$\boxed{\text{Use a differential to estimate } \frac{1}{49}.}$

$$y = \frac{1}{x} = x^{-1}$$

(old) $x = 50$ $\quad y = \frac{1}{50} = 0.02$

(change) $\Delta x = -1$ $\quad \Delta y = ?$

(new) $x + \Delta x = 49$ $\quad y + \Delta y = ?$

$$\Delta y \approx dy \underset{\uparrow}{=} (-1)\, x^{-1-1}\, dx = -x^{-2}\, dx$$

power rule $\qquad (dx = \Delta x)$

$$dy = \frac{-dx}{x^2} = \frac{-(-1)}{50^2} = \frac{1}{2500} = \frac{4}{10{,}000}$$

$$dy = 0.0004$$

$$\frac{1}{49} = y + \Delta y \approx y + dy = 0.02 + 0.0004$$

$$\frac{1}{49} \approx 0.0204$$

$$\begin{array}{r} 0.0204081\ldots \\ 49\,\overline{)\,1.00} \\ 98 \\ \hline 0200 \\ 196 \\ \hline 400 \\ 392 \\ \hline 80 \\ 49 \\ \hline 31 \end{array}$$

$\sqrt{50} \approx ?$ $\qquad x = 49 \quad \Delta x = 1$

$$x + \Delta x = 50$$

$$dx = \Delta x = 1$$

$$y = x^{1/2} = \sqrt{49} = 7$$

$$\Delta y \approx dy = \frac{1}{2} x^{\overbrace{1/2 - 1}^{-1/2}} dx = \frac{dx}{2\sqrt{x}} = \frac{1}{2\sqrt{49}} = \frac{1}{2 \cdot 7}$$

$$dy = \frac{1}{14} \Rightarrow \Delta y \approx \frac{1}{14} \Rightarrow \sqrt{50} = y + \Delta y \approx 7 + \frac{1}{14}$$

$$d(fg) = (df)(g) + (f)(dg)$$
$$(fg)' = f'g + fg'$$

Product

$$d\left(\frac{f}{g}\right) = \frac{(df)(g) - (f)(dg)}{g^2}$$
$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Quotient

$f'(x)\,dx = df \qquad g'(x)\,dx = dg$

$$d\left(\left(x^{1/3}-7x^{1/7}\right)\left(x^2+2x+3\right)\right)$$

$$=\left(\left(x^{1/3}-7x^{1/7}\right)\left(x^2+2x+3\right)\right)'dx$$

$$=\Big[\left(x^{1/3}-7x^{1/7}\right)'\left(x^2+2x+3\right)$$

$$+\left(x^{1/3}-7x^{1/7}\right)\left(x^2+2x+3\right)'\Big]dx$$

$$=\Big[\left(\tfrac{1}{3}x^{-2/3}-7\left(\tfrac{1}{7}\right)x^{-6/7}\right)\left(x^2+2x+3\right)$$

$$+\left(x^{1/3}-7x^{1/7}\right)\left(2x^1+2\underbrace{(1)}_{\frac{dx}{dx}=1}+\underbrace{0}_{\frac{d3}{dx}=\frac{0}{dx}}\right)\Big]dx$$

$$\left(\frac{\sqrt{x}}{1+x}\right)'=\frac{\sqrt{x}'(1+x)-\sqrt{x}(1+x)'}{(1+x)^2}$$

$$=\frac{\frac{1}{2\sqrt{x}}(1+x)-\sqrt{x}(0+1)}{(1+x)^2}$$

$$=\frac{1+x-\sqrt{x}(1)(2\sqrt{x})}{2(1+x)^2\sqrt{x}}$$

$$=\frac{1-x}{2\sqrt{x}(1+x)^2}\Rightarrow d\left(\frac{\sqrt{x}}{1+x}\right)=\frac{(1-x)\,dx}{2\sqrt{x}(1+x)^2}$$

The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. If a balloon is expanding and roughly spherical, what is the instantaneous rate of change of volume with respect to radius when $r = 8\text{cm}$?

