Standard maximization problem:

Maximize P subject to:

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le b_m \\
c_1x_1 + c_2x_2 + \cdots + c_nx_n &= P \\
x_1, \ldots, x_n &\ge 0
\end{aligned}$$

Must be ≥ 0.

Standard minimization problem:

Minimize C subject to:

$$\begin{aligned}
a_{11}y_1 + a_{21}y_2 + \cdots + a_{m1}y_m &\ge c_1 \\
a_{12}y_1 + a_{22}y_2 + \cdots + a_{m2}y_m &\ge c_2 \\
&\vdots \\
a_{1n}y_1 + a_{2n}y_2 + \cdots + a_{mn}y_m &\ge c_n \\
b_1y_1 + b_2y_2 + \cdots + b_my_m &= C \\
y_1, \ldots, y_m &\ge 0.
\end{aligned}$$

Must be ≥ 0

Big M method (6-4) for mixed constraints.

Maximize $P = 3x_1 + 4x_2$

subject to:

$$\begin{cases} x_1, x_2 \geq 0 \\ 2x_1 + x_2 \leq 100 \\ x_1 + x_2 \geq 30 \\ x_1 + 3x_2 \leq 110 \end{cases}$$

$$\begin{cases} x_1, x_2 \geq 0 \\ 2x_1 + x_2 \leq 100 \\ -x_1 - x_2 \leq -30 \\ x_1 + 3x_2 \leq 110 \end{cases}$$

not all ≥ 0

Slack variables

$2x_1 + x_2 + s_1 = 100$ & $s_1 \geq 0$

$x_1 + 3x_2 + s_3 = 110$ & $s_3 \geq 0$

$x_1 + x_2 \geq 30 \longrightarrow x_1 + x_2 - s_2 = 30$

$\& \; s_2 \geq 0$

surplus variable

Maximize $P = 3x_1 + 4x_2$

subject to:

$x_1, x_2, s_1, s_2, s_3 \geq 0$

$2x_1 + x_2 + s_1 = 100$

$x_1 + x_2 - s_2 = 30$

$x_1 + 3x_2 + s_3 = 110$

$-3x_1 - 4x_2 + P = 0$

$$\left[\begin{array}{cccccc|c} 2 & 1 & (1) & 0 & 0 & 0 & 100 \\ 1 & 1 & 0 & -1 & 0 & 0 & 30 \\ 1 & 3 & 0 & 0 & (1) & 0 & 110 \\ \hline -3 & -4 & 0 & 0 & 0 & (1) & 0 \end{array}\right]$$

$x_1 \quad x_2 \quad s_1 \quad s_2 \quad s_3 \quad P$

$-R_2 \rightarrow R_2$

$$\left[\begin{array}{cccccc|c} 2 & 1 & (1) & 0 & 0 & 0 & 100 \\ -1 & -1 & 0 & (1) & 0 & 0 & -30 \\ 1 & 3 & 0 & 0 & (1) & 0 & 110 \\ \hline -3 & -4 & 0 & 0 & 0 & (1) & 0 \end{array}\right]$$

negative ∴ unfeasible

$$
\begin{aligned}
2x_1 + x_2 + s_1 &= 100 \\
x_1 + x_2 - s_2 + a_2 &= 30 \\
x_1 + 3x_2 + s_3 &= 110 \\
-3x_1 - 4x_2 + Ma_2 + P &= 0
\end{aligned}
$$

$$P = 3x_1 + 4x_2 \underbrace{- Ma_2}_{\text{penalty}}$$

$$x_1, x_2, s_1, s_2, s_3, a_2 \geq 0$$

a_2 ↑ artificial variable

HW #37, 38, 39, 40, 46 (6-3)

#1A, 2A (6-4)