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↑ lecture notes, etc. will be here

$y = x^2$ $\qquad \frac{dy}{dx} = 2x$

What does this mean?
What is it good for?

$103^2 \approx 10600$
$103^2 = 10609$

Derivatives approximate small changes

$x = 100$ (old x)

$\Delta x = 3$ (change in x)

$x + \Delta x = 103$ (new x)

$y = x^2 = 10{,}000$ (old y) = (old x^2)

$\Delta y = 609$ (change in x^2)

$y + \Delta y = 10{,}609$ (new x^2)

Derivatives estimate Δy, given Δx.

$\frac{dy}{dx} = 2x \qquad dy = 2x\ dx$

dx & dy are estimates of Δx & Δy

$dx = \Delta x = 3$

$dy = 2\underset{\substack{\uparrow \\ 100}}{x}\ \underset{\substack{\nwarrow \\ 3}}{dx} = 2(100)(3) = 600$

$dy = 600 \approx 609 = \Delta y$

$y + dy = 10{,}600 \approx 10{,}609 = y + \Delta y$

Another: $x = 1000 = 10^3$ $\quad y = x^2 = 10^6$

$\Delta x = -15$ $\quad \Delta y = ?$

$x + \Delta x = 985$ $\quad \underbrace{y + \Delta y}_{985^2} = ?$

$\Delta y \approx dy = 2x\,dx$ $\qquad dx = \Delta x = -15$

$\Delta y \approx dy = 2(1000)(-15) = -30{,}000$

$y + \Delta y \approx y + dy = 1{,}000{,}000 - 30{,}000$

$y + \Delta y = 985^2 \approx 970{,}000$

$y + \Delta y = 970{,}225$ (from calculator)

This is the meaning of

"$y = x^2 \Rightarrow \frac{dy}{dx} = 2x$":

If Δx is small, then, setting $dx = \Delta x$, we have

$$\Delta y = \Delta(x^2) \approx \underbrace{2x\,dx}_{= dy} = 2x\,\Delta x$$

HW #1 Use derivatives to estimate 1009^2, 5.08^2, and $(-7.94)^2$.

Use a calculator to get the exact squares.

Where did $\underbrace{dy = 2x\,dx}$, come from? ← when $y = x^2$

same as

$$y' = \frac{dy}{dx} = 2x$$

→ Binomial theorem

$$(a+b)^2 = (a+b)(a+b) = a^2 + \underbrace{ab + ba}_{2ab} + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$y = x^2$ $\quad$ $y + \Delta y = (x + \Delta x)^2$

old = old $\quad$ new = new

Put these equations together to solve for Δy.

$$\begin{array}{rl} y + \Delta y &= (x+\Delta x)^2 \\ -(y &= x^2 \quad) \\ \hline \Delta y &= (x+\Delta x)^2 - x^2 \end{array}$$

new - old = new - old

$$\Delta y = \cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{x^2}$$

$$\Delta y = 2x\Delta x + (\Delta x)^2$$

E.g. $x = 100$ & $\Delta x = 3$

$$\Delta y = 2(100)(3) + 3^2 = 609$$

$$103^2 = y + \Delta y = 10{,}000 + 609 = 10{,}609$$

Estimate: $dy = 2x\,dx = 2x\Delta x$

$$\Delta y \approx 2x\Delta x$$

$$dy = 2(100)(3) = 600$$

$$y + dy = 10000 + 600 = 10{,}600$$

Why is $\Delta y \approx 2x\Delta x$ a good estimate of $\Delta y = 2x\Delta x + (\Delta x)^2$?

