Optimization (12-6) Today

Related rates (11-6) Friday

Review / Q&A Monday

Final 8AM-11AM Next Friday

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x = # times plant set up per year (p. 690, 12-6 #27)

y = # cans produced each time

C = cost $= 4y + 500x$ Minimize C.

Constraint: $xy = 16000$

↓ solve for one of the variables.

$$y = \frac{16000}{x}$$

↓

Minimize $C = \frac{64000}{x} + 500x$

E.g.

x	C
1	64500
50	26,280
12	11,333

$C = 64000x^{-1} + 500x$

$dC = 64000(-1)x^{-2}dx + 500\,dx$

$d(x^n) = nx^{n-1}\,dx$

$dC/dx = -64000/x^2 + 500$

Solve $0 = -64000/x^2 + 500$

$64000/x^2 = 500$

$64000 = 500x^2$

$128 = x^2$

$\sqrt{128} = x$

$11.3137\ldots = 8\sqrt{2} = x$

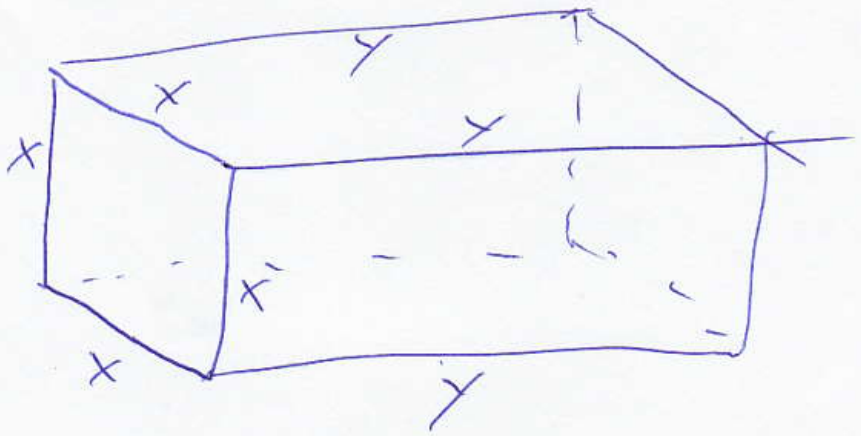
$\downarrow$

$x = 11$ or 12

x	$C = \frac{64000}{x} + 500x$
12	11,333
11	11,318

To minimize cost, set up the plant 11 times per year.

Box with square side



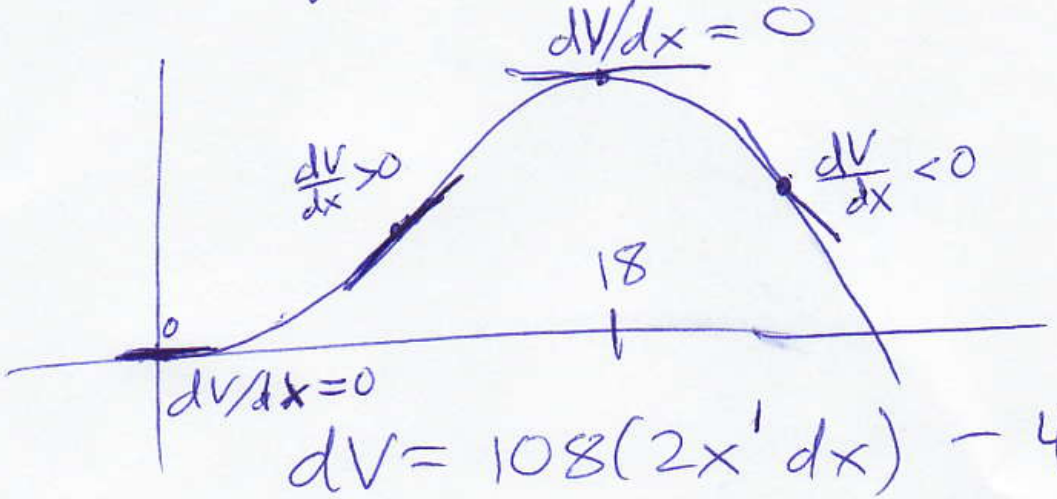
Maximize V = volume $= x^2 y$

Constraint: $\underbrace{\text{length}}_{y} + \underbrace{\text{girth}}_{4x} = 108$ inches