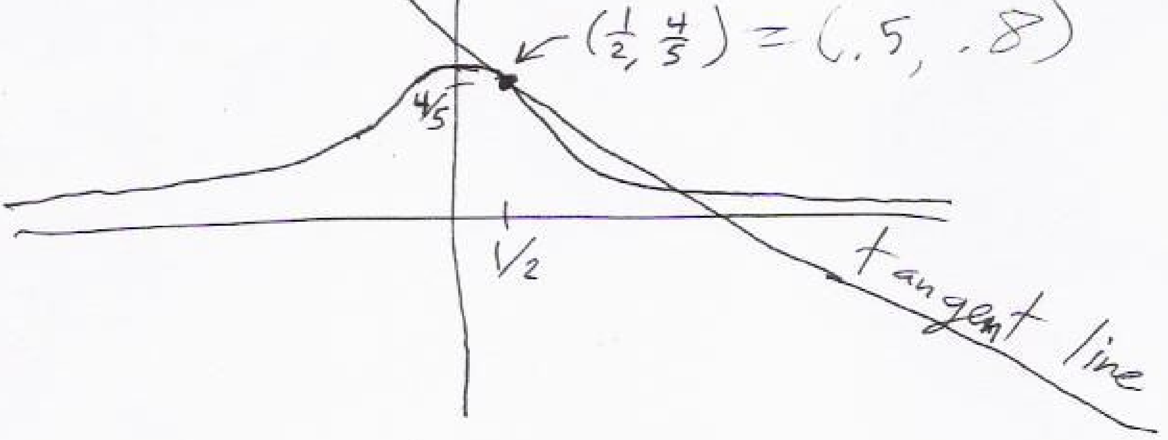
$$r = 8\text{ cm} \Rightarrow V = \frac{4}{3}\pi(8\text{cm})^3 = \frac{2048\pi}{3}\text{cm}^3$$

$$\frac{dV}{dr} = ? \qquad \frac{dV}{dr} = V' = \frac{4}{3}\pi(3r^2) = 4\pi r^2$$

$$r = 8\text{cm} \Rightarrow \frac{dV}{dr} = 4\pi(8\text{cm})^2 = 256\pi\text{ cm}^2$$

$$y = \frac{1}{x^2+1}$$

Find an equation for the tangent line at $\left(\frac{1}{2}, \frac{4}{5}\right)$



$$\frac{dy}{dx} = y' = \left(\frac{1}{x^2+1}\right)' = \frac{1'(x^2+1) - (1)(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{0(x^2+1) - (1)(2x+0)}{(x^2+1)^2}$$

