Review session: BH 204
(today) 4:30-5:30

Extended office hours tomorrow:
9:30 – 3:00 BVC 321

Test Thursday:
topics since last test
(similar to homework)

Bring calculator & 1 sheet of notes.

Next week:
I'll be in Toronto.

Tuesday: video lecture

Thursday: Fall Break (no class)

Today we'll start Ch. 13
(not on Thursday's test).

Today's material corresponds to
(parts of) 13-4 & 13-5

Look at #79 (13-5) [p. 752]

$R = \frac{100}{t+1} + 5$ = (instanteous) rate of production at t years from start

P = total production from first t years

What is P when $t = 10$?

$$R = \frac{dP}{dt}$$

R is (instantaneous) rate of change of P.

$dP = R\,dt$ When $dt = \Delta t$ is small, $\Delta P \approx dP$

Over a small time interval, the ~~average rate of~~ change in P is approximately the differential of P.

$\left(\dfrac{\Delta P}{\Delta t} \approx \dfrac{dP}{dt}\right.$ too: the instantaneous rate of change dP/dt is $\approx$ the average rate of change $\Delta P/\Delta t$ over a small time interval.)

$$\underbrace{\Delta P \approx dP}_{\substack{1000\\ \text{barrels}}} = \underbrace{R}_{\frac{1000 \text{ barrels}}{\text{year}}} \; \underbrace{dt}_{\text{year}}$$

units: 1000 barrels

Estimate ΔP from $t=0$ to $t=10$:

Break it up into smaller changes.

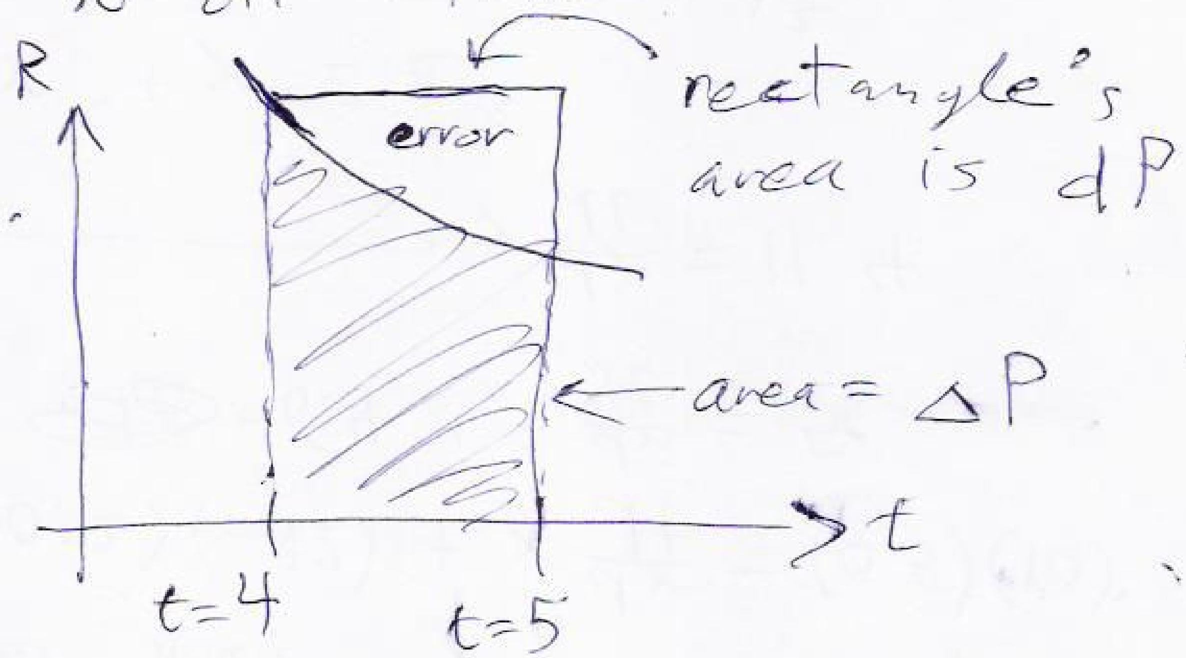
E.g., yearly:

$$(\Delta P \text{ from } t=0 \text{ to } t=1)$$
$$+(\Delta P \text{ from } t=1 \text{ to } t=2)$$
$$+(\Delta P \text{ from } t=2 \text{ to } t=3)$$
$$+\cdots+(\Delta P \text{ from } t=9 \text{ to } t=10).$$

