Friday, 8AM – 11AM, here:

Bring calculator & $\underbrace{\text{2 sheets}}_{\text{4 pages}}$ notes

Extra practice problems: p. 553 – 557

Review Exercises at end of Ch. 10

" " " " " Ch. 11

Exercises of 12 – 6

p. 688 – 691 p. 612 – 613

Also review topics from midterms I – III.

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f, g functions of x; k constant

$$f' = \frac{df}{dx}$$

$$d(f \pm g) = df \pm dg$$

$$d(f^k) = k f^{k-1} df$$

$$d(f^2) = 2f\,df$$

$$d(x^2) = 2x\,dx$$

$$d(e^g) = e^g\,dg$$

$$d(\ln g) = dg/g$$

$$d(kg) = k\,dg$$

$$d(g/k) = dg/k$$

$$dk = 0$$

$$d(f^{-1}) = d\left(\frac{1}{f}\right) = -1 f^{-2} df = -\frac{df}{f^2}$$

$$d(f^{1/2}) = d(\sqrt{f}) = \frac{1}{2} f^{-\frac{1}{2}} df = \frac{df}{2\sqrt{f}}$$

$$d(f \cdot g) = df \cdot g + f \cdot dg$$

$$d(f/g) = (df \cdot g - f \cdot dg)/g^2$$

$$d\left(\frac{x^2 e^{3x} - 5}{\ln(x^2+3)}\right) = (df \cdot g - f \cdot dg)/g^2$$

where $f = x^2 e^{3x} - 5$

$g = \ln(x^2+3)$

$$df = d(x^2 e^{3x}) - d5$$

$$= d(x^2)e^{3x} + x^2 d(e^{3x}) - 0$$

$$= 2x(dx)e^{3x} + x^2 e^{3x} d(3x)$$

$$= 2x(dx)e^{3x} + x^2 e^{3x}(3dx)$$

$$dg = (d(x^2+3))/(x^2+3)$$

$$= (d(x^2) + d3)/(x^2+3)$$

$$= (2x\,dx + 0)/(x^2+3)$$

$$= \left(\frac{e^{3x}(3x^2+2x)\ln(x^2+3) - (x^2 e^{3x} - 5)2x/(x^2+3)}{\ln^2(x^2+3)}\right)dx$$

Estimates using differentials:

$\sqrt[3]{7}$ is hard, but $\sqrt[3]{8}$ is easy: 2.

$x = 8 \qquad \sqrt[3]{x} = 2$

$dx = \Delta x = -1$

$x + dx = 7$

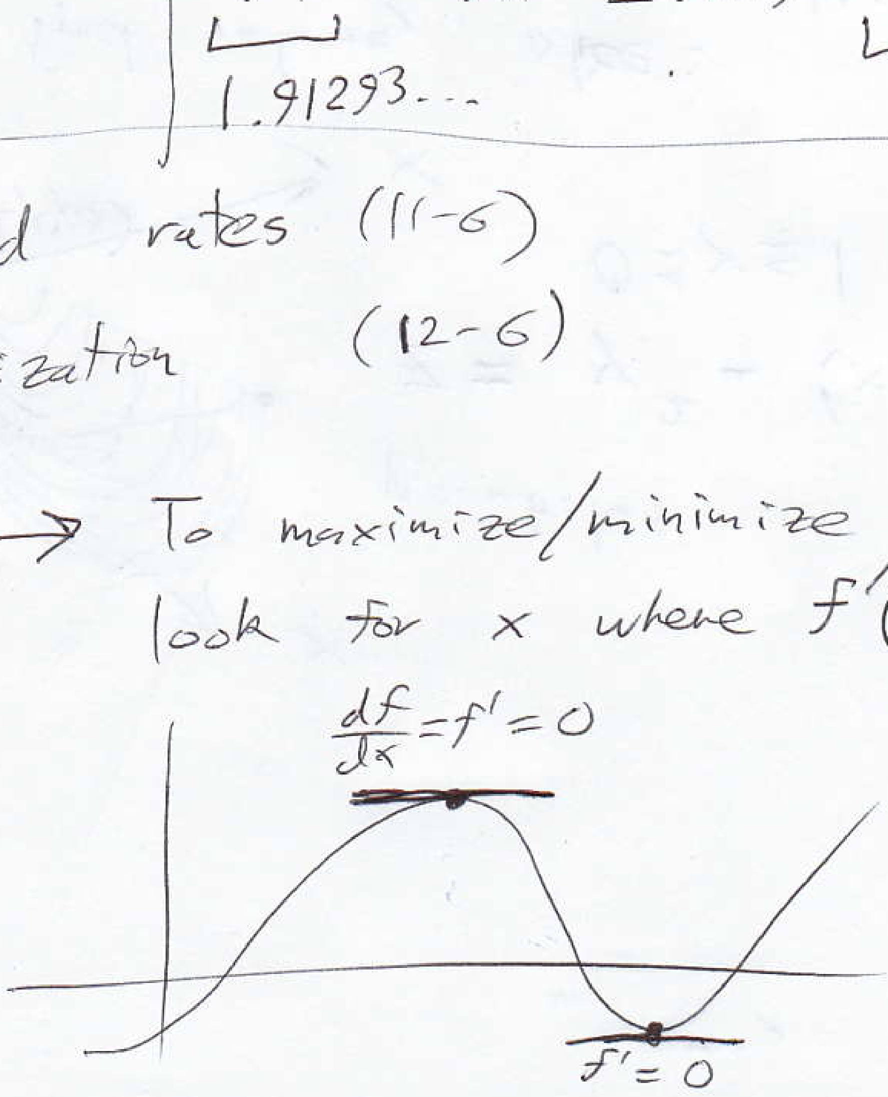
$$\Delta\left(\sqrt[3]{x}\right) \approx d\sqrt[3]{x} = d\left(x^{1/3}\right) = \frac{1}{3}x^{\left(-\frac{2}{3}\right)}dx = \frac{dx}{3\left(\sqrt[3]{x}\right)^2} = \frac{-1}{3\cdot 4}$$

