8/30 Basic derivative rules

y	$y' = \frac{dy}{dx}$	dy	
x^2	$2x$	$2x\,dx$	
x^3	$3x^2$	$3x^2dx$	
x^4	$4x^3$	$4x^3dx$	
x^n	nx^{n-1}	$nx^{n-1}dx$	$\leftarrow$ power rule
$\sqrt{x}$	$1/(2\sqrt{x})$	$dx/(2\sqrt{x})$	
$x^{1/2}$	$\frac{1}{2}x^{-1/2}$	$\frac{1}{2}x^{-1/2}dx$	
$x^{-1/2}$	$-\frac{1}{2}x^{-3/2}$	$-\frac{1}{2}x^{-3/2}dx$	

derivative (column y') differential (column dy)

$dy \approx \Delta y$ when $dx = \Delta x$ is small

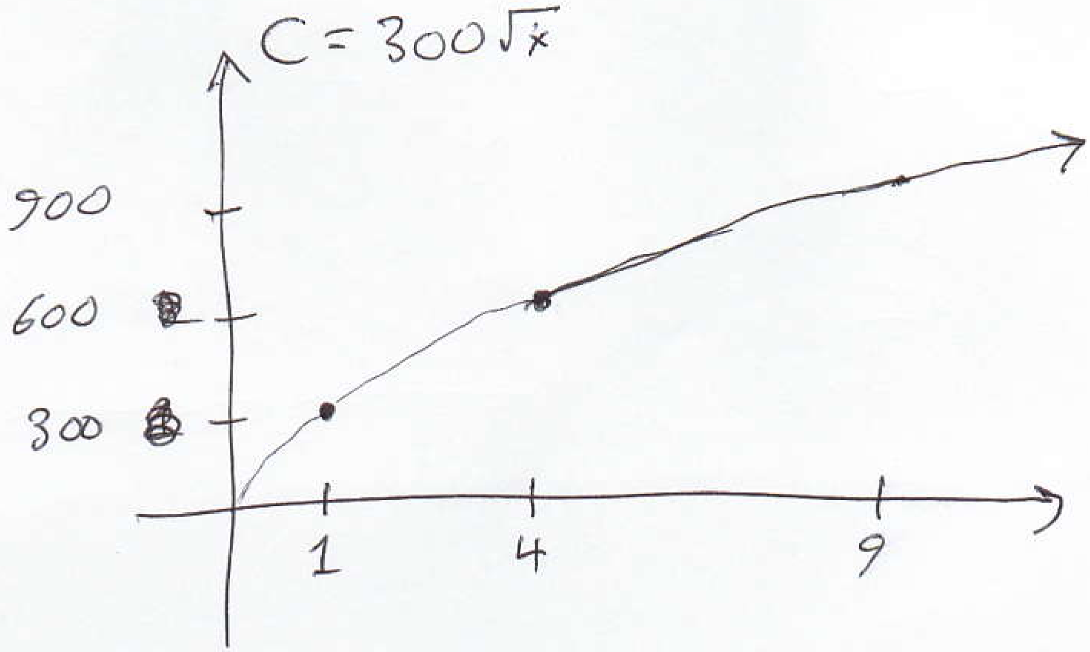
HW #1 Use a differential to estimate $\sqrt[3]{500}$.

Compare to the exact value from calculator.

HW #2

Suppose the cost of making x shoes per day in a factory is $300\sqrt{x}$ dollars.

Estimate ~~the~~ how much more it would cost to increase daily production from 2500 to 2600 shoes.



Marginal cost = cost of making one more shoe

$= dC$ when $dx = 1$

What is the marginal cost when $x = 2500$?

What is the marginal cost when $x = 2600$?

$(x^2)' = 2x$

$d(x^2) = 2x\,dx$

$(5x^2)' = 10x$

$d(5x^2) = 5\,d(x^2) = 5(2x\,dx)$

$d(5x^2) = 10x\,dx$

$$(5x^2)' = \frac{d(5x^2)}{dx} = 10x$$

<u>Constant multiple rule:</u>

If k is constant, then

$$d(k f(x)) = k\,d(f(x))$$

$$(k f(x))' = k f'(x)$$

$$\left(-\frac{x^3}{7}\right)' = \left(\left(-\frac{1}{7}\right)x^3\right)' = \left(-\frac{1}{7}\right)(x^3)'$$

$$= \left(-\frac{1}{7}\right)3x^2 = -\frac{3x^2}{7}$$

$$d\left(-\frac{x^3}{7}\right) = d\left(\left(-\frac{1}{7}\right)x^3\right) = \left(-\frac{1}{7}\right)d(x^3)$$

