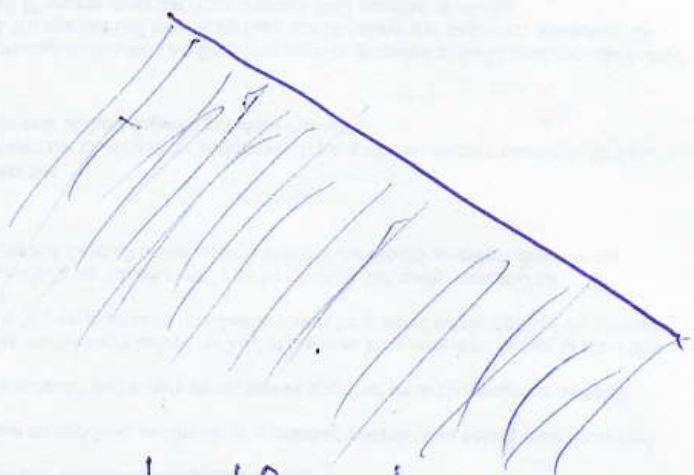# Starting Chapter 5

Linear inequalities in 2D.

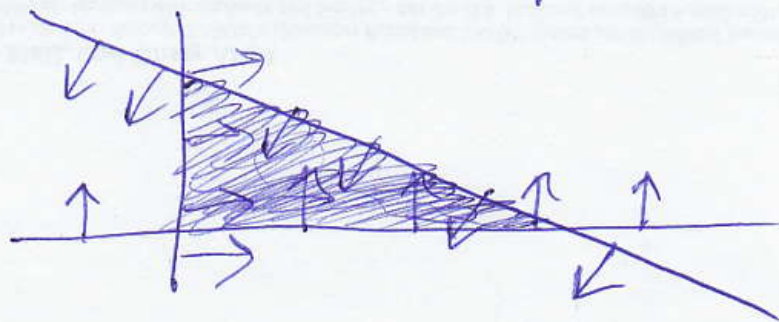$$3x + 4y = 5 \qquad\qquad 3x + 4y \leq 5$$

line   half plane

- Plotting lines my way (the easy way?)
- Plotting half planes
- Systems of linear inequalities:
    intersection of half-planes

$$Ax + By = C$$

$$x = 0 \Rightarrow By = C$$
$$y = C/B$$

$$(x, y) = (0, C/B)$$

$$y = 0 \Rightarrow Ax = C$$
$$x = C/A$$

$$(x, y) = (C/A, 0)$$

• Draw a line through two points satisfying the equation.

• Get points by plugging in $x = 0$ & solving for y; plugging in $y = 0$ & solving for x.
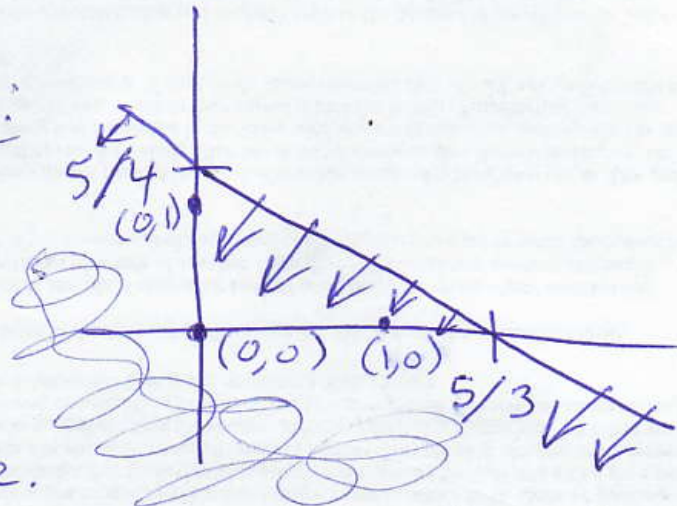

