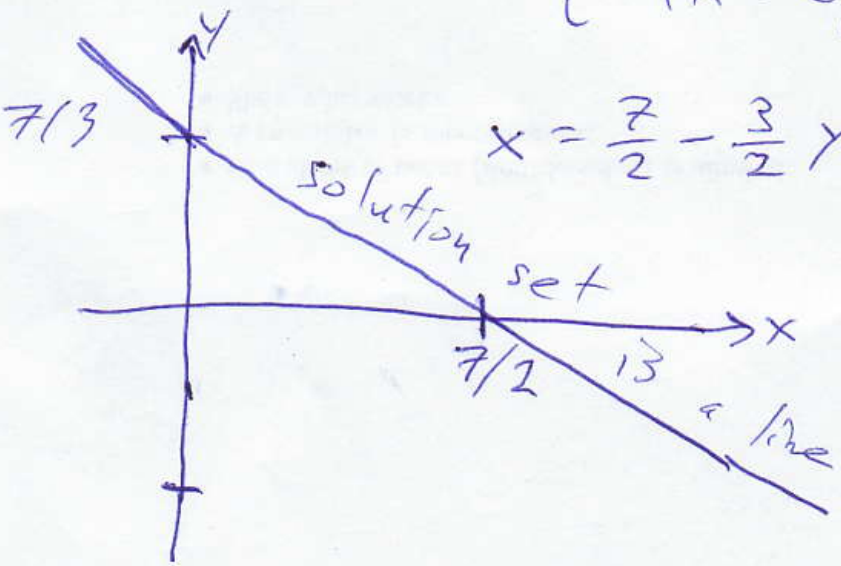
Example:

t	x	y
-4	$19/2$	-4
18	$-47/2$	18
0	$7/2$	0
3	-1	3

$(x,y) = (19/2, -4)$, or $(-47/2, 18)$, or $(7/2, 0)$ or

$(-1, 3)$ or ...

↑ All satisfy $\begin{cases} 2x + 3y = 7 \\ -4x - 6y = -14 \end{cases}$



$x = \frac{7}{2} - \frac{3}{2}y$

$$\begin{cases} x - 3y = 4 \\ y + z = 1 \\ x - 5y - 2z = 2 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 1 & 1 \\ 1 & -5 & -2 & 2 \end{array}\right] \quad R_3 - R_1 \to R_3$$

Goal: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 \end{array}\right] \quad R_1 + 3R_2 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 7 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 \end{array}\right] \quad R_3 + 2R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 7 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{cases} x + 3z = 7 \\ y + z = 1 \\ 0 = 0 \end{cases}$$

$$\begin{cases} x = 7 - 3t \\ y = 1 - t \\ z = t \text{ arbitrary} \end{cases} \leftarrow \text{Solution set}$$

Examples:

t	x	y	z
-2	13	3	-2
17	-44	-16	17
4	-5	-3	4
1	4	0	1

$(x, y, z) = (13, 3, -2)$ or
$(-44, -16, 17)$ or
$(-5, -3, 4)$ or
$(4, 0, 1)$ or ...
are all solutions.

HW: Find the solution set for:

$$\begin{cases} 2x - 3y + 4z = 5 \\ -x + y = 6 \\ x - 3y + 8z = 28 \end{cases}$$

And do the same for:

$$\begin{cases} x + 4y - 3z = 5 \\ -2x - 8y + 6z = 10 \\ 3x + 12y - 9z = 15 \end{cases}$$

$$\begin{cases} -2x - 6y + 4z = 12 \\ 3x + 9y - 6z = -18 \\ -5x - 15y + 10z = 30 \end{cases}$$

$$\begin{bmatrix} -2 & -6 & 4 & 12 \\ 3 & 9 & -6 & -18 \\ -5 & -15 & 10 & 30 \end{bmatrix}$$

$-R_1/2 \to R_1$

Goal:

$$\begin{bmatrix} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & -6 \\ 3 & 9 & -6 & -18 \\ -5 & -15 & 10 & 30 \end{bmatrix} R_2 - 3R_1 \to R_2$$

$$\begin{bmatrix} 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 \\ -5 & -15 & 10 & 30 \end{bmatrix} R_3 + 5R_1 \to R_3$$

$$\begin{bmatrix} 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x + 3y - 2z &= -6 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

$$x = -3y + 2z - 6$$

Solution set:

$$\begin{cases} x = -3s + 2t - 6 \\ y = s \quad \text{arbitrary} \\ z = t \quad \text{arbitrary} \end{cases}$$

both solutions

Examples: $(-17, 5, 2)$, $(26, 0, 16)$...

s	t	x	y	z
5	2	-17	5	2
0	16	26	0	16
-3	11	25	-3	11
8	5	-20	8	5