Chain rule (11-4)
Related rates (11-5)

3. $f(x) = \dfrac{1+x^2}{1-x^3}$ $\quad g(x) = x + \sqrt{x}$

$$f(g(x)) = \frac{1 + (x+\sqrt{x})^2}{1 - (x+\sqrt{x})^3}$$

If you know f', g', what is $(f(g(x)))'$?

$df = f'(x)\,dx \qquad dg = g'(x)\,dx$

$d(f(g(x))) = (f(g(x)))'\,dx$

$d(f(g(x))) = f'(g(x))\,d(g(x))$

$d(f(g(x))) = f'(g(x))\,g'(x)\,dx$

$$(f(g(x)))' = \frac{d(f(g(x)))}{dx} = f'(g(x))\,g'(x)$$

Chain rule: $(f(g(x)))' = f'(g(x))\,g'(x)$

Chain rule: $d(f(g(x))) = f'(g(x))\, d(g(x))$

Just like $d(f(x)) = f'(x)\, dx$

This course's functions are built up from $+, -, \cdot, /, \underbrace{x^n, e^x, \ln x}$

3 special cases of chain rule:

$$d\left((g(x))^n\right) = n g(x)^{n-1} d(g(x))$$

Just like $d(x^n) = n x^{n-1} dx$

$$d\left(e^{g(x)}\right) = e^{g(x)} d(g(x))$$

Just like $d(e^x) = e^x dx$

$$d(\ln(g(x))) = [d(g(x))] / g(x)$$

Just like $d(\ln x) = dx / x$

$$d(f \pm g) = df \pm dg$$

$$d(f \cdot g) = df \cdot g + f \cdot dg$$

$$d(f/g) = (df \cdot g - f \cdot dg)/g^2$$

$$d(g^n) = n g^{n-1} dg$$

$$d(e^g) = e^g dg$$

$$d(\ln g) = (dg)/g$$

k constant $\Rightarrow dk = 0$

Shortcuts:

$$\begin{cases} d(k \cdot f) = k \cdot df \\ d(f/k) = df/k \end{cases}$$

$$d(x^1) = dx = 1\,dx \Rightarrow (x)' = 1$$

$$d(x^{1/2}) = d(\sqrt{x}) = \frac{dx}{2\sqrt{x}}$$

$$d(g^{1/2}) = d(\sqrt{g}) = dg/(2\sqrt{g})$$

$f(x) = x^2+1$, $g(x) = x^3+1$, $h(x) = x^4+1$

$f(g(h(x))) = ((x^4+1)^3+1)^2+1$

$d(((x^4+1)^3+1)^2+1) = d(((x^4+1)^3+1)^2) + \underbrace{d1}_{0}$

$= 2((x^4+1)^3+1)\, d((x^4+1)^3+1)$

Just like $d(x^2) = 2x\,dx$

$= 2((x^4+1)^3+1)\left[d((x^4+1)^3) + \underbrace{d1}_{0}\right]$

$= 2((x^4+1)^3+1)\underbrace{(3(x^4+1)^2\, d(x^4+1))}$

Just like $d(x^3) = 3x^2dx$

$= 2((x^4+1)^3+1)(3)(x^4+1)^2(\underbrace{d(x^4)}_{4x^3dx} + \underbrace{d1}_{0})$

$= 2((x^4+1)^3+1)(3)(x^4+1)^2(4x^3\,dx)$

Divide by dx to get derivative:

$[((x^4+1)^3+1)^2+1]' = 2((x^4+1)^3+1)(3)(x^4+1)^2(4x^3)$

$= 24((x^4+1)^3+1)(x^4+1)x^3$

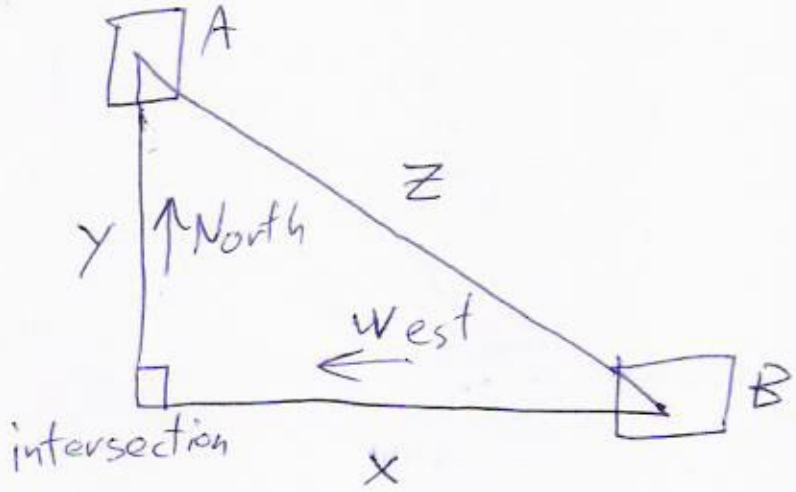
HW#1: Find the derivative of

$$\sqrt{1+e^{x}\ln x} + \left(e^{(1/\ln x)} + 1\right)^{2}$$

Many more practice problems in 11-4.

Related rates 11-5:

Car A is heading north, away from an intersection, while car B is heading west, approaching the intersection. The driver of car B knows that his speed is 32 mph and he's 0.2 miles from the intersection. The passenger of car B is tracking car A: the distance between the cars is increasing at a rate of 10mph and the current distance is 0.5 miles. What is the speed of car A?



"Right now": $x = 0.2$ $\quad \frac{dx}{dt} = -32$

$z = 0.5$ $\quad \frac{dz}{dt} = 10$

$dy/dt = ?$

At all times: $x^2 + y^2 = z^2$

Take the differential of the equation(s) that are always true.

$$d(x^2 + y^2) = d(z^2)$$

$$2x\,dx + 2y\,dy = 2z\,dz$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$

true all the time

Right now:

$$(0.2)(-32) + y\frac{dy}{dt} = (0.5)(10)$$

$$-6.4 + y\frac{dy}{dt} = 5$$

$$y\frac{dy}{dt} = 11.4$$

$$x^2 + y^2 = z^2$$

$$(0.2)^2 + y^2 = (0.5)^2$$

$$0.04 + y^2 = 0.25$$

$$y^2 = 0.21$$

$$y = \sqrt{0.21} \approx 0.45826$$

$$(0.45826)\frac{dy}{dt} = 11.4$$

$$dy/dt \approx 24.877$$

Right now, the speed of car A is ≈ 24.877 mph.

HW#2: If two cars are currently approaching an intersection, car 1 From the north and car 2 from the west, with car 1 0.7mi from the intersection (right now) and at speed 36mph & car 2 0.8mi from the intersection (right now) and at speed 41mph. What is the current rate of change of the distance between these cars? If the cars continue at these speed, what is the ~~closest~~ least ~~the~~ distance between them that will occur?

HW #3

If you're selling 10,000 socks per day currently, but this quantity is increasing at a rate of 1% per day while your total costs are increasing at a rate of 0.5% per day with current total cost at $4,000/day, and your current price is $0.43 per sock and you're lowering the price 0.7% per day, then what is the current daily profit, and what is its rate of change?

Hint: if x = quantity sold, then right now $dx/dt = (1\%)x$.

Many more related rates exercises in 11-5.