$$\underline{3x + 4y = 5:}$$

$$3x + 4y \leq 5:$$

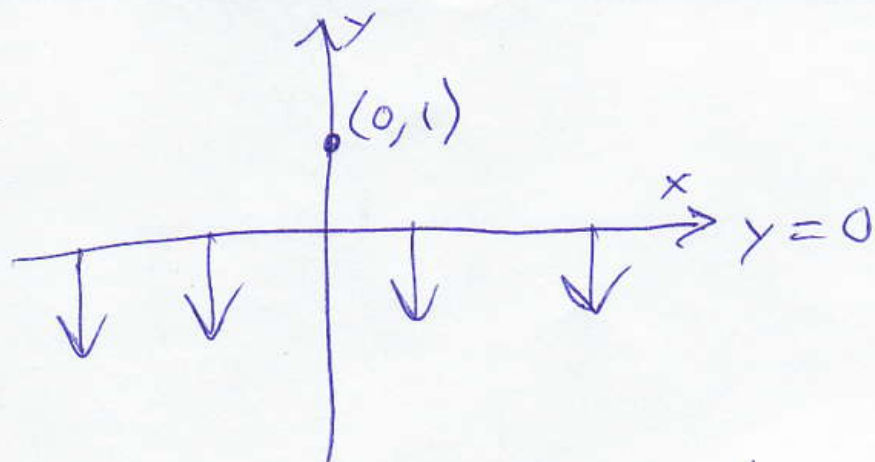Test a point not on the line.

Easiest pts: $(0, 0), (1, 0), (0, 1)$

$$3(0) + 4(0) = 0 < 5 \Rightarrow (0, 0) \text{ in half-plane}$$

Bad test point: $(48/5, 36/5)$

$$3(48/5) + 4(36/5) =$$

$y \leq 0$.

$(0,0)$ is a bad test point:

$0 = 0 \Rightarrow$ is on the line.

$(1,0)$ is a bad test point:

$0 = 0 \Rightarrow$ is on the line

$(0,1)$ is a good test point:

$1 > 0 \Rightarrow$ no $(0,1)$ not in half plane

Plotting $Ax = C$:

vertical ~~horizontal~~ line $x = C/A$ ———

$$By = C$$

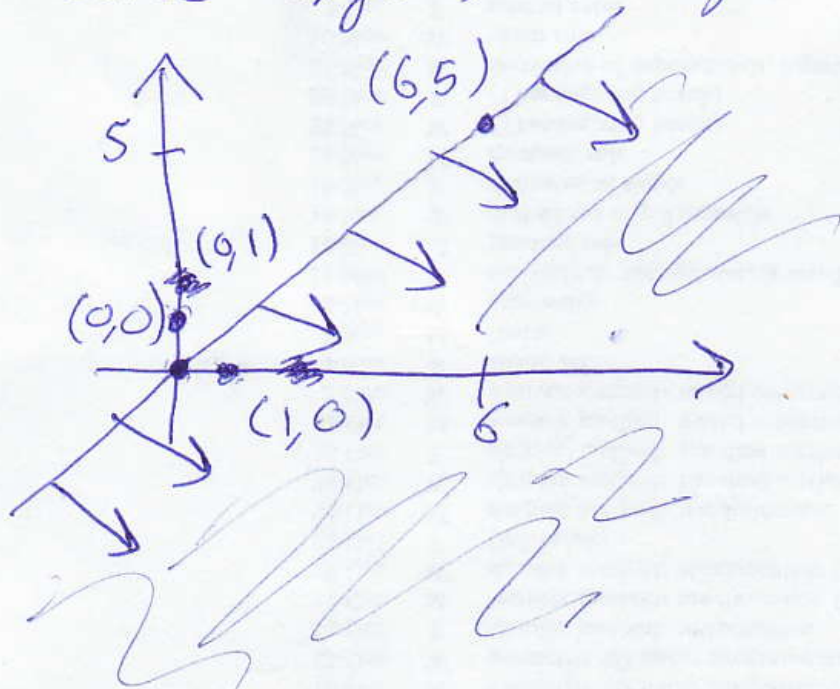horizontal ~~vertical~~ line $y = C/B$ |

Plotting $Ax + By = 0$

Draw the line through $(0,0)$ & $(-B, A)$

$A(0) + B(0) = 0$ ✓

$A(-B) + B(A) ~~= 0~~ = -AB + AB = 0$ ✓

$5x - 6y \geq 0$:

Line goes through $(0,0)$ & $(6,5)$

$\left(\begin{array}{l} \text{Line is} \\ 5x - 6y = 0 \end{array} \right)$

Test point:

$(1,0)$:

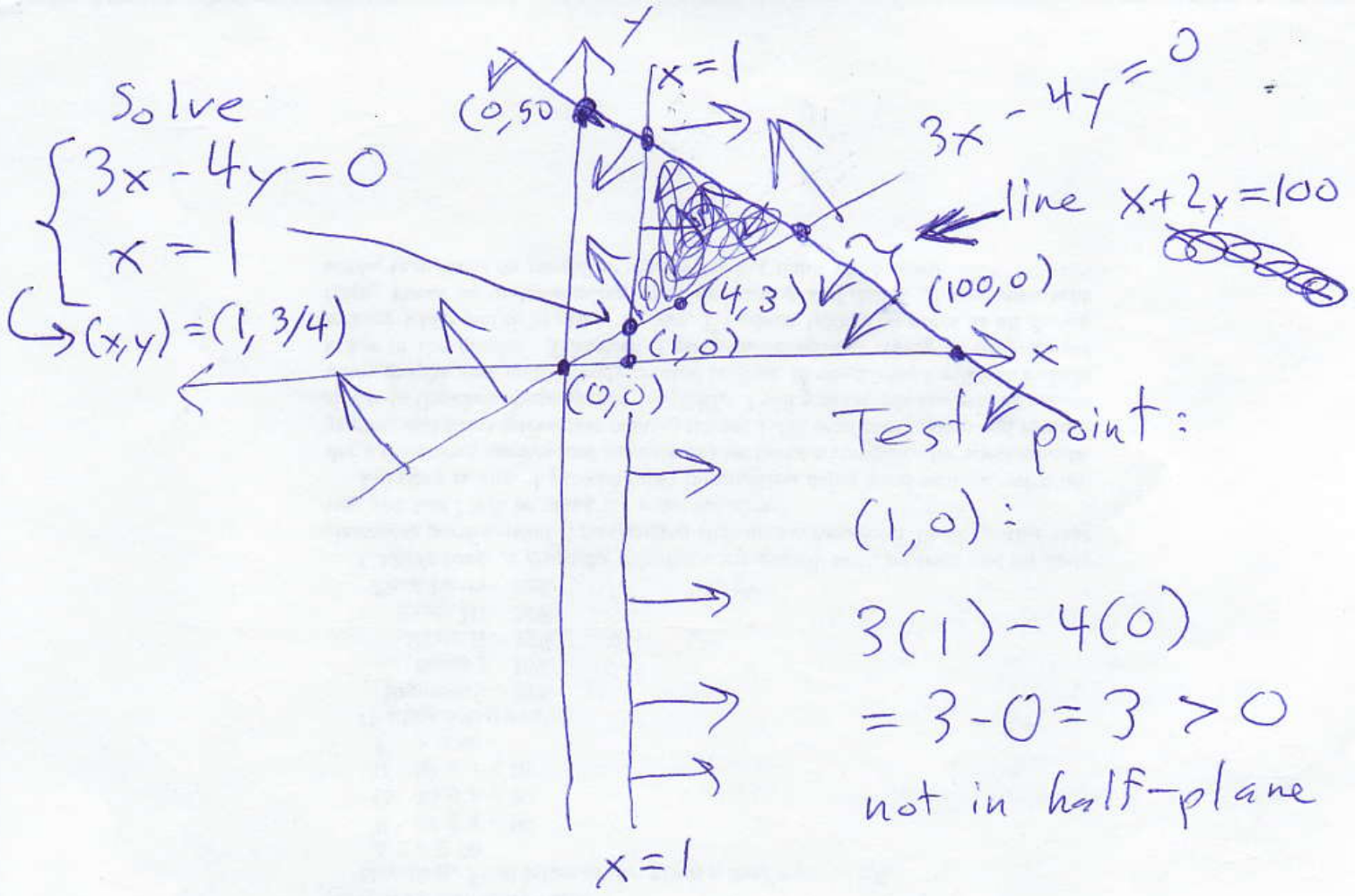$5(1) - 6(0)$

$= 5 - 0 = 5 > 0 \checkmark$

half plane $5x - 6y \geq 0$.
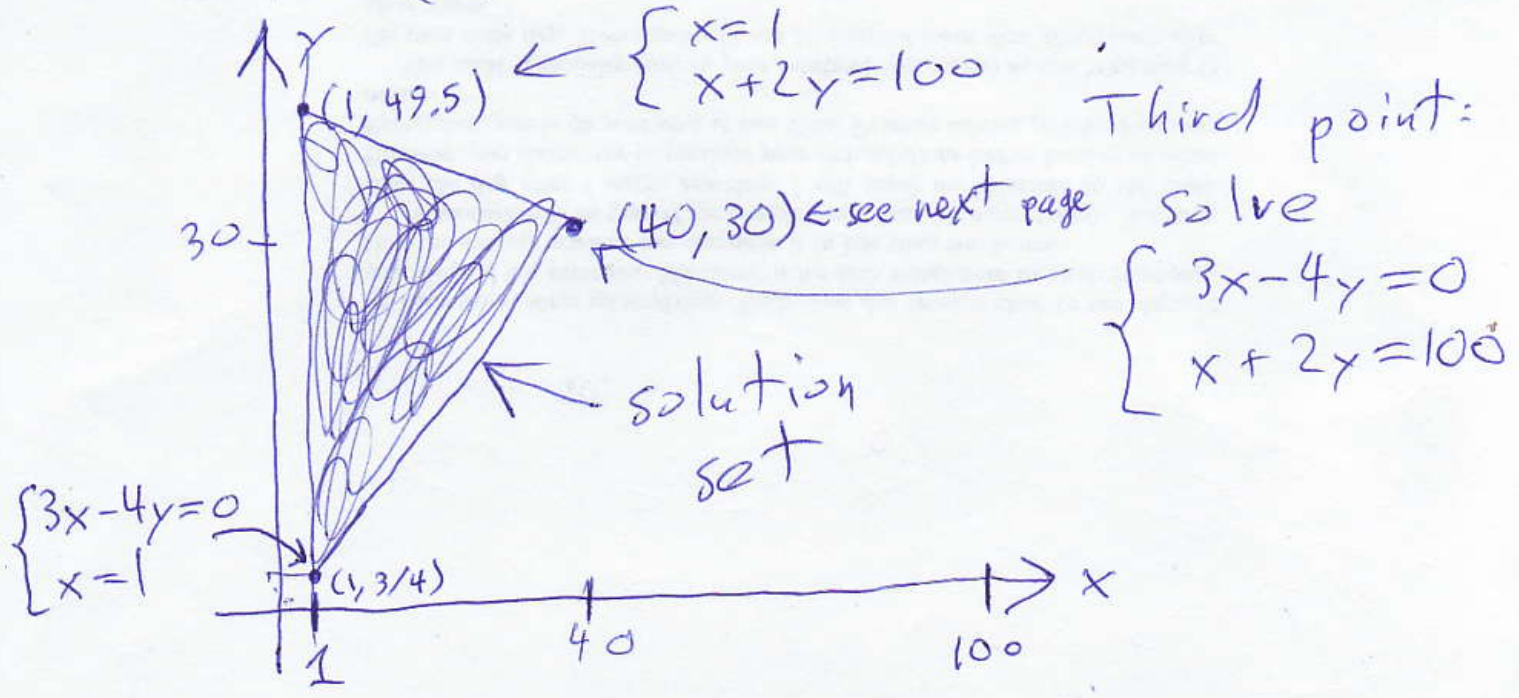

Plot the solution set of

$\begin{cases} 3x - 4y \leq 0 \\ x \geq 1 \\ x + 2y \leq 100 \end{cases}$

Strategy: • plot the half planes
          • Find where the
            pairs of lines intersect.

Solve
$$\begin{cases} 3x - 4y = 0 \\ x = 1 \end{cases}$$

$\rightarrow (x,y) = (1, 3/4)$

(0,50)

$x = 1$

$3x - 4y = 0$

line $x + 2y = 100$

(100, 0)

(4, 3)

(1, 0)

(0, 0)

Test point:

(1, 0):

$3(1) - 4(0)$

$= 3 - 0 = 3 > 0$

not in half-plane

$x = 1$

$x + 2y = 100$ goes through (0, 50) & (100, 0)

$x + 2y \leq 100$: test pt : (0, 0)

$0 + 2(0) = 0 < 100$

$\begin{cases} x = 1 \\ x + 2y = 100 \end{cases}$

(1, 49.5)

