Slack variables (6-1)

Standard form for maximization:

Maximize $P = c_1x_1 + c_2x_2 + c_3x_3 + \cdots c_nx_n$

subject to constraints of the form

$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n \leq b,$

where $b \geq 0$, and subject to the "nonnegative constraints": $x_1, x_2, \ldots, x_n \geq 0$

Example:

Maximize $P = 3x + 4y + z$

subject to $x, y, z \geq 0$ and

$$2x + 8y + 7z \leq 1000,$$
$$15x + 4z \leq 800,$$
$$4y + 3z \leq 500,$$
$$17x + y + 3z \leq 900$$

are ≥ 0

Minimize $4x - y + 6z$

subject to $x \le 2y + z + 5$,

$x + y + z \ge \cancel{-7}0$, and

$14x + 6y \le +5$, and $x, y, z \ge 0$

Put in standard form:

<u>Maximize</u> $-4x + y - 6z$

subject to: $x - 2y - z \le 5$,

$-x - y - z \le \cancel{7}0$,

$14x + 6y \le 5$,

all ≥ 0

$x, y, z \ge 0$

Maximize $5x - y$ subject to:

$$\begin{cases} 4x + 3y \le 100, \\ x + 2y \le 80, \\ x, y \ge 0 \end{cases}$$

(in standard form already)

Example possible pictures

boundary lines:

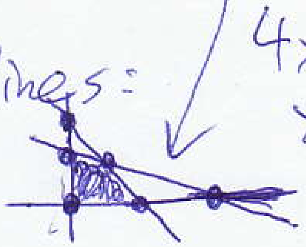
$4x + 3y = 100$

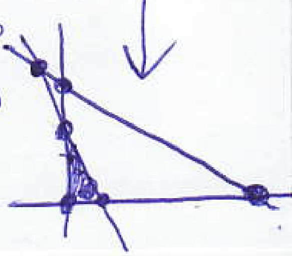
$x + 2y = 80$

$x = 0$

$y = 0$

6 intersections:





Slack variable introduction:

Maximize $5x - y$ subject to:

$$4x + 3y + s_1 = 100$$
$$x + 2y + s_2 = 80$$

$$x, y, s_1, s_2 \geq 0$$

Select as many variables as there are equations (2) as <u>basic</u> and set the other variables to 0.

Do this for all possible combinations of basic variables (6 possibilities).

Solve the system of equations you get from setting the nonbasic variables to 0. (You'll do this 6 times.)

Check that your solution satisfies the $x, y, s_1, s_2 \geq 0$ constraints.

If it does, it's a corner of the feasible set.

Pick the corner that maximizes your objective function $(5x-y)$.

	basic	equations	solutions x	y	s_1	s_2	feasible	$5x-y$
A	x, y	$4x+3y=100$ $x+2y=80$	-8	44	0	0	no	
B	x, s_1	$4x+s_1=100$ $x=80$	80	0	-220	0	no	
C	x, s_2	$4x=100$ $x+s_2=80$	25	0	0	55	yes	125
D	y, s_1	$3y+s_1=100$ $2y=80$	0	40	-20	0	no	
E	y, s_2	$3y=100$ $2y+s_2=80$	0	$\frac{100}{3}$	0	$\frac{40}{3}$	yes	$-\frac{100}{3}$
F	s_1, s_2	$s_1=100$ $s_2=80$	0	0	100	80	yes	0

HW: #9, #12 (6-1) (p. 285)

Plot: