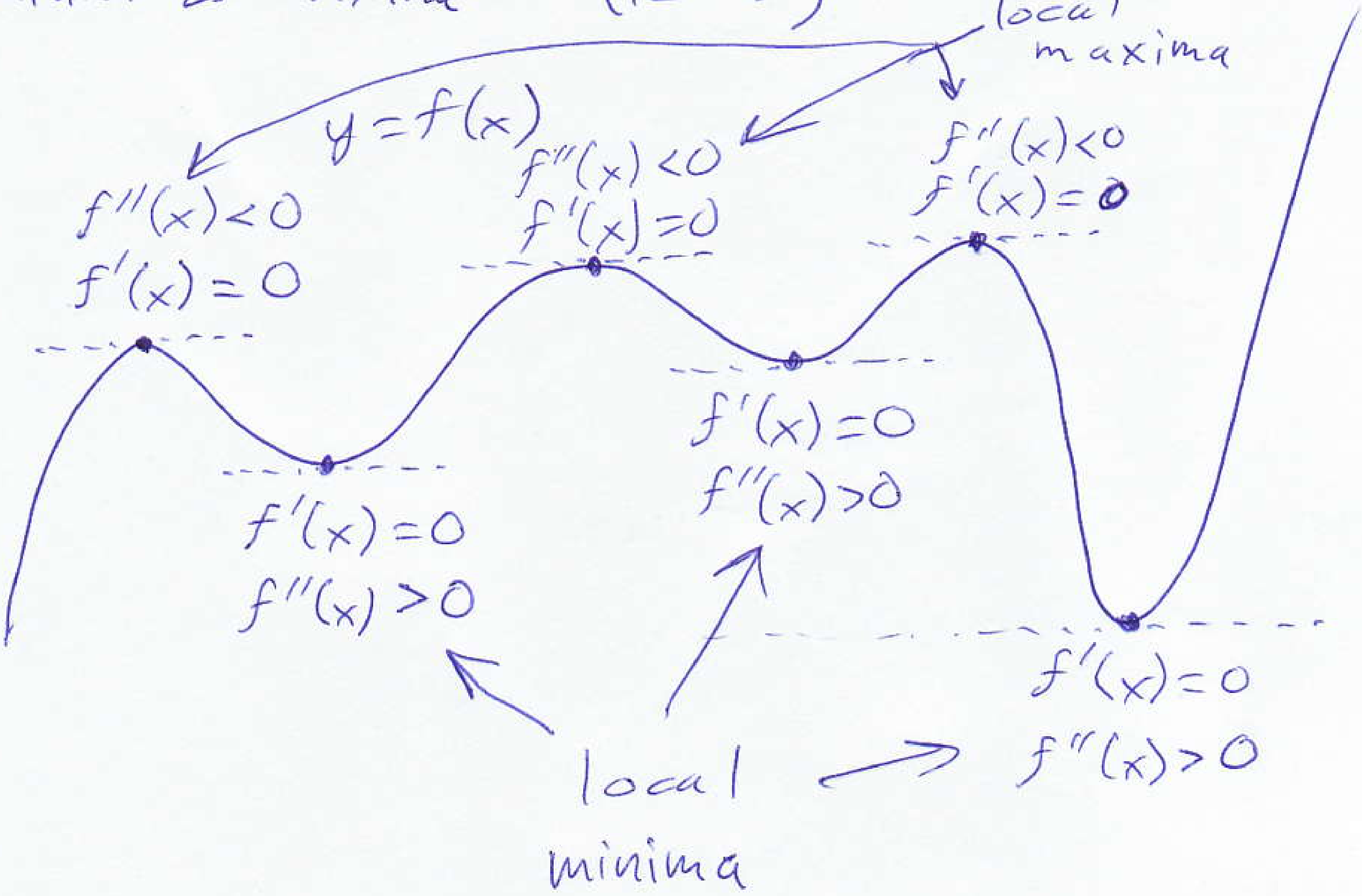
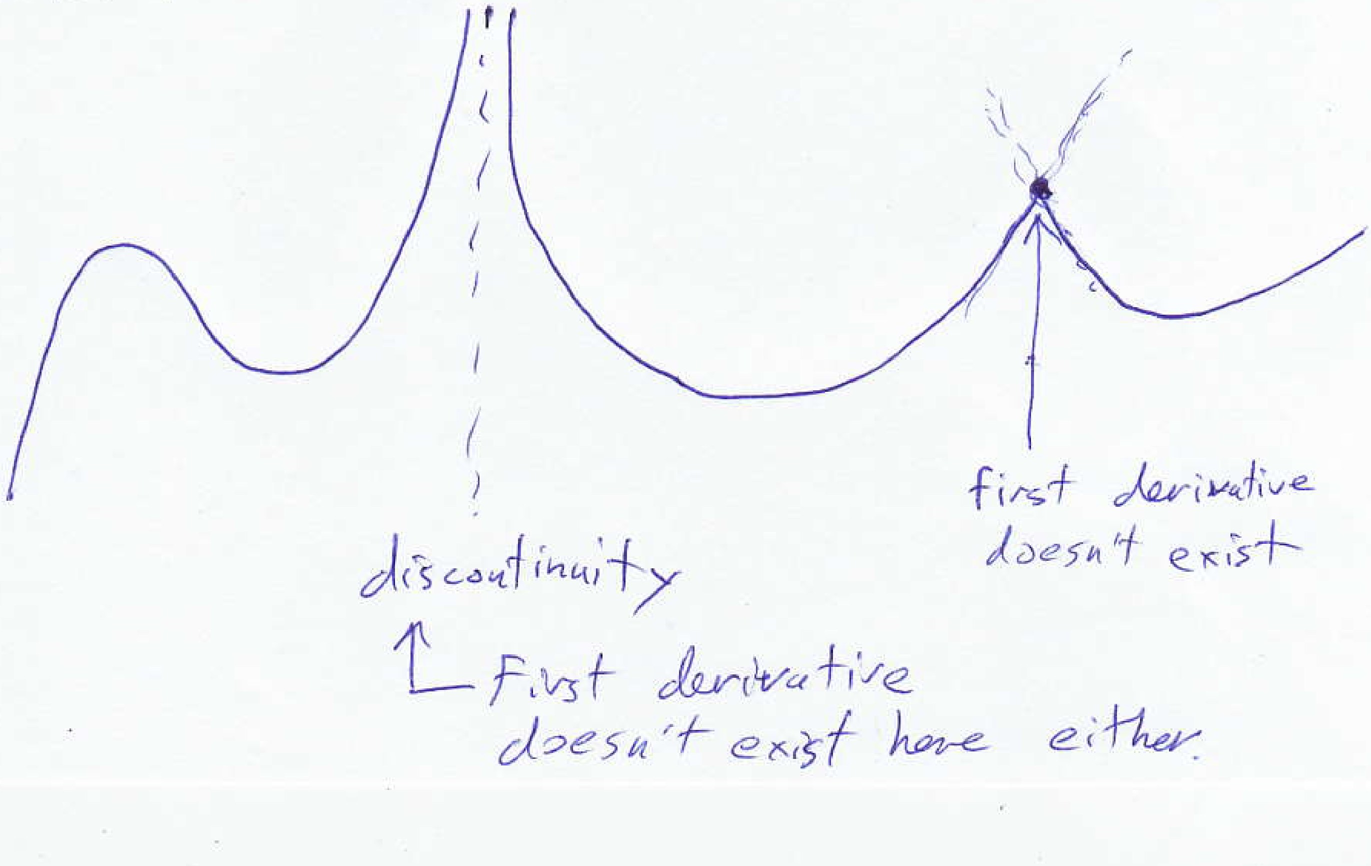
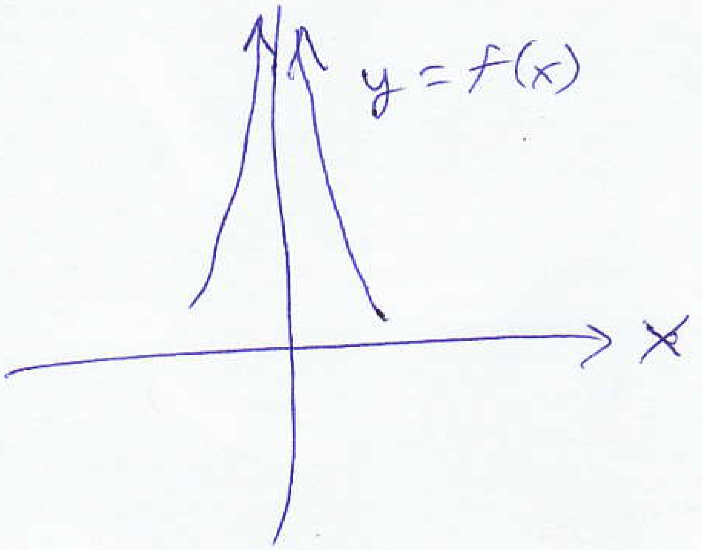
Minima & Maxima (12-5)



