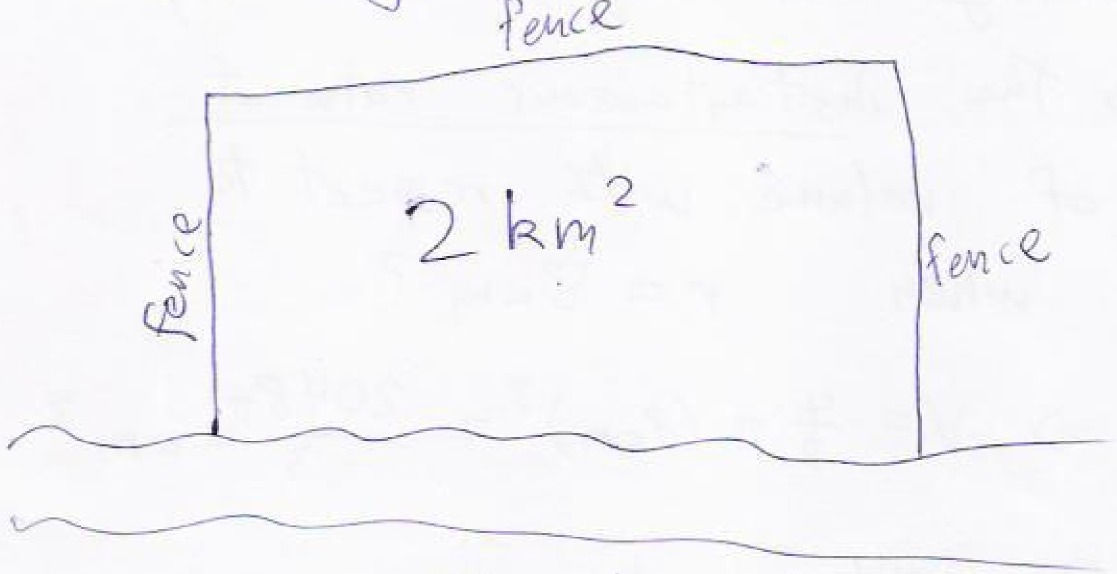
Optimization (12-6)

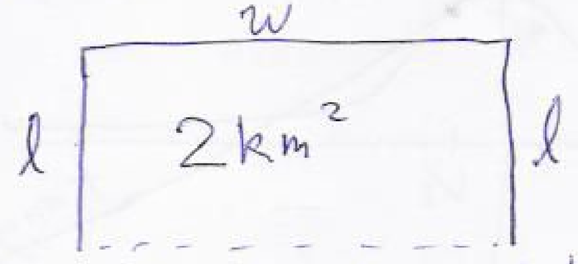
Fencing



What is the cheapest rectangle?

<u>Minimize</u> fence length,
but enclose an area of $2\ km^2$.

Strategy: • ~~Label~~ Draw picture &
label with variables & constants



• Write formula for what you want to optimize: minimize $l+w+l$
• Write formula(s) for constraints.

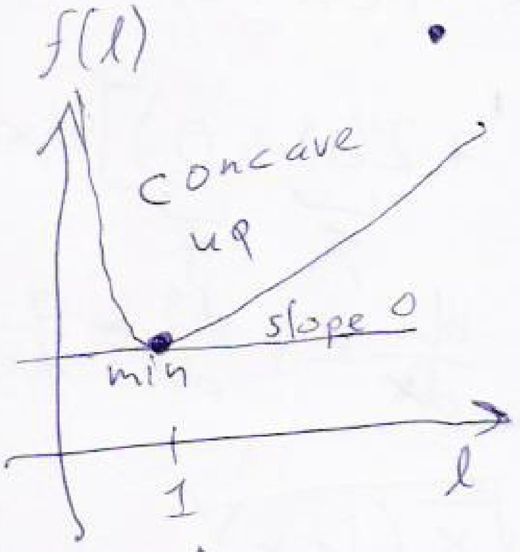
Constraint: $lw = 2$

- Express the formula you're optimizing with just 1 variable by using the constraint formula(s).