$$= \left(-\frac{1}{7}\right)3x^2dx = -\frac{3x^2dx}{7}$$

$$d\left(\frac{2}{3}x^{-3/5}\right) = \frac{2}{3}\,d\left(x^{-3/5}\right)$$

$$= \frac{2}{3}\left(-\frac{3}{5}\right)x^{(-3/5)-1}dx = \frac{2}{3}\left(-\frac{3}{5}\right)x^{-8/5}dx$$

$$= -\frac{2}{5}x^{-8/5}dx$$

Sum rule: $\begin{cases} (f+g)' = f' + g' \\ d(f+g) = df + dg \\ \Delta(f+g) = \Delta f + \Delta g \end{cases}$

$$(x^2 + x^3)' = (x^2)' + (x^3)' = 2x + 3x^2$$

Same for subtraction:

$$(x^2 - x^3) = (x^2)' - (x^3)' = 2x - 3x^2$$

$$d\left(5x^4 - \sqrt{x} + \frac{x^3}{8}\right) = d(5x^4) - d(\sqrt{x}) + d\left(\frac{x^3}{8}\right)$$

$$= 5\,d(x^4) - d(\sqrt{x}) + \frac{1}{8}d(x^3)$$

$$= 5(4x^3dx) - \frac{dx}{2\sqrt{x}} + \frac{1}{8}(3x^2dx)$$

same as $d(x^{1/2}) = \frac{1}{2}x^{-\frac{1}{2}}dx = \dfrac{dx}{2\sqrt{x}}$

$$\left(20x^3 - \frac{1}{2\sqrt{x}} + \frac{3}{8}x^2\right)dx$$

HW #3

Estimate the change in $y = \frac{3}{x} - \frac{1}{x^2}$ for the following values of x & Δx. Use a calculator to get exact changes

x	$\Delta x = dx$	dy	Δy
1	0.1		
1	0.01		
1	0.001		
1	-0.001		
1	-0.01		
1	-0.1		

Hint: $\frac{3}{x} = 3x^{-1}$

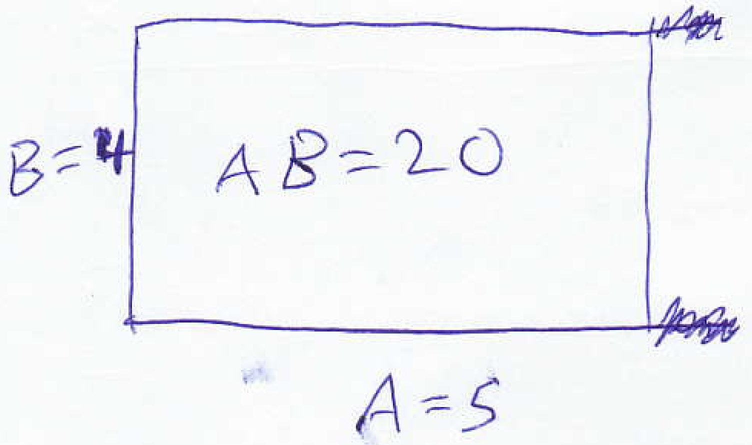
$A = 5$	$B = 4$	(old)
$\Delta A = 0.01$	$\Delta B = -0.01$	(change)
$A + \Delta A = 5.01$	$B + \Delta B = 3.99$	(new)

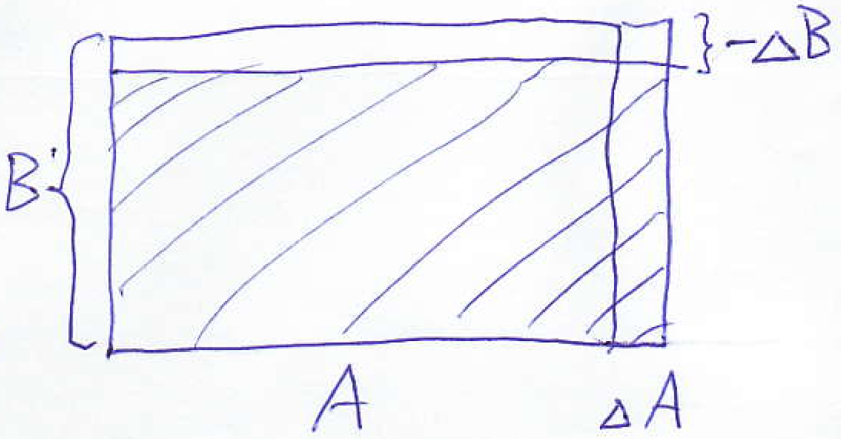
$AB = 20$ (old)

$19.9899 - 20 = -0.0101$ ← (change)

(new)

