f, f', f'' and the shape of the curve

(12-1 & 12-2 in book)

$y = f(x)$	$f'(x)$
$\nearrow$	big, positive
$\nearrow$	small, positive
$\rightarrow$	0
$\searrow$	small, negative
$\downarrow$	big, negative

HW:

12-1

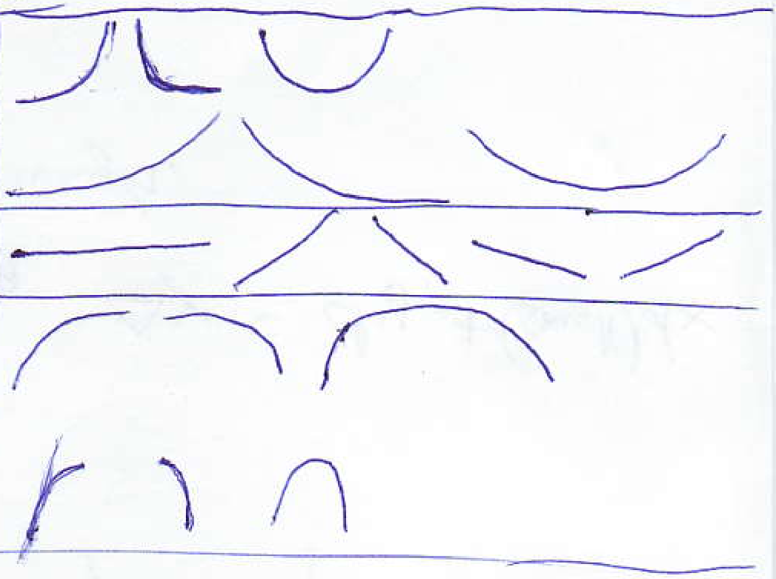
#63-68

$f''(x)$ (2nd derivative) is the derivative of the derivative of f

$$f(x) = x^5 + x^2 - 3x + 4$$

$$f'(x) = 5x^4 + 2x - 3 \qquad (x' = 1;\ 4' = 0)$$

$$f''(x) = 5(4x^3) + 2 = 20x^3 + 2$$

$y = f'(x)$	$f''(x)$	$y=f(x)$
↗	+ big	
→	+ small	
⟶	0	
↘	− small	
↘	− big	

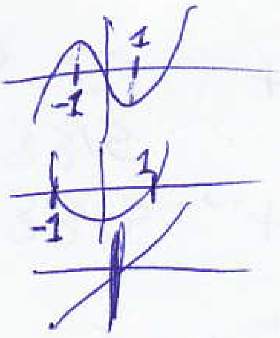
f''	f'	f	names for shape of f
+	+ ↗		increasing, concave up
+	− ↗		decreasing, concave up
−	+ ↘		increasing, concave down
−	− ↘		decreasing, concave down

HW: 12-2 #1, 2 parts A–F

$$x^3 - 3x = f(x)$$

$$3x^2 - 3 = f'(x)$$

$$\underbrace{3(2x)}_{6x} = f''(x)$$



$f(x)$ is increasing on $(-\infty, -1]$ & $[1, \infty)$

$f(x)$ is decreasing on $[-1, 1]$

$f(x)$ is concave down on $(-\infty, 0]$

$f(x)$ is concave ~~down~~ up on $[0, \infty)$

$f'(x)$ is decreasing on $(-\infty, 0]$

$f'(x)$ is increasing on $[0, \infty)$

$f''(x)$ is negative on $(-\infty, 0)$

$f''(x)$ is positive on $(0, \infty)$

$f'(x)$ is positive on $(-\infty, -1)$ & $(1, \infty)$

$f'(x)$ is negative on $(-1, 1)$

$y = f(x) \qquad \frac{dy}{dx} = f'(x) \qquad dy = f'(x)\,dx$

$dy = f'(x)\,dx$ means that if

$dx = \Delta x$ is small, then

$\Delta y \approx dy$; specifically,

$$\left.\begin{aligned} \Delta y &= dy + (\text{small})\,dx \\ \Delta y &= f'(x)\,\Delta x + (\text{small})\,\Delta x \end{aligned}\right\} \text{same}$$

Version for 2nd derivatives:

$y = f(x) \qquad \frac{d^2y}{dx^2} = f''(x) \qquad d^2y = f''(x)\,dx^2$

$dx^2 = (dx)^2 \neq d(x^2)$

$d^2y = d(dy)$

If $y = x^3$, then

$d^2y = d^2(x^3) = d(d(x^3)) = d(3x^2\,dx)$

$= d(3x^2)\,dx = 3\,d(x^2)\,dx = 3(2x\,dx)\,dx$

$= 6x\,dx^2$

Compare to $(x^3)'' = (3x^2)' = 6x$

If $dx = \Delta x$ is small, then

$\Delta^2 y \approx d^2 y$; specifically,

$$\left.\begin{aligned} \Delta^2 y &= d^2 y + (\text{small})\, dx^2 \\ \Delta^2 y &= f''(x)\,\Delta x^2 + (\text{small})\,\Delta x^2 \end{aligned}\right\} \text{same}$$

$$\Delta x^2 = (\Delta x)^2 \neq \Delta(x^2)$$

$$\Delta^2 y = \Delta(\Delta y) = \Delta(\Delta f(x))$$

$$\Delta^2 y = \Delta(f(\underbrace{x + \Delta x}_{\text{new } x}) - f(\underbrace{x}_{\text{old } x}))$$

$\cancel{\Delta^2 y = [f(x + \Delta x) + \Delta x}$

$$\Delta^2 y = \underbrace{\Delta f(x + \Delta x)}_{\boxed{f(x+2\Delta x) - f(x+\Delta x)}} - \underbrace{\Delta f(x)}_{\boxed{f(x+\Delta x) - f(x)}}$$

$$\Delta^2 y = (f(x+2\Delta x) - f(x+\Delta x)) - (f(x+\Delta x) - f(x))$$

$y = f(x) = x^3 \qquad \Delta x = dx = 0.1$

x	Δx	y	Δy	$\Delta^2 y$	dy	$d^2 y$
2.8	0.1	21.952	2.437	0.174	2.352	0.168
2.9	0.1	24.389	2.611	0.18	2.523	0.174
3.0	0.1	27	2.791	0.186	2.7	0.18
3.1	0.1	29.791	2.977		2.883	
3.2		32.768				

$$dy = f'(x)\,dx = 3x^2 dx = 3x^2 \Delta x$$

E.g. $x = 2.8 \Rightarrow dy = 3(2.8)^2(0.1)$

$$d^2y = f''(x)\,dx^2 = 6x\,dx^2 = 6x(\Delta x)^2$$

E.g $x = 2.8 \Rightarrow d^2y = 6(2.8)(0.1)^2$

HW Let $y = x^2 + \frac{1}{x}$

For the following x-values & Δx-values, ~~comple~~ compute Δy, dy, $\Delta^2 y$, $d^2 y$:

x	Δx
2.00	
	0.05
2.05	
	0.05
2.10	
	0.05
2.15	

&

x	Δx
−3.01	
	0.01
−3.00	
	0.01
−2.99	
	0.01
−2.98	

Some optimization:

Maximizing profit

x = # items produced

$p(x)$ = price at which x items get sold

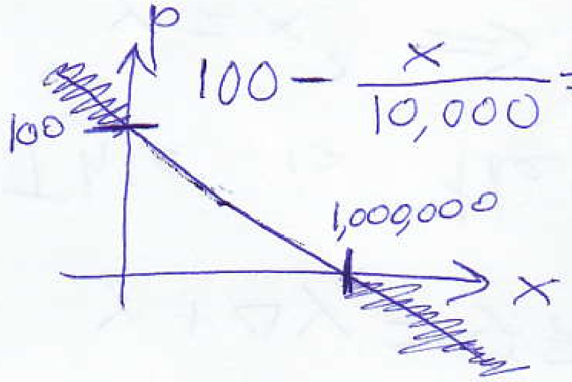
~~higher~~ higher prices $\Leftrightarrow$ lower sales
lower prices $\Leftrightarrow$ higher sales

$C(x)$ = cost to produce x items

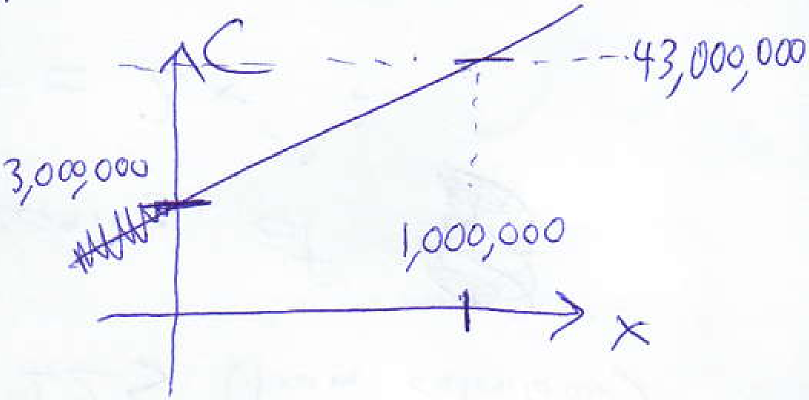
$R(x) = x\,p(x)$ = quantity · price = revenue