$$lw = 2 \Rightarrow w = 2/l$$

minimize $\quad l + \frac{2}{l} + l = 2l + \frac{2}{l}$

- Use derivatives to find your optimum. $\quad f(l) = 2l + \frac{2}{l}$



$$f(l) = 2l + 2l^{-1}$$

$$f'(l) = 2 + 2(-1)l^{-2}$$

Solve $f'(l) = 0$

$$0 = 2 - 2/l^2$$

$$0 = 1 - 1/l^2$$

$$1/l^2 = 1$$

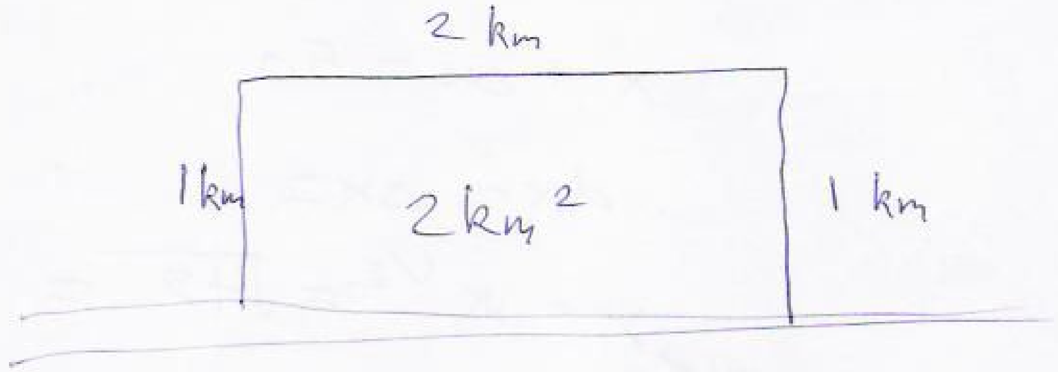
$$1 = l^2$$

$$\pm 1 = l$$

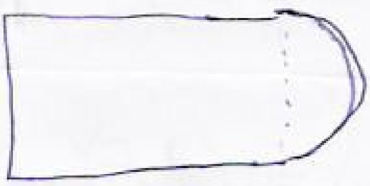
$+1 = l$ (no negative ~~[illegible]~~ lengths)

(You can check that $f''(l) > 0$ when $l > 0$.)

$\ell = 1$ is best $\quad w = 2/\ell = 2/1 = 2$

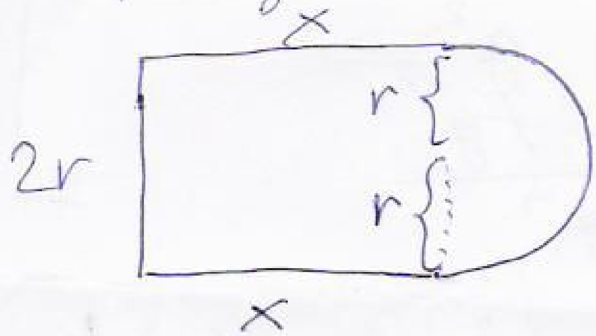


Another fencing problem:

Desired shape 

rectangle + semicircle

Maximize area given 5000 ft of fencing.



5000 = perimeter

Maximize area $= 2rx + \frac{1}{2}\pi r^2$

Constraint: $5000 = x + 2r + x + \underbrace{\frac{1}{2}(2\pi r)}_{\pi r}$

Solve constraint for x & plug that into area formula.

$$5000 = 2x + (2+\pi)r$$

$$5000 - (2+\pi)r = 2x$$

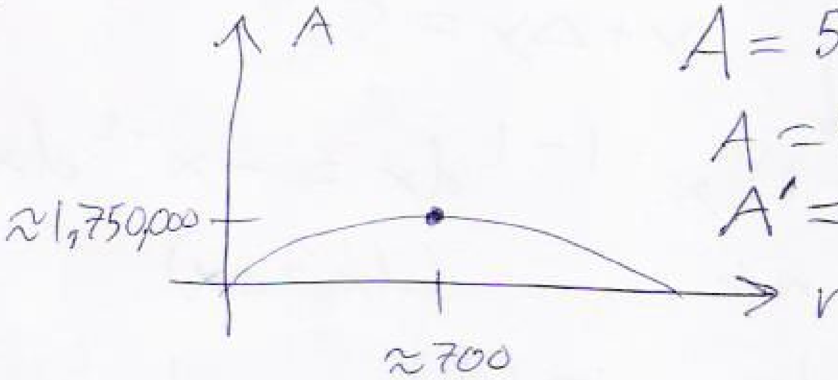
$$2500 - \left(\frac{2+\pi}{2}\right)r = x$$

