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Antiderivatives (13-1, 13-2)

$\int$ undoes d.

c constant

$\int dx = x + c$ because $d(x+c) = dx + 0$
$= dx$

$\int 2x\,dx = \int d(x^2) = x^2 + c$

$\int x\,dx = \int \frac{1}{2} \cdot 2x\,dx = \int \frac{1}{2} d(x^2) = \int d\left(\frac{x^2}{2}\right)$
$= \frac{x^2}{2} + c$

$\int 5x^3\,dx = ?$ $\quad d(x^4) = 4x^3\,dx$

$d\left(\frac{5}{4}x^4\right) = \frac{5}{4} d(x^4) = 5x^3\,dx$

$\left[\int 5x^3\,dx = \frac{5}{4}x^4 + c\right]$

same as: $\left[\left(\frac{5}{4}x^4 + c\right)' = 5x^3\right]$

also same as: $\left[d\left(\frac{5}{4}x^4 + c\right) = 5x^3\,dx\right]$

n constant

$$\int x^n dx = \int \frac{1}{n+1} \cdot (n+1) x^n \, dx = \int \frac{1}{n+1} d(x^{n+1})$$

need $n+1 \neq 0$

need $n \neq -1$

$$d(x^{n+1}) = (n+1)\underbrace{x^{n+1-1}}_{x^n} dx$$

$$\int x^n dx = \int d\left(\frac{x^{n+1}}{n+1}\right) = \frac{x^{n+1}}{n+1} + c$$

Check: $n=-4$ & $c=5$:

$$d\left(\frac{x^{-4+1}}{-4+1} + 5\right) = d\left(-\frac{1}{3}x^{-3} + 5\right)$$

$$= -\frac{1}{3} d(x^{-3}) + \underbrace{d5}_{0} = -\frac{1}{3}(-3)x^{-3-1} dx$$

$$= x^{-4} dx \quad \checkmark$$

Power rule for integrals: $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

for n constant and $n \neq -1$.

$c = 4c_1 + 5c_2$ is constant

Simpler

$$\int \left(4x^2 + \frac{5}{x}\right) dx = \frac{4}{3}x^3 + 5\ln x + c$$

Avoid those c_1's & c_2's; just do this:

$$4\int x^2\, dx + 5\int \frac{dx}{x} = 4\left(\frac{x^3}{3}\right) + 5\ln x + c$$

Warnings:
$$\int fg\, dx \neq \int f\, dx \int g\, dx$$
$$\int \frac{f}{g}\, dx \neq \int f\, dx \Big/ \int g\, dx$$

$$\int \underbrace{x^2}_{} dx = \frac{x^3}{3} + c \neq \underbrace{\left(\frac{x^2}{2}+c_1\right)}_{\int x\, dx}\underbrace{\left(\frac{x^2}{2}+c_2\right)}_{\int x\, dx}$$

$x^2 = x \cdot x$

$x^2 = \frac{x^3}{x}$

$$\neq \left(\frac{x^4}{4}+c_1\right) \Big/ \left(\frac{x^2}{2}+c_2\right)$$

$$\int (1+x)(3-x^2)\, dx \neq \left(x + \frac{x^2}{2}\right)\left(3x - \frac{x^3}{3}\right) + c$$

$$= \int (3 - x^2 + 3x - x^3)\, dx = \int (3\,dx - x^2\,dx + 3x\,dx - x^3\,dx)$$

$$= 3\int dx - \int x^2\, dx + 3\int x\, dx - \int x^3\, dx$$

$$d(\ln x) = \frac{dx}{x} = \frac{1}{x}\,dx = x^{-1}dx$$

$$d(\ln x + c) = \frac{dx}{x} + 0 = \frac{1}{x}\,dx = x^{-1}dx$$

$$\int \frac{dx}{x} = \int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln x + c$$

Special, $n=-1$ case of the power rule for integrals.

$$\int \left(4x^2 + \frac{5}{x}\right)dx = ?$$

$$d(f+g) = df + dg \iff \int (df + dg) = \int df + \int dg$$

$$d(kf) = k\,df \iff \int k\,df = k\int df$$

k constant

$$\int \Big(\underbrace{4x^2\,dx}_{df} + \underbrace{5\,\frac{dx}{x}}_{dg}\Big) = \int 4x^2\,dx + \int 5\,\frac{dx}{x}$$

$$= 4\int x^2\,dx + 5\int \frac{dx}{x} = 4\left(\frac{x^3}{3} + c_1\right) + 5(\ln x + c_2)$$

$$= \frac{4}{3}x^3 + 4c_1 + 5\ln x + 5c_2$$

$$= 3x - \frac{x^3}{3} + 3\left(\frac{x^2}{2}\right) - \frac{x^4}{4} + c$$

$$\int \frac{x^3+5x+1}{\underbrace{x\sqrt{x}}_{x^{3/2}}}\,dx \neq \frac{\int (x^3+5x+1)\,dx}{\int x\sqrt{x}\,dx}$$

$$= \int \frac{x^3+5x+1}{x^{3/2}}\,dx = \int \left(x^{3/2} + 5x^{-\frac{1}{2}} + x^{-\frac{3}{2}}\right)dx$$

$$= \frac{x^{5/2}}{5/2} + 5\left(\frac{x^{1/2}}{1/2}\right) + \frac{x^{-1/2}}{-1/2} + c$$

$$= \frac{2}{5}x^2\sqrt{x} + 10\sqrt{x} - \frac{2}{\sqrt{x}} + c$$

HW: Compute the following integrals:

