More on related rates (11−6; not 11−5)

Mexico: $\frac{dM}{dt} = (1\%)M = \frac{1}{100} M$

Japan: $\frac{dJ}{dt} = (-0.1\%) J = -\frac{1}{1000} J$

Libya: $\frac{dL}{dt} = (2\%)L = \frac{2}{100} L$

current relative population growth rates

t measured in years

$$\left.\begin{aligned} L &= 6.42\cdot 10^6 \\ J &= 127.56\cdot 10^6 \\ M &= 107.43\cdot 10^6 \end{aligned}\right\}$$ current population.

What is the rate of change of the fraction of Mexico's population out of the total population of these 3 countries?

$$\frac{d(M/(L+J+M))}{dt} = ? \qquad d\left(\frac{f}{g}\right) = \frac{df\cdot g - f\,dg}{g^2}$$

$$d\left(\frac{M}{L+J+M}\right) = \frac{(dM)(L+J+M) - M\,d(L+J+M)}{(L+J+M)^2}$$

$$= \frac{(dM)(L+J+M) - M(dL + dJ + dM)}{(L+J+M)^2}$$

$$\frac{d(M/(L+J+M))}{dt} = \frac{\frac{dM}{dt}(L+J+M) - M\left(\frac{dL}{dt} + \frac{dJ}{dt} + \frac{dM}{dt}\right)}{(L+J+M)^2}$$

We differentiated first; now we plug in current data.

$$= \frac{\frac{M}{100}(L+J+M) - M\left(\frac{2L}{100} - \frac{J}{1000} + \frac{M}{100}\right)}{(L+J+M)^2}$$

$$= 2.47 \cdot 10^{-3} \longrightarrow = 0.00247$$

↑ This is the current annual rate of the fraction $\frac{M}{L+J+M}$

Right now $\frac{M}{L+J+M} = 0.44501$

In one year, $\frac{M}{L+J+M} \approx 0.44748$.

11-6 #13

The radius of a spherical balloon is increasing at 3cm/min. What is the rate of change of the volume of balloon when the radius is 10cm?

(Appendix C: $V = \frac{4}{3}\pi r^3$)

$$dV = d\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi d(r^3)$$

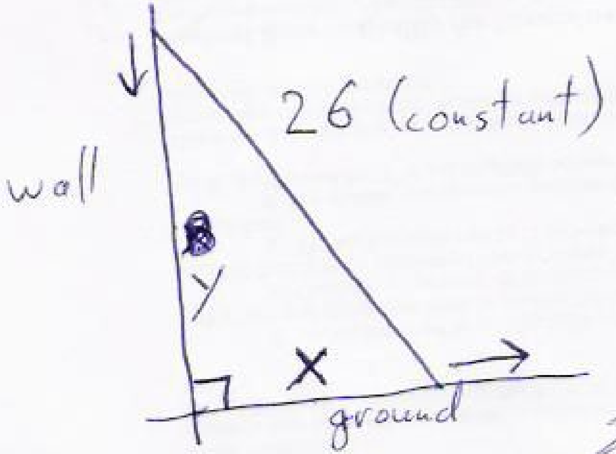
$$dV = \frac{4}{3}\pi \cdot 3r^2 dr = 4\pi r^2 dr$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(10)^2(3)$$

$$\frac{dV}{dt} = 1200\pi \text{ cm}^3/\text{min}.$$

$$\approx 3770 \text{ cm}^3/\text{min} = 3.77 \text{ L}/\text{min}$$

p. 598 A 26-foot ladder is placed against a wall. If it is sliding down the wall at 2 ft/sec, at what rate is the bottom of the ladder moving away from the wall when the bottom of the ladder is 10 feet away from the wall?



$x^2 + y^2 = 26^2$

true all the time

When $x = 10$, $\frac{dx}{dt} = ?$

and when $\frac{dy}{dt} = -2$

Right now: $x = 10$, $\frac{dy}{dt} = -2$, $\frac{dx}{dt} = ?$

All the time:

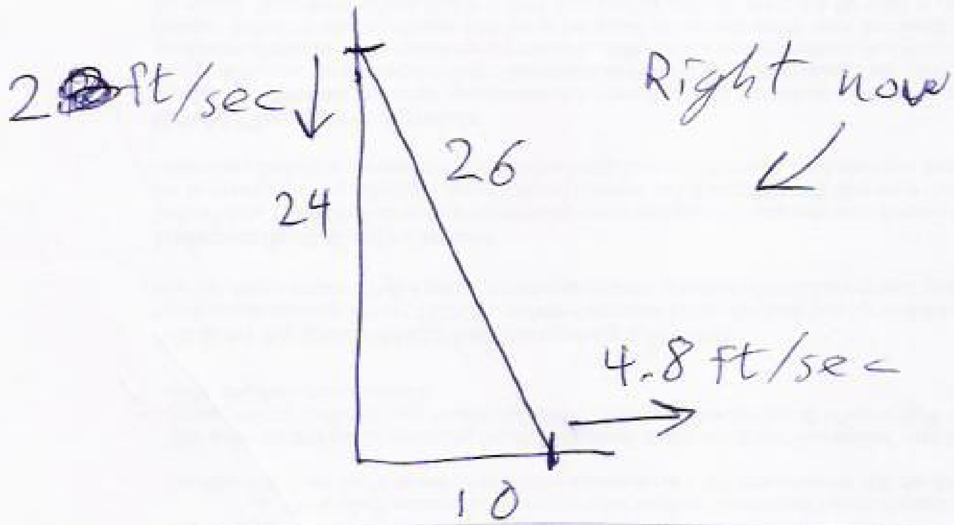
$$d(x^2 + y^2) = \underbrace{d(26^2)}_{0}$$

$$2x\,dx + 2y\,dy = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} = -y\frac{dy}{dt}$$

Right now: $\left\{\begin{array}{l} 10\dfrac{dx}{dt} = -y(-2) = 48 \\ 10^2 + y^2 = 26^2 \\ y = \sqrt{26^2 - 10^2} = 24 \\ dx/dt = \boxed{4.8 \text{ ft/sec}} \end{array}\right.$



11-6 #15 Boyle's law for enclosed gases states that if the volume is kept constant, the pressure P and temperature T satisfy $\frac{P}{T} = k$ where k is a constant. If the temperature is increasing at 3 K/hr, what is the rate of change of pressure when T = 250K and P = 500 lbs/in^2

$P/T = k$ = constant always

Right now: $dT/dt = 3$, $T = 250$, $P = 500$

$P = kT$ always

$dP = d(kT) = k\,dT$

$\dfrac{dP}{dt} = k\dfrac{dT}{dt} = 3k \longrightarrow \dfrac{dP}{dt} = \boxed{6 \dfrac{\text{pounds}}{\text{in}^2 \cdot \text{hr}}}$

Right now

Right now: $500 = P = kT = k(250)$

$2 = k$

HW: 9, 16, 31, 32 from 11-6.

Upcoming: Test 10/13

Review session TBA