Today: related rates (11-6)

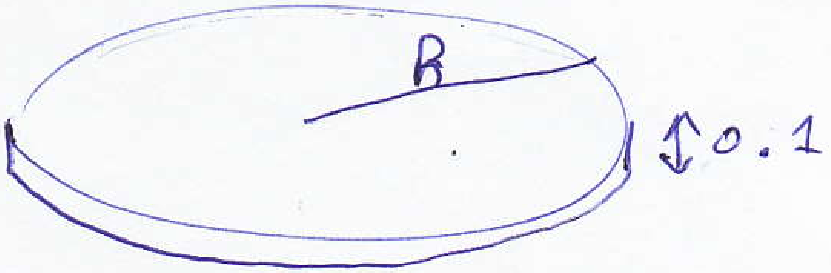
Monday: Q & A / review

Friday, 8AM-11AM, Dec 9:

Final Exam, emphasizing calculus, but with some older-topic questions too.

For final exam, bring calculator & 2 sheets of notes.

#31 (11-~~7~~6):



$\text{area} = \pi R^2 \qquad V = 0.1 \cdot \pi R^2 = \pi R^2/10$

When $R = 500$, $dR/dt = 0.32$ (feet/minute)

what is dV/dt at this time?

At all times, $V = \pi R^2/10$.

Differentiate what is true at all times.

Then plug in #s for a particular.

$$dV = d(\pi R^2/10) = \frac{\pi}{10} d(R^2) = \frac{\pi}{10}(2R\,dR)$$

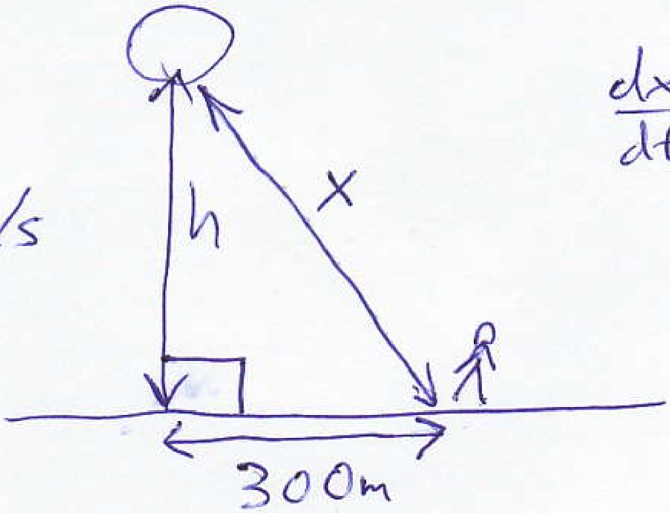
$dV = \frac{\pi}{5} R\, dR \qquad \frac{dV}{dt} = \frac{\pi}{5} R \frac{dR}{dt}$

$\frac{dV}{dt} = \frac{\pi}{5}(500)(0.32) = 32\pi \ (ft^3/min)$

$\approx 100 ft^3/min$

#18 (11-6)

$\frac{dh}{dt} = 5 m/s$



$\frac{dx}{dt} = ?$ when $h = 400m$

Picture labels things that change with variables.

$300^2 + h^2 = x^2$ true all the time.

$d(300^2 + h^2) = d(x^2)$

$0 + 2h\,dh = 2x\,dx$

$2h\frac{dh}{dt} = 2x\frac{dx}{dt}$

Now examine instant when $h = 400$:

$2(400)(5) = 2x\frac{dx}{dt}$

$300^2 + 400^2 = x^2$

$500 = \sqrt{90000 + 160000} = x$

$$2(400)(5) = 2(500)\frac{dx}{dt}$$

$$4 \text{ m/s} = dx/dt$$

#25 (11-6)

true at all times
$$\begin{cases} C = 90{,}000 + 30x \\ R = 300x - x^2/30 \\ P = R - C \end{cases}$$

Find $\frac{dC}{dt}, \frac{dR}{dt}, \frac{dP}{dt}$ at the instant when $x = 6000$ & $dx/dt = 500$.

$d(x^2) = 2x\,dx$

all the time
$$\begin{cases} dC = 0 + 30\,dx \\ dR = 300\,dx - (2x\,dx)/30 \\ dP = dR - dC \end{cases}$$

$$dC/dt = 30\,dx/dt = 30(500) = 15000$$

$$dR/dt = 300\,dx/dt - \left(\frac{2}{30}\right)x\,dx/dt$$

$$dP/dt = dR/dt - dC/dt$$

Cost increasing at \$15,000/week

$$\frac{dR}{dt} = \underbrace{300(500)}_{150,000} - \underbrace{\left(\frac{2}{30}\right)(6000)(500)}_{200,000} = -50,000$$

Revenue decreasing at \$50,000/week

$$dP/dt = \underbrace{dR/dt}_{-50000} - \underbrace{dC/dt}_{15000} = -65,000$$

Profit decreasing at \$65,000/week.

HW #16, 17, 27 (11-6) p. 602-603