(40, 30) ← see next page

Third point:

solve

$$\begin{cases} 3x - 4y = 0 \\ x + 2y = 100 \end{cases}$$

30

solution set

$\begin{cases} 3x - 4y = 0 \\ x = 1 \end{cases}$
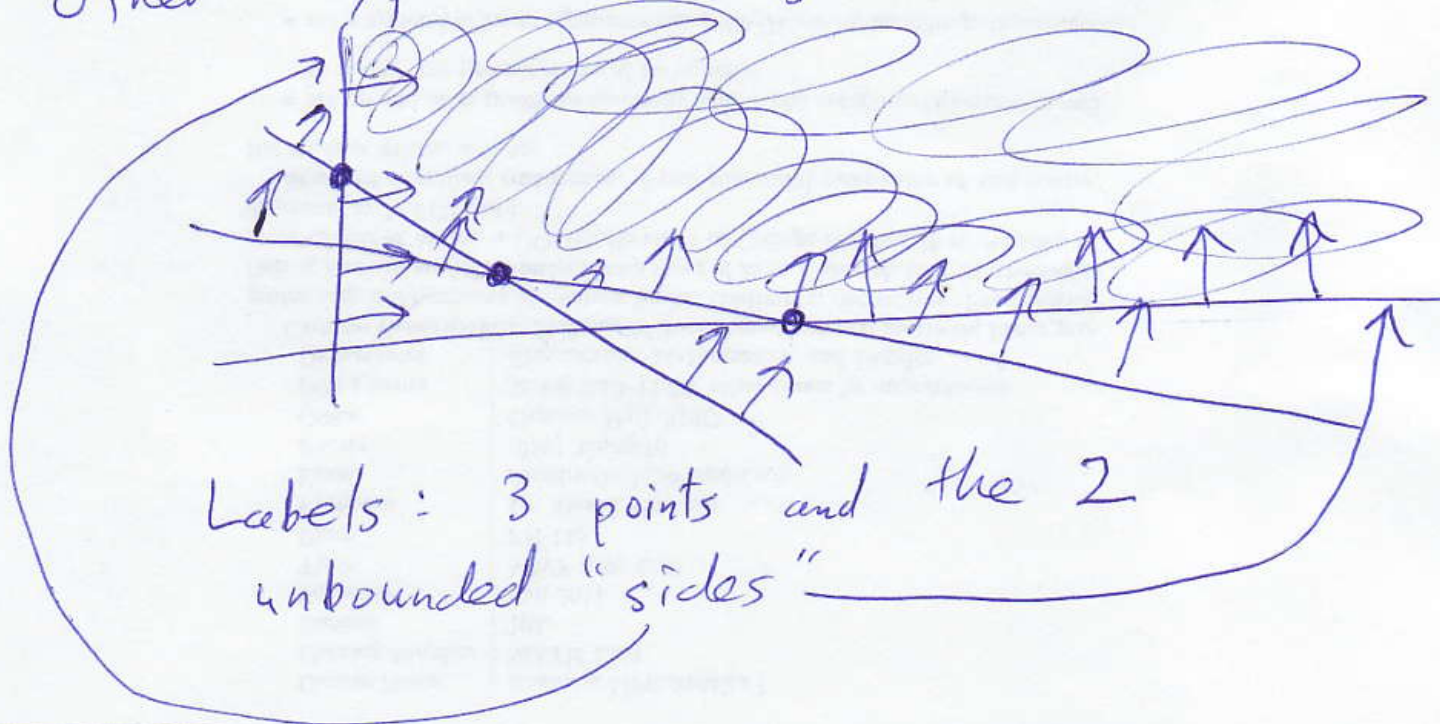
(1, 3/4)

1

40

100

x

$$\begin{bmatrix} 3 & -4 & 0 \\ 1 & 2 & 100 \end{bmatrix} \xrightarrow[\text{elim.}]{G-J} \begin{bmatrix} 1 & 0 & 40 \\ 0 & 1 & 30 \end{bmatrix}$$

"rref" $(x,y) = (40, 30)$

Alternative

$$\begin{cases} ax + by = p \\ cx + dy = q \end{cases} \Rightarrow \begin{cases} x = (dp - bq)/(ad - bc) \\ y = (aq - cp)/(ad - bc) \end{cases}$$

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other types of solution sets:

Labels: 3 points and the 2 unbounded "sides"

HW: Plot the solution sets:

① $\begin{cases} x \geq 0 \\ y \geq 0 \\ 5x + y \leq 1000 \end{cases}$

② $\begin{cases} x + 3y \geq 500 \\ 2x + y \geq 700 \\ x \geq y \end{cases}$

③ $\{ x \leq 5; \quad y \leq 2 + x; \quad x + y \leq 4 \}$