$$\underbrace{\sqrt[3]{7}}_{1.91293\ldots} = \sqrt[3]{x} + \Delta\left(\sqrt[3]{x}\right) \approx \underbrace{2 - \frac{1}{12}}_{1.91666\ldots}$$

Related rates (11-6)

Optimization (12-6)

$\hookrightarrow$ To maximize/minimize $f(x)$, look for x where $f'(x) = 0$.



(10-7)

x = quantity p = price $\overset{\text{revenue}}{R} = px$ C = cost

$MC = \frac{dC}{dx}$ $MR = \frac{dR}{dx}$ $\underbrace{P}_{\text{profit}} = R - C$

$MP = \frac{dP}{dx}$

When $dx = \Delta x = 1$, $\begin{cases} MC \approx \Delta C \\ MR \approx \Delta R \\ MP \approx \Delta P \end{cases}$

From #44 (10-7):

$p = 60 - 2\sqrt{x}$ $C = 3000 + 5x$

When $x = 100$, what are p, C, R, P, MC, MR, MP? $p = 60 - 2\sqrt{100} = 60 - 20$

$p = 40$ $C = 3000 + 5(100) = 3500$ $R = px = 4000$

$dC = d3000 + 5dx = 0 + 5dx = 5dx$ $P = 500$

$dC/dx = 5$ $MC = 5$

$dR = d(px) = dp\,x + p\,dx$

$dp = d60 - 2\underbrace{d\sqrt{x}}\ = 0 - 2\left(\frac{dx}{2\sqrt{x}}\right) = -\frac{dx}{\sqrt{x}}$

$d(x^{1/2}) = \frac{1}{2}x^{-1/2}dx$

$dR = -\frac{dx}{\sqrt{x}}\,x + (60 - 2\sqrt{x})dx$

$MR = dR/dx = \underbrace{-\sqrt{x} + 60 - 2\sqrt{x}}_{60 - 3\sqrt{x}} = 30$ at $x = 100$

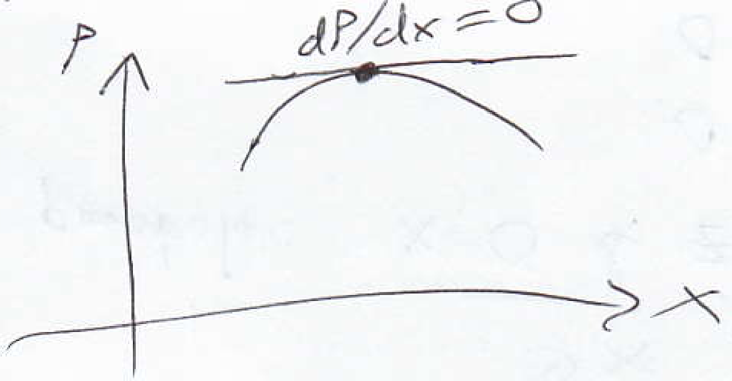
$$MP = \frac{dP}{dx} = \frac{dR - dC}{dx} = \frac{dR}{dx} - \frac{dC}{dx} = MR - MC$$

At $x = 100$, $MP = 30 - 5 = 25$

We could optimize: At what x is profit maximized?

$$MP = dP/dx = dR/dx - dC/dx = 60 - 3\sqrt{x} - 5$$

Solve $60 - 3\sqrt{x} - 5 = 0$ for x.



$$55 = 3\sqrt{x}$$

$$\left(\frac{55}{3}\right)^2 = x$$

$$336 = x$$

maximizes P

max(P) = ?

$$P = R - C = px - C = (60 - 2\sqrt{x})x - (3000 + 5x)$$

max(P) = \$ 3,126

Related rates:

You're given a situation and asked to find some dX/dt at some instant in time; X is some changing quantity and t is time.

You have equation relating X to other quantities Y, Z, etc that are true all the time

You differentiate those, divide by dt, then plug in #'s for ~~___~~ that instant in time to find dX/dt.

Continuing example above, if

$x = 100$ & and $\frac{dx}{dt} = 10$ per day,

then what is dP/dt at that moment?

$$dP = (60 - 3\sqrt{x} - 5)\,dx$$

$$\frac{dP}{dt} = (60 - 3\sqrt{x} - 5)\frac{dx}{dt}$$

$$\frac{dP}{dt} = (60 - \underbrace{3\sqrt{100}}_{30} - 5)(\underbrace{10}_{\text{per day}}) = 250 \text{ per day}$$