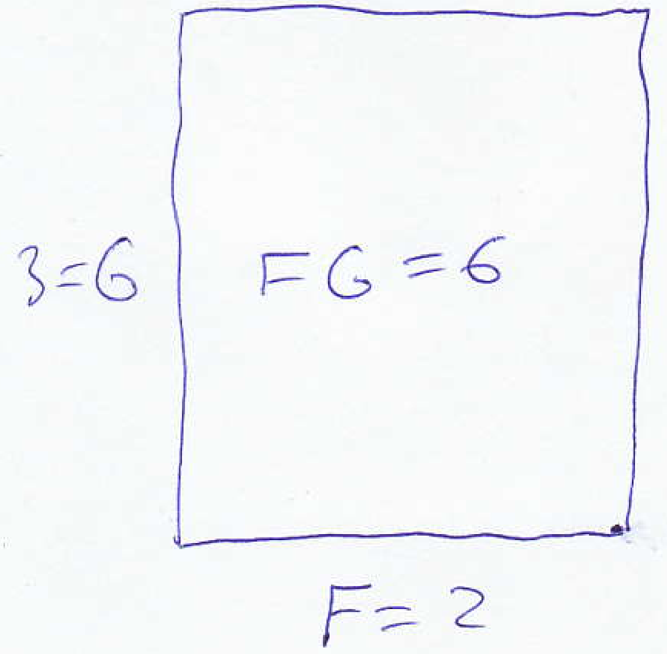
$(A + \Delta A)(B + \Delta B) = (5.01)(3.99) = 19.9899$

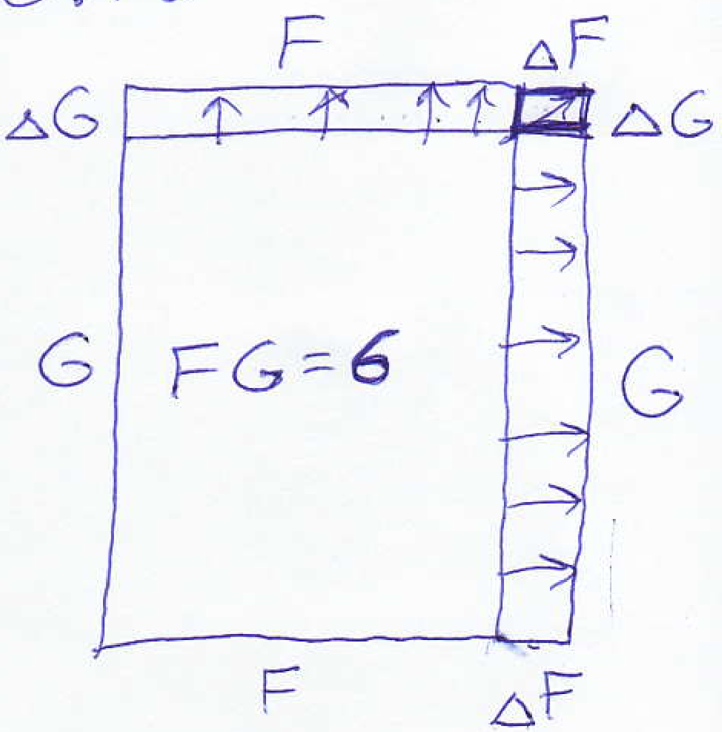




Another: $F = 2$, $G = 3$

$\Delta F = 0.25$, $\Delta G = 0.15$

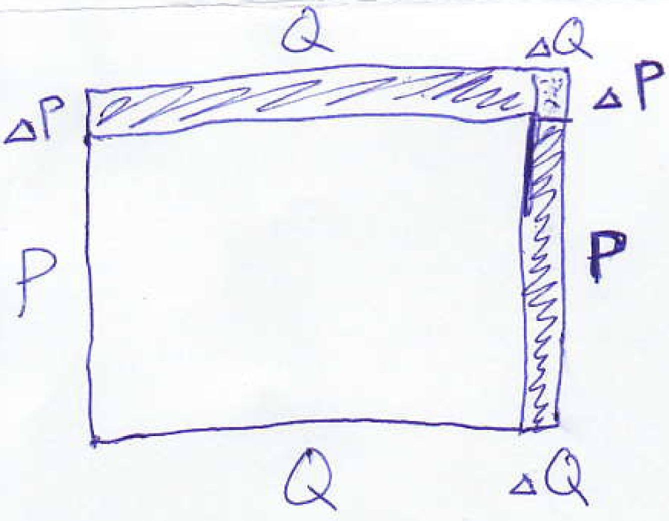




$$\Delta(FG) = (\Delta F)(G) + (\Delta F)(\Delta G) + (F)(\Delta G)$$
$$= (0.25)(3) + (0.25)(0.15) + (2)(0.15)$$
$$= 0.75 + 0.0375 + 0.3$$
$$= 1.0875$$