$P(x)$ = profit = $R(x) - C(x)$ = revenue − cost

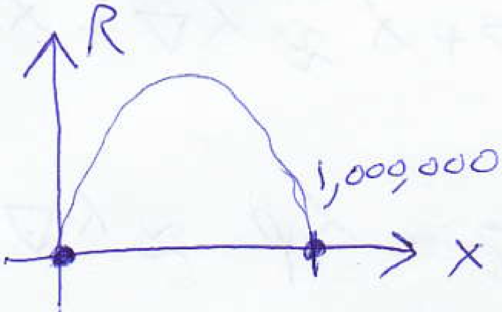
Simplest version: $p(x)$ & $C(x)$ are linear



$$100 - \frac{x}{10,000} = p$$



$$\frac{dp}{dx} < 0 \qquad \frac{dC}{dx} > 0$$



$$C = 3,000,000 + 40x$$

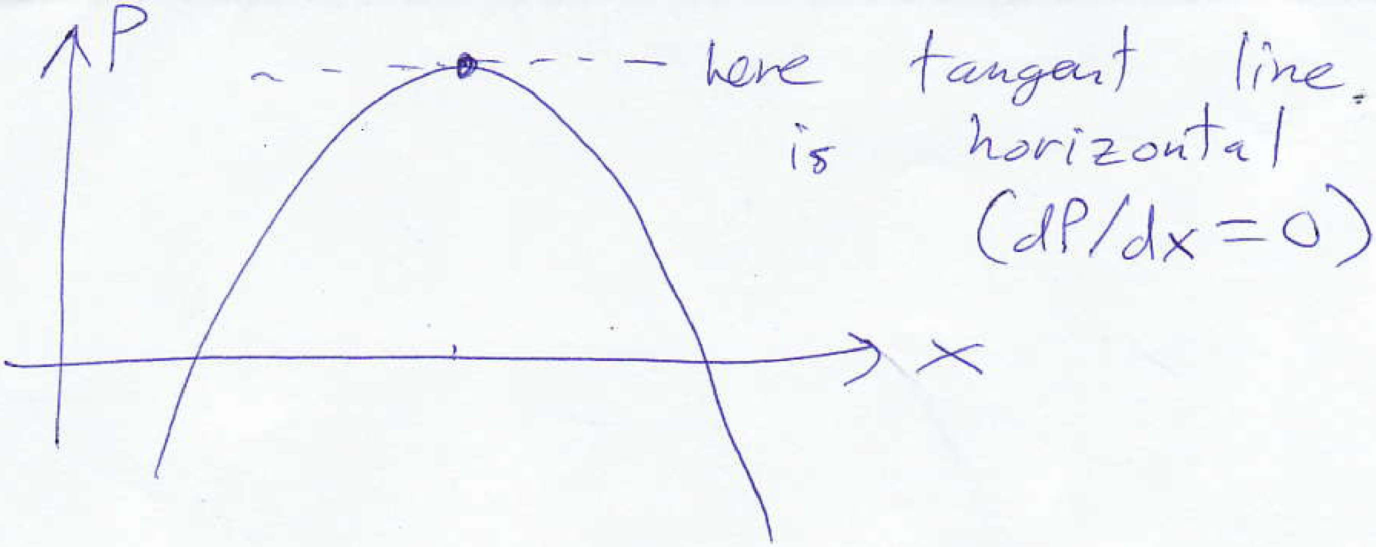
$$R(x) = x\,p(x)$$

$$R = 100x - \frac{x^2}{10,000}$$

$$P = R - C = 60x - \frac{x^2}{10,000} - 3,000,000$$

$$\frac{dP}{dx} = 60 - \frac{2x}{10,000}$$

$$\frac{d^2P}{dx^2} = -\frac{2}{10,000} \Rightarrow$$ graph of P is concave down



HW: Find x where $dP/dx = 0$.
What is the maximum possible profit?