$y + 4x = 108$

$y = 108 - 4x$

$V = x^2(108 - 4x) = 108x^2 - 4x^3$



$dV = 108(2x^1 dx) - 4(3x^2 dx)$

$dV/dx = 216x - 12x^2$

Solve $0 = 12x(18 - x)$

$12x = 0$ or $18 - x = 0$

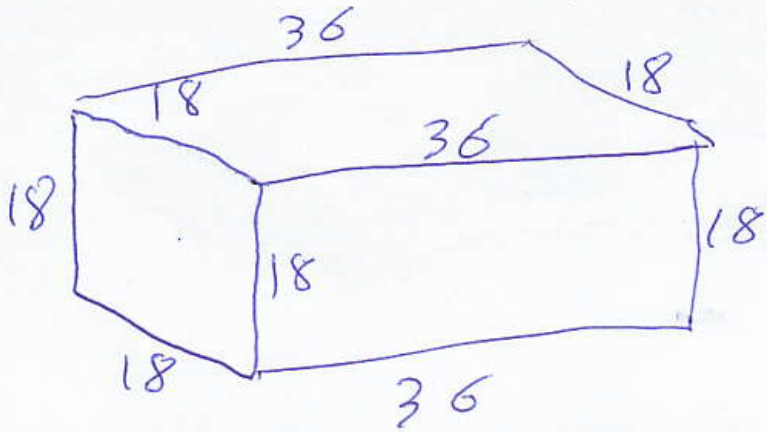
$\Downarrow$ $\Downarrow$

$x = 0$ or $x = 18$

$\uparrow$ $\Downarrow$

no

$y = 108 - 4(18) = 36$



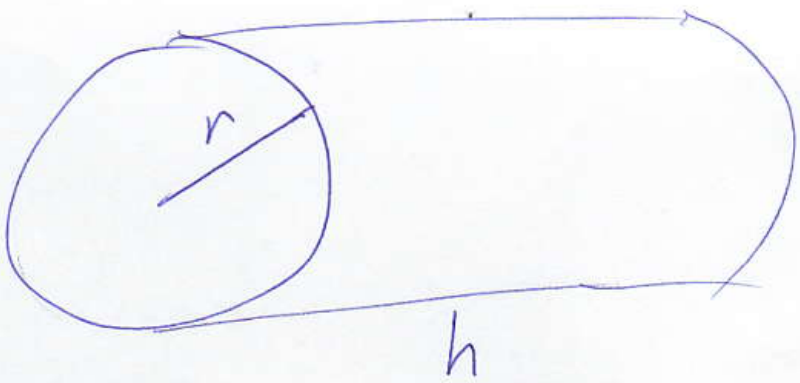
$V = x^2 y$

$\max(V) = 11664 \text{ in}^3$

$=$

6.75 ft^3

Cylindrical:



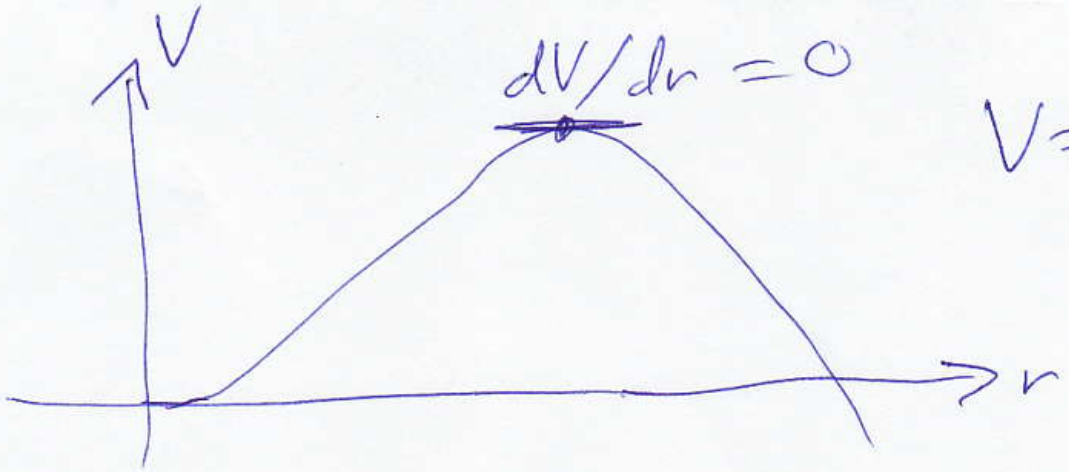
(Appendix C p. 925)

maximize $V = \pi r^2 h$

Constraint: $\underbrace{\text{length}}_{h} + \underbrace{\text{girth}}_{2\pi r} = 108$

$h + 2\pi r = 108$

$h = 108 - 2\pi r$ $\quad V = \pi r^2 (108 - 2\pi r)$



$V = 108\pi r^2 - 2\pi^2 r^3$

$dV = 108\pi(2r\,dr) - 2\pi^2(3r^2\,dr)$

Solve $0 = dV/dr = 216\pi r - 6\pi^2 r^2$

$0 = 6\pi r(36 - \pi r)$

$6\pi r = 0$ or $36 - \pi r = 0$

$r = 0$

nonsense

$36 = \pi r$

$36/\pi = r$

$11.459\ldots$

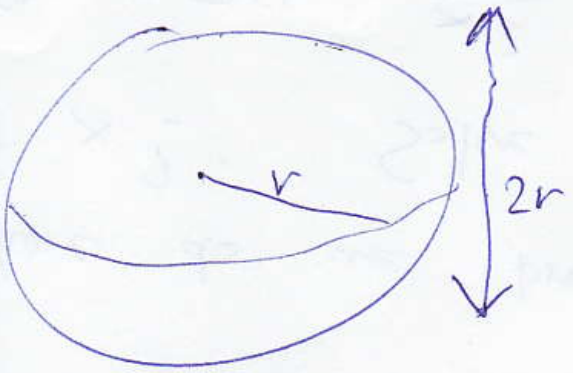
$h = 108 - 2\pi\left(\frac{36}{\pi}\right) = 36$

$V = \pi r^2 h = \pi\left(\frac{36}{\pi}\right)^2 36 = \pi\left(\frac{36^2}{\pi^2}\right)36$

$\max(V) = \frac{36^3}{\pi} = 14{,}851\ in^3 = 8.59\ ft^3$

11.459 36

Sphere $V = \frac{4\pi r^3}{3}$



length $= 2r$

girth $= 2\pi r$

Constraint: $108 = 2r + 2\pi r$

$54 = r + \pi r = (1+\pi)r$

$13.0384\ldots = 54/(1+\pi) = r$

$V = \frac{4\pi r^3}{3} = 9,284\ in^3$

HW: #25, 28, 30 (12-6).