Basic

Differential rules (10-5, 10-6)

$$3.01^2 = 9.0601$$

$$3.01^2 \approx 9.06$$

old: $x = 3$ $\qquad x^2 = 9$

small change $\Delta x = .01$ $\qquad \Delta(x^2) = 0.0601$

new: $x + \Delta x = 3.01$ $\qquad x^2 + \Delta(x^2) = 9.0601$

$$\Delta(x^2) \approx 0.06 = 2(3)(0.01) = 2x\Delta x$$

$\Delta(x^2) \approx 2x\Delta x$ in general, if Δx is small

Why? $(a+b)^2 = a^2 + 2ab + b^2$

$$(3 + .01)^2 = 3^2 + 2(3)(.01) + .01^2$$

$$= \underbrace{9}_{x^2} + \underbrace{\underbrace{.06}_{2x\Delta x} + .0001}_{\Delta(x^2)}$$

Estimate 97^2

$x = 100$ $\qquad$ $x^2 = 10{,}000$

$\Delta x = -3$ $\qquad$ $\Delta(x^2) = 97^2 - 100^2 = ?$

$x + \Delta x = 97$ $\qquad$ $x^2 + \Delta(x^2) = 97^2 = ?$

$$\Delta(x^2) \approx 2x\Delta x = 2(100)(-3) = -600$$

$$x^2 + \Delta(x^2) \approx 10{,}000 - 600 = 9{,}400$$

$$97^2 \approx 9400$$

$$\begin{array}{r} {\scriptstyle 64} \\ 97 \\ \times\ 97 \\ \hline 679 \\ 873 \\ \hline 9409 \end{array}$$

$$97^2 = 9409$$

HW Estimate 1015^2 using differentials.

$$\Delta(x^2) \approx d(x^2) = 2x\,dx, \text{ when } dx = \Delta x \text{ is small}$$

small change

Differentials

small change

Recall $d\sqrt{x} = \dfrac{dx}{2\sqrt{x}}$ from Wednesday.

Why does this work?

Because of $d(x^2) = 2x\,dx$:

$$dx = d(\sqrt{x}^{\,2}) = 2\sqrt{x}\,d\sqrt{x}$$

when $x \geq 0$

$$dx = 2\sqrt{x}\,d\sqrt{x}$$

$$\boxed{\frac{dx}{2\sqrt{x}} = d\sqrt{x}}$$

$$4.01^3 = 64.481201$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$4.01^3 \approx 64.48$$

$x = 4$ $\quad x^3 = 64$

$\Delta x = .01$ $\quad \Delta(x^3) = .481201 \approx .48$

$x + \Delta x = 4.01$ $\quad x^3 + \Delta(x^3) = 64.481201 \approx 64.48$

$$\Delta(x^3) \approx 3x^2 \Delta x$$

$$d(x^3) = 3x^2 dx$$

HW
Estimate 11^3 using $d(x^3) = 3x^2 dx$.

In general, $\underbrace{\boxed{d(x^n) = n x^{n-1} dx}}_{\text{Power rule}}$

$$d(\sqrt{x}) = d(x^{1/2}) = \frac{1}{2} x^{\frac{1}{2}-1} dx$$

$$d(\sqrt{x}) = \frac{1}{2} x^{-\frac{1}{2}} dx = \frac{1}{2} \frac{1}{x^{1/2}} dx$$

$$d(\sqrt{x}) = \frac{1}{2} \frac{1}{\sqrt{x}} dx = \frac{dx}{2\sqrt{x}} \checkmark$$

HW: Use the power rule to estimate

$\sqrt[3]{65}$, $\frac{1}{95}$ $(= 95^{-1})$, and $\frac{1}{\sqrt{10}}$.

Other basic rules:

Constants: $d5 = \Delta 5 = 0$
(5 never changes)

Constant multiples: $d(7x^2) = 7\, d(x^2)$

$$d(kf) = k\, df$$

$\uparrow$ if k constant

Sums & differences: $d(x^2+x^3)=d(x^2)+d(x^3)$

$$d(f+g)=df+dg$$

$$d(x^2-x^3)=d(x^2)-d(x^3)$$

$$d(f-g)=df-dg$$

$$d(x^3-4x^2+5x-6)$$

$$=d(x^3)-d(4x^2)+d(5x)-d6$$

$$=d(x^3)-4\,d(x^2)+5dx-d6$$

$$=d(x^3)-4d(x^2)+5dx-0$$

$$=3x^2dx-4(2x\,dx)+5dx$$

$$=(3x^2-8x+5)dx$$

This means

$$\Delta(x^3-4x^2+5x-6)\approx(3x^2-8x+5)\Delta x$$

when Δx is small.

Try $x=1$ $\quad\Delta x=-0.02$

$$x+\Delta x=0.98$$

Let $y=x^3-4x^2+5x-6$

$$dy=(3x^2-8x+5)dx$$

x	$\Delta x = dx$	y	Δy
1	-0.02	-4	$-.000408$
0.98		-4.000408	

$$dy = \underbrace{\left(3(1)^2 - 8(1) + 5\right)}_{0}(-0.02) = 0 \approx \Delta y$$

$$d\left(\frac{x^3 - 4x^2 + 5x - 6}{x^5}\right) = d\left(\frac{x^3}{x^5} - \frac{4x^2}{x^5} + \frac{5x}{x^5} - \frac{6}{x^5}\right)$$

$$= d\left(x^{-2} - 4x^{-3} + 5x^{-4} - 6x^{-5}\right)$$

$$= d(x^{-2}) - 4d(x^{-3}) + 5d(x^{-4}) - 6d(x^{-5})$$

$$= -2x^{-3}dx - 4(-3x^{-4}dx) + 5(-4x^{-5}dx) - 6(-5x^{-6}dx)$$

$$= (-2x^{-3} + 12x^{-4} - 20x^{-5} + 30x^{-6})dx$$

Notice that a dx always factors out.

The <u>derivative</u> is what you get by dividing the <u>differential</u> by dx.

$$f'(x) = \frac{df}{dx} = \frac{d(f(x))}{dx}$$

$$(x^2)' = \frac{d(x^2)}{dx} = \frac{2x\,dx}{dx} = 2x$$

The differential estimates a small change.

The derivative estimates the rate (or slope) of a small change.