Global maximum: highest point everywhere
Global minimum: lowest point everywhere



$f(x) = +\text{big} \pm \text{small} = +\text{big}$



when $x = \pm\text{small}$

" $\lim_{x \to 0} f(x) = \infty$ "

What about those ~~the~~ 2 local minima we see in the graph?

There should be two x-values a, b where $f'(a) = f'(b) = 0$ &

$f''(a) > 0$ & $f''(b) > 0$.

$$f'(x) = \frac{-2}{x^3} + 2x - 2$$

Solve $0 = \frac{-2}{x^3} + 2x - 2$:

$$x^3 \cdot 0 = x^3\left(\frac{-2}{x^3} + 2x - 2\right)$$

$$0 = -2 + 2x^4 - 2x^3$$

$$0 = -1 + x^4 - x^3$$

$$\underbrace{x = 1.380\ldots \quad \text{or} \quad x = -0.819\ldots}_{f'(x)=0 \text{ here}}$$

$$f''(x) = (f'(x))' = (-2x^{-3} + 2x - 2)'$$

$$= -2(-3)x^{-4} + 2 = \underbrace{\frac{6}{x^4} + 2}_{\substack{\text{always positive} \\ \text{except undefined} \\ \text{at } x=0}}$$

f is concave up on $(-\infty, 0)$ & $(0, \infty)$. $\Leftarrow$

f is concave up at the critical pts, $x = 1.38\ldots$ & $x = -0.819\ldots$, so $f(1.38\ldots)$ & $f(-0.819\ldots)$ are local minima

$$f(1.38\ldots) = \underbrace{\frac{1}{(1.38\ldots)^2} + (1.38\ldots - 2)(1.38\ldots)}_{-0.33\ldots}$$

$$f(-0.819\ldots) = 3.7996\ldots$$

An x-value c is a critical point, if $f'(c)=0$ or $f'(c)$ doesn't exist.

To find global minima/maxima, look at all the critical points and endpoints.

$$f(x)=\frac{1}{x^2}+(x-2)x = x^{-2}+x^2-2x$$

$$f'(x)=-2x^{-3}+2x-2$$
$$f'(x)=\frac{-2}{x^3}+2x-2$$

$f'(0)$ is undefined: division by 0.

0 is a critical point.

When x is near 0, say $x=\pm$small,

$$f(x)=\underbrace{\frac{1}{(\pm \text{small})^2}}_{+\text{big}\ \left\{\frac{1}{+\text{small}}\right.} + \underbrace{\underbrace{(\pm\text{small}-2)}_{\approx -2}\underbrace{(\pm\text{small})}_{\approx 0}}_{\approx(-2)(0)=0}$$

local/global minima/maxima are y-values

Is $-0.33...$ the global minimum?

Yes, if we restrict x to $[-3, 3]$:

$$\begin{cases} f(-3) = 1/(-3)^2 + (-3-2)(-3) = 15 + \frac{1}{9} \\ f(3) = 1/3^2 + (3-2)(3) = 3 + \frac{1}{9} \end{cases}$$

$\hookrightarrow$ both higher than -0.33.

If x is unrestricted, then $-0.33...$ is still the global minimum:

Look at $f(+\text{big})$ & $f(-\text{big})$:

$$f(+\text{big}) = \underbrace{\underbrace{\frac{1}{(+\text{big})^2}}_{+\text{small}} + \underbrace{\underbrace{(+\text{big}-2)}_{+\text{big}}(+\text{big})}_{+\text{big}}}_{+\text{small} + \text{big} = +\text{big}}$$

$$\lim_{x \to \infty} f(x) = \infty$$

when x is $+$big, $f(x)$ is $+$big

HW: Argue that when x is $-$big, $f(x)$ is $+$big

"$\lim_{x \to -\infty} f(x) = \infty$"

HW Let $y = g(x) = x^5 - x^3 + x + \frac{1}{(x-7)}$

Find (approximately) all the critical points.

Find $g'(x)$ & $g''(x)$.

Find the local minima & local maxima (approximate).

Find approximately ~~(approximately) where g is~~ the intervals on which g is $\nearrow$, $\searrow$, C.U., C.D.

↳ Hint: approximately solve $g'(x) = 0$ and $g''(x)$ for x.

Is there a global maximum?
If so, what is it?
Is there a global minimum?
If so, what is it?

Justify your answer.

A factory makes shoes at a cost of $4x + 200\sqrt{x} + 5{,}000{,}000$, for x pairs of shoes.

Market research suggests a price model $p = 100 - \frac{x}{18{,}000}$.

What is the maximum possible profit (approximately)?