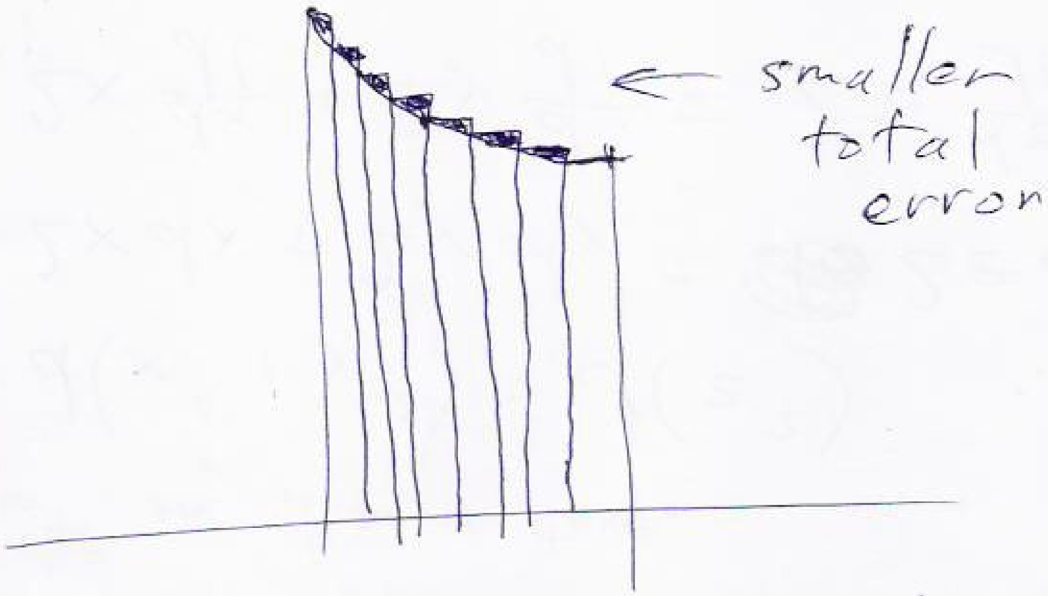
Each of these little ΔP's

is $\approx dP = R\,dt$

E.g.



For better estimates, use more and thinner rectangles.



(See spreadsheet...)

A better way:

If we could find a formula $P(t)$ such that $dP = R\,dt$, then we could just plug in $t=0$ & $t=10$ and compute $P(10)-P(0)$ to get 1st 10 years' production.

$$dP = \left(\frac{100}{t+1} + 5\right) dt$$

$$dP = \underbrace{\frac{100\,dt}{t+1}}_{} + \underbrace{5\,dt}_{}$$

$$dP = 100\left(\frac{dt}{t+1}\right) + d(5t) \qquad d1 = 0$$

$$dP = 100\,\frac{dt + d1}{t+1} + d(5t)$$

$$dP = 100\,\frac{d(t+1)}{t+1} + d(5t)$$

$$dP = 100\,d(\ln(t+1)) + d(5t)$$

$$dP = d(100\ln(t+1)) + d(5t)$$

$$dP = d(100\ln(t+1) + 5t)$$

$$P = 100\ln(t+1) + 5t$$

satisfies $dP = R\,dt$

For any constant c, since $dc = 0$,

$P = 100\ln(t+1) + 5t + c$ also satisfies $dP = R\,dt$.

Integral notation:

$$\int R\,dt = 100\ln(t+1) + 5t + c$$

$$\int dP = P + c$$

"$\int$" undoes "d"

Back to ΔP from $t=0$ to $t=10$:

Integral notation:

$$\Delta P = P\Big|_0^{10} = \int_0^{10} dP = \int_0^{10} R\,dt$$

$$= P(10) - P(0)$$

$$= \left(100\ln(t+1) + 5t + c\right)\Big|_0^{10}$$

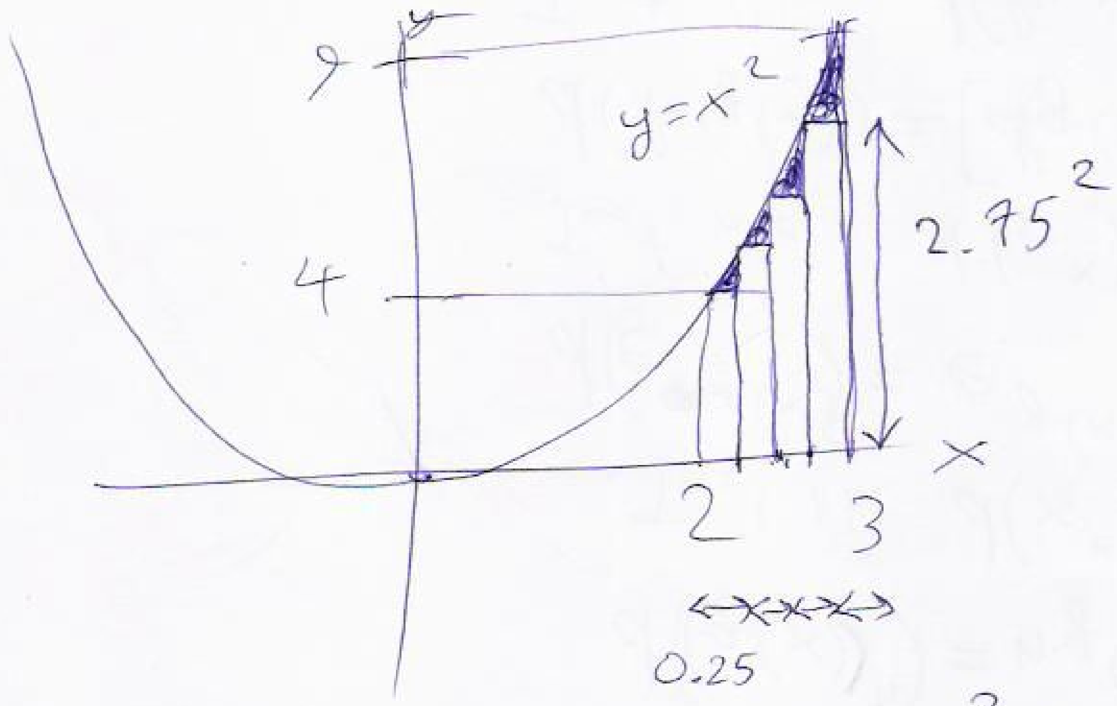
$$= \left[100\ln(11) + 50 + c\right] - \left[100\overbrace{\ln 1}^{0} + 0 + c\right]$$

$$= 100\ln(11) + 50 \approx 289.7895$$

Abstract example:

Area under parabola $y = x^2$

from $x = 2$ to $x = 3$ is ___.



$$\text{Area} \approx 2^2(0.25) + 2.25^2(0.25)$$
$$+ 2.5^2(0.25) + 2.75^2(0.25)$$
$$\approx 5.71875$$

For true area, you need

to find z where $dz = y\,dx = x^2 dx$

$$3\,dz = 3x^2 dx = d(x^3)$$

$$dz = d\left(\frac{x^3}{3}\right). \qquad z = \frac{x^3}{3} \text{ works}$$

$$\int x^2\,dx = \frac{x^3}{3} + c$$

$$\text{area} = \int_2^3 x^2\,dx = \left(\frac{x^3}{3} + c\right)\Bigg|_2^3$$

$$\left(\frac{3^3}{3} + c\right) - \left(\frac{2^3}{3} + c\right) = \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$$

6.3333...

HW: due next Tuesday (turn in over ANGEL or email):

Do a spreadsheet like mine for #81 of 13-5, using time intervals of ~~months~~, minutes, and then repeat for seconds

(so, $dt = \frac{1}{60}$ hour, then $dt = \frac{1}{3600}$ hour).