When Δx is small compared to x, $(\Delta x)^2$ is small compared to $2x\Delta x$

E.g. $x = 100$, $\Delta x = 3$, $(\Delta x)^2 = 9$,

$2x\Delta x = 600$

$y = x^3$; Δx is small (compared to x)

E.g. $x = 2$

$\Delta x = -0.04$ $\quad x + \Delta x = 1.96$

$y = 8$ $\quad \underbrace{y + \Delta y}_{1.96^3} = ?$ $\quad \underbrace{\Delta y}_{2^3 - 1.96^3} = ?$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\Delta y = (x + \Delta x)^3 - x^3$

new-old

$\Delta x = \cancel{x^3} + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - \cancel{x^3}$

$$\Delta y = 3x^2 \Delta x + \underbrace{(3x\Delta x + \Delta x^2)}_{\text{small}} \Delta x$$

$$\Delta y + 3x^2 \Delta x + (\text{small})\, \Delta x$$

$$dy = 3x^2 dx \Rightarrow y' = \frac{dy}{dx} = 3x^2$$

$$x = 2 \qquad dx = \Delta x = -0.04$$

$$\Delta y = \underbrace{3x^2 dx}_{dy} + \underbrace{(\text{small})\, dx}_{\text{much smaller than } dx} \approx \underbrace{3x^2 dx}_{dy}$$

$$\Delta y \approx dy = 3(2^2)(-0.04) = -0.48$$

$$\underbrace{y + \Delta y}_{1.96^3} \approx 8 - 0.48 = 7.52$$

$$y + \Delta y = 1.96^3 = 7.529536$$

General idea: $y = f(x)$

$dy = f'(x)\, dx$ (same as $\frac{dy}{dx} = f'(x)$)

means when Δx is small enough,

$$\Delta y = f(x+\Delta x) - f(x) \approx f'(x)\, \underbrace{dx}_{dx = \Delta x}$$

new − old

and the estimate is good in the sense that

$$\Delta y = \underbrace{f'(x)\,dx}_{dy} + \underbrace{(\text{small})\,dx}_{\text{much smaller than } dx}$$

In fancy terms,

$$\lim_{\Delta x \to 0} \frac{\Delta y - f'(x)\,\Delta x}{\Delta x} = 0$$

(same as $\displaystyle\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x)$.)

I claim $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$.

What I mean is if $y = \sqrt{x}$ and Δx is small enough, then

$$\Delta y = (\sqrt{x})' \underbrace{\Delta x}_{=dx} + (\text{small}) \underbrace{\Delta x}_{=dx}$$

Another way to write it:

$dx = \Delta x \qquad \frac{dy}{dx} = (\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$dy = \frac{dx}{2\sqrt{x}} \qquad \Delta y = dy + (\text{small})\, dx$

$x = 9 \quad (\text{old}) \qquad \Delta x = 0.2$

$x + \Delta x = 9.2 \quad (\text{new})$

$(\text{old})\ y = \sqrt{x} = 3 \qquad (\text{new})\ y + \Delta y = \sqrt{x + \Delta x}$

$(\text{new})\ y + \Delta y = \sqrt{9.2} = 3.03315\ldots$

$\Delta y = 0.03315\ldots$

$dy = \frac{dx}{2\sqrt{x}} = \frac{0.2}{2\sqrt{9}} = \frac{0.2}{2 \cdot 3} = \frac{0.2}{6}$

$dy = 0.0333333\ldots$

Is $\Delta y = dy + (\text{small})\, dx$?

$0.033333\ldots = 0.3315\ldots + (\text{small})(0.2)$?

$-0.03315\ldots \qquad -0.3315\ldots$

$0.00018\ldots = 0 + (\text{small})(0.2)$?

$$\frac{0.00018\ldots}{0.2} = \frac{(\text{small})(0.2)}{0.2} = \text{small}?$$

$0.0009\ldots = \text{small}$? yes, compared to our ~~thr~~ starting numbers

$$\begin{array}{ccc} 9, & 3, & 0.2, \\ \uparrow & \uparrow & \uparrow \\ x & y & \Delta x \end{array}$$

y	dy	Δy	dx	Δx
x^2	$2x\,dx$	$(x+dx)^2 - x^2$	Δx	dx
x^3	$3x^2\,dx$	$(x+dx)^3 - x^3$	Δx	dx
$\sqrt{x}$	$\dfrac{dx}{2\sqrt{x}}$	$\sqrt{x+dx} - \sqrt{x}$	Δx	dx
x^4	$4x^3\,dx$	$(x+dx)^4 - x^4$	Δx	dx

All examples where $\Delta y = dy + (\text{small})dx$ if dx is small enough.