Maximize $A = 2r\left(2500 - \left(\frac{2+\pi}{2}\right)r\right) + \frac{\pi r^2}{2}$

$$A = 5000r - (2+\pi)r^2 + \frac{\pi r^2}{2}$$

$$A = 5000r - \left(2 + \frac{\pi}{2}\right)r^2$$

$$A' = 5000 - \left(2 + \frac{\pi}{2}\right)(2r)$$



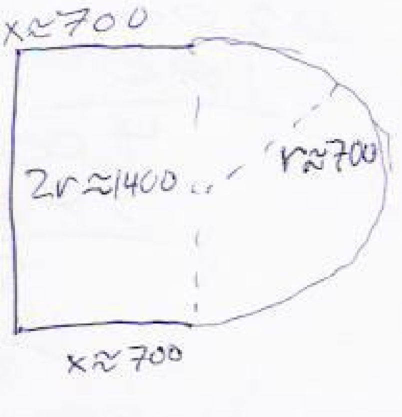
Solve $0 = A'$: $0 = 5000 - \left(2 + \frac{\pi}{2}\right)(2r)$

$$0 = 5000 - (4+\pi)r \Rightarrow (4+\pi)r = 5000$$

$$\Rightarrow r = \frac{5000}{4+\pi} \approx 700.12394\ldots \text{ ft}$$

Find x

$$x = 2500 - \left(\frac{2+\pi}{2}\right)\left(\frac{5000}{4+\pi}\right)$$



$$x \approx 700.12394\ldots \text{ ft}$$

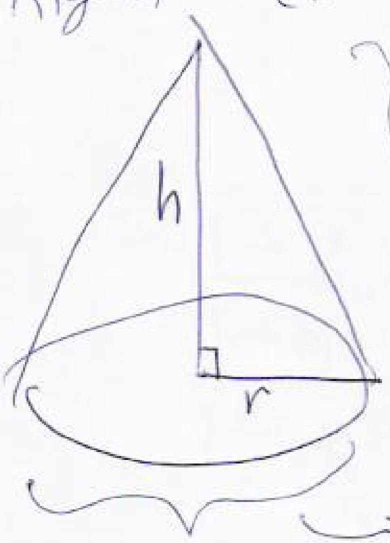
$$A = 5000r - \left(2 + \frac{\pi}{2}\right)r^2$$

$$\max(A) = \frac{12500000}{\pi + 4} \approx 1,750,309.85\ldots \text{ ft}^2$$

HW #31 (12-6)
#26 (12-6)
#24 (12-6)

Geometry: See Appendix C.

Right circular cone



$V = \frac{1}{3}\pi r^2 h$

lateral area $= \pi r\sqrt{r^2+h^2}$

base area $= \pi r^2$

Given a fixed lateral area of $50\ cm^2$, maximize volume.

$A = 50$ $\qquad \underbrace{50 = \pi r\sqrt{r^2+h^2}}_{\text{constraint}}$

maximize $V = \frac{1}{3}\pi r^2 h$

Solve constraint for h:

$$50^2 = \left(\pi r \sqrt{r^2+h^2}\right)^2$$

$$2500 = \pi^2 r^2 (r^2+h^2)$$

$$\cancel{2500 = \pi^2 r^4 + \pi^2 r^3 h}$$

$$\frac{2500}{\pi^2 r^2} = r^2 + h^2$$

$$\frac{2500}{\pi^2 r^2} - r^2 = h^2$$

$$\sqrt{\frac{2500}{\pi^2 r^2} - r^2} = h$$

Plug into $V = \frac{1}{3}\pi r^2 h$

Maximize $$V = \frac{1}{3}\pi r^2 \sqrt{\frac{2500}{\pi^2 r^2} - r^2}$$

$$dV = d\left(\frac{1}{3}\pi r^2\right)\sqrt{\frac{2500}{\pi^2 r^2} - r^2} + \left(\frac{1}{3}\pi r^2\right) d\left(\sqrt{\frac{2500}{\pi^2 r^2} - r^2}\right)$$

Product rule

$$dV = \frac{1}{3}\pi(2r\,dr)\sqrt{\frac{2500}{\pi^2 r^2} - r^2} + \frac{1}{3}\pi r^2 \left(\frac{d\left(\frac{2500}{\pi^2 r^2} - r^2\right)}{2\sqrt{\frac{2500}{\pi^2 r^2} - r^2}}\right)$$

$$d(\sqrt{x}) = \frac{dx}{2\sqrt{x}}$$

$$V' = \frac{dV}{dr} = \ldots$$

Solve $V' = 0$ with a calculator.