$$= -2x/(x^2+1)^2$$

Alternative: $d\left(\frac{1}{x^2+1}\right) = d\left((x^2+1)^{-1}\right)$

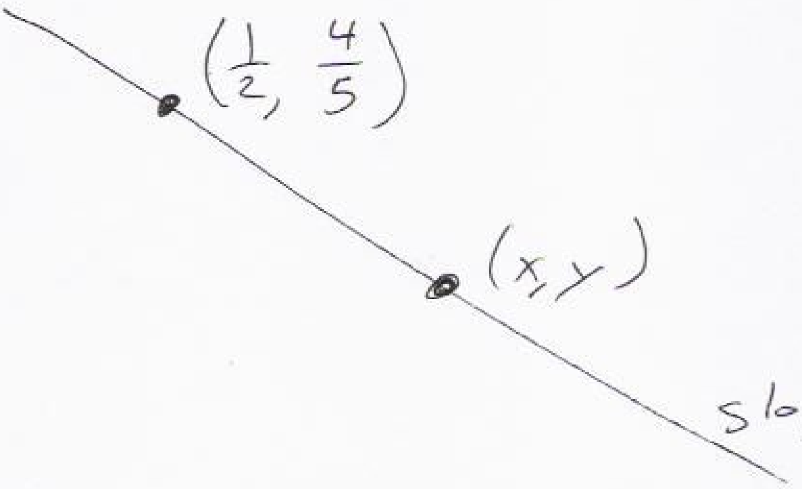
$$= (-1)(x^2+1)^{-2} d(x^2+1)$$

$$= (-1)(x^2+1)^{-2}(2x+0)dx$$

$$= \left[-2x/(x^2+1)^2\right]dx$$

tangent line slope: $\frac{dy}{dx}$ at $x = \frac{1}{2}$

$$\underbrace{-2(1/2)}_{-1} / \underbrace{\underbrace{(\underbrace{(1/2)^2}_{1/4} + 1)}_{5/4}{}^2}_{(25/16)} = -\frac{16}{25}$$



$-\frac{16}{25} = m = \frac{\Delta y}{\Delta x} = \frac{y - 4/5}{x - 1/2} \Rightarrow \boxed{y - \frac{4}{5} = -\frac{16}{25}\left(x - \frac{1}{2}\right)}$ ✓

Graph $y = \frac{1}{x^2+1}$ & $y = \frac{4}{5} - \frac{16}{25}\left(x - \frac{1}{2}\right)$ to check answer

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$y = x^5 - 10x^3 - 4$

$y' = 5x^4 - 10(3x^2) - 0 = 5x^4 - 30x^2$

$y'' = 5(4x^3) - 10(3)(2x) = 20x^3 - 60x$

Solve $0 = 5x^4 - 30x^2 = 5x^2(x^2 - 6)$

$a^2 - b^2 = (a+b)(a-b)$ $6 = \sqrt{6}^2$

$0 = 5x^2(x + \sqrt{6})(x - \sqrt{6})$

$0 = x^2$ or $0 = x + \sqrt{6}$ or $0 = x - \sqrt{6}$

$0 = x$ $x = -\sqrt{6}$ $x = \sqrt{6}$

critical points

$y' = 5x^4 - 30x^2$

x	T.P. -3	$-\sqrt{6}$	T.P. -1	0	T.P. 1	$\sqrt{6}$	T.P. 3
y'	135	0	-25	0	-25	0	135
y	$\nearrow$	local max ≈ 54.78	$\searrow$		$\searrow$	local min ≈ -62.78	$\nearrow$

Concavity: $y'' = 20x^3 - 60x = 20x(x^2-3)$

Solve $0 = 20x(x^2-3) = 20x(x+\sqrt{3})(x-\sqrt{3})$

$0 = x$ or $0 = x + \sqrt{3}$ or $0 = x - \sqrt{3}$

$x = -\sqrt{3}$ $\qquad$ $x = \sqrt{3}$

x	T.P. -2	$-\sqrt{3}$	T.P. -1	0	T.P. 1	$\sqrt{3}$	T.P. 2
y''	-40	0	40	0	-40	0	40
y	C.D.	inflection point	C.U.	inflection pt.	C.D.	Inflect. pt.	C.U.

$y \approx 32.37$ $\qquad$ $y \approx -40.37$

Intervals:

C.U. on $[-\sqrt{3}, 0]$ & $[\sqrt{3}, \infty)$

C.D. on $(-\infty, -\sqrt{3}]$ & $[0, \sqrt{3}]$

$\nearrow$ on $(-\infty, -\sqrt{6}]$ & $[\sqrt{6}, \infty)$

$\searrow$ on $[-\sqrt{6}, \sqrt{6}]$

If x is restricted to $[-4, 4]$,

then $y = 380$ at $x = 4$ &

$y = -388$ at $x = -4$, so

y	~~380~~ -388 = global min.	54.78		-62.78	global max = 380 ~~-388~~
x	-4	$-\sqrt{6}$	0	$\sqrt{6}$	4