$$FG + \Delta(FG) = 6 + 1.0875 = 7.0875$$

$$(F + \Delta F)(G + \Delta G) = (2.25)(3.15) = 7.0875 \checkmark$$



ΔP, ΔQ

small

Most of the change $\Delta(PQ)$ of the product PQ is in the two long skinny rectangles. The area of the 2 long rectangles, in total, is

$$(\Delta P)(Q) + (P)(\Delta Q)$$

$$\Delta(PQ) \approx (\Delta P)(Q) + (P)(\Delta Q)$$

$$d(PQ) = (dP)(Q) + (P)(dQ)$$

where $dP = \Delta P \quad dQ = \Delta Q$

$$(PQ)' = P'Q + PQ'$$

$\uparrow$ Product rule

$$[(x^2 - 5x^5)(x^3 + 2x^4)]'$$

$$= (x^2 - 5x^5)'(x^3 + 2x^4) + (x^2 - 5x^5)(x^3 + 2x^4)'$$

$$= (2x - 5(5x^4))(x^3 + 2x^4) + (x^2 - 5x^5)(3x^2 + 2(4x^3))$$

$\hookrightarrow$ Same as $[x^5 + 2x^6 - 5x^8 - 10x^9]'$.

HW #4

$$\left[(x^2 + x^3 + x^4)\left(\frac{1}{x} - \frac{1}{x^2} + \frac{3}{7\sqrt{x}}\right)\right]' = ?$$

You do not need to simplify your answer

$$dx = dx = 1\, dx$$

$$x' = \frac{dx}{dx} = 1$$

$d5 = \Delta 5 = 0$ (5 is always 5; it never changes)

$$d(3x+7) = 3\,dx + \underbrace{d7}_{0} = 3\,dx$$

$$(3x+7)' = 3\underbrace{x'}_{1} + \underbrace{7'}_{0} = 3 \cdot 1 + 0 = 3$$

HW#5

$$d\left[(1+x-5x^2)(2-4x+6\sqrt{x})\right] = ?$$

Again, you don't need to simplify your answer

Optional: why is $d(\sqrt{x}) = \frac{dx}{2\sqrt{x}}$?

$$y = \sqrt{x} \Rightarrow y^2 = x \Rightarrow d(y^2) = dx$$

$$dx = d(y^2) = 2y\,dy = 2\sqrt{x}\,d(\sqrt{x})$$

$$\Rightarrow dx = 2\sqrt{x}\,d(\sqrt{x})$$

$$\Rightarrow \frac{dx}{2\sqrt{x}} = d(\sqrt{x})$$

Similarly, $d(x^{1/3}) = \frac{1}{3}x^{-2/3}dx$

because:

$$y = x^{1/3} \Rightarrow y^3 = x \Rightarrow d(y^3) = dx$$

$$dx = d(y^3) = 3y^2 dy = 3(x^{1/3})^2 d(x^{1/3})$$

$$\Rightarrow dx = 3x^{2/3} d(x^{1/3})$$

$$\Rightarrow \frac{1}{3}x^{-2/3}dx = d(x^{1/3})$$

Optional:

Why is $d(x^{7/3}) = \frac{7}{3}x^{4/3}dx$?

(Note: $\frac{4}{3} = \frac{7}{3} - 1$.)

$$y = x^{7/3} \Rightarrow y^3 = x^7 \Rightarrow d(y^3) = d(x^7)$$

$$\Rightarrow 3y^2 dy = d(y^3) = d(x^7) = 7x^6 dx$$

$$\Rightarrow 3(x^{7/3})^2 d(x^{7/3}) = 7x^6 dx$$

$$\Rightarrow 3x^{14/3} d(x^{7/3}) = 7x^6 dx$$

$$\Rightarrow d(x^{7/3}) = \frac{7x^6}{3x^{14/3}} dx = \frac{7x^{18/3}}{3x^{14/3}} dx$$

$$\Rightarrow d(x^{7/3}) = \frac{7}{3}x^{4/3} dx$$

~~Similarly~~ $d(x^{-5}) = -5x^{-6}dx$ because:

(Note: $-6 = -5 - 1$.)

$$y = x^{-5} \Rightarrow yx^5 = 1 \Rightarrow d(yx^5) = d1$$

$$\Rightarrow (dy)x^5 + y\,d(x^5) \underset{\substack{\uparrow \\ \text{product rule}}}{=} d(yx^5) = d1 \underset{\substack{\uparrow \\ \text{1 never changes}}}{=} 0$$

$$\Rightarrow (dy)x^5 + y\,d(x^5) = 0 \Rightarrow x^5 dy = -y\,d(x^5)$$

$$\Rightarrow x^5 dy = -y(5x^4 dx) \Rightarrow x^5 d(x^{-5}) = -x^{-5}(5x^4)dx$$

$$\Rightarrow x^5 d(x^{-5}) = -5x^{-1}dx \Rightarrow d(x^{-5}) = -5x^{-6}dx$$