1) $\int 5\sqrt[3]{x}\,dx$ 2) $\int (4-3x+2x^2)\,dx$

3) $\int \frac{5x+7x^3}{x^2}\,dx$ 4) $\int (x^2-3)(5-2x^8)\,dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \text{for} \quad n \neq 1$$

$$\int \frac{dx}{x} = \ln x + c \qquad \int (df \pm dg) = \int df \pm \int dg$$

k constant $\Rightarrow \int k\,df = k \int df; \quad \int \frac{df}{k} = \frac{1}{k} \int df$

$$\int e^x dx = e^x + c$$

because $d(e^x) = e^x dx$

$d(e^x + c) = e^x dx + 0$

$$\int e^{5x} dx \neq e^{5x} + c = \int e^{5x} d(5x)$$

$$\int e^{5x} dx = \int \frac{1}{5} e^{5x} \cdot 5\,dx = \frac{1}{5} \int e^{5x} d(5x)$$

$$= \frac{1}{5} e^{5x} + c$$

Check: $d\left(\frac{1}{5} e^{5x} + c\right) = \frac{1}{5} e^{5x} d(5x)$

$= \frac{1}{5} e^{5x} \cdot 5\,dx = e^{5x} dx$ ✓

$$\int (x^2+1)^{50} x\,dx = \frac{1}{2}\int (x^2+1)^{50}\,d(x^2+1)$$

$$d(x^2+1) = 2x\,dx + 0$$

$$\frac{1}{2}\,d(x^2+1) = x\,dx$$

$$\int (x^2+1)^{50} x\,dx = \frac{1}{2}\cdot\frac{(x^2+1)^{51}}{51} + c$$

Check: $d\left(\frac{1}{2}\cdot\frac{(x^2+1)^{51}}{51} + c\right) = \frac{1}{2}\cdot\frac{1}{51}\cdot 51(x^2+1)^{50}\cdot d(x^2+1)$

$$= \frac{1}{2}\cdot\cancel{\frac{1}{51}\cdot 51}\,(x^2+1)^{50}(2x\,dx\cancel{+0})$$

$$= (x^2+1)^{50}\,dx \checkmark$$

$$\int \frac{5x+10}{x^2+4x}\,dx \qquad \int \frac{dx}{x} = \ln x + c$$

$$d(x^2+4x) = 2x+4$$

$$\frac{5}{2}\,d(x^2+4x) = 5x+10$$

$$\int \frac{5x+10}{x^2+4x}\,dx = \frac{5}{2}\int \frac{d(x^2+4x)}{x^2+4x}$$

$$= \frac{5}{2}\ln(x^2+4x) + c \quad \text{only when } x^2+4x>0$$

$\frac{dx}{x}$ makes sense when $x>0$ or $x<0$ (not when $x=0$)

$\ln x$ only makes sense when $x>0$

$\int \frac{dx}{x} = \ln x$ only for when $x>0$

$x<0 \Rightarrow -x>0 \Rightarrow \ln(-x)$ makes sense

$$d(\ln(-x)) = \frac{d(-x)}{-x} = \frac{d(-1x)}{-x} = \frac{-1\,dx}{\cancel{-1}x}$$

$$= \frac{dx}{x}$$

when $x<0$, $\int \frac{dx}{x} = \ln(-x) + c.$

$$|x| = \begin{cases} x : & x \geq 0 \\ -x : & x<0 \end{cases}$$

"absolute value"

$\int \frac{dx}{x} = \ln|x| + c$ for all $x \neq 0$

$\int \frac{5x+10}{x^2+4x}\,dx = \frac{5}{2}\ln|x^2+4x| + c$

$\int \frac{x^3}{x^4+5}\,dx = \int \frac{x^3 dx}{x^4+5} = \int \frac{\frac{1}{4}d(x^4+5)}{x^4+5} = \frac{1}{4}\int \frac{d(x^4+5)}{x^4+5}$

$d(x^4+5) = 4x^3\,dx + 0$

$\frac{1}{4}d(x^4+5) = x^3\,dx$

$= \frac{1}{4}\ln|x^4+5| + c$

$\int \frac{e^{3x}\,dx}{(e^{3x}+2)^4} = \int (e^{3x}+2)^{-4} e^{3x}\,dx$

$d(e^{3x}+2) = e^{3x}\,d(3x) + 0 = 3e^{3x}\,dx$

$\frac{1}{3}d(e^{3x}+2) = e^{3x}\,dx$ $\quad \int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int \frac{e^{3x}\,dx}{(e^{3x}+2)^4} = \frac{1}{3}\int (e^{3x}+2)^{-4}\,d(e^{3x}+2)$

$= \frac{1}{3}\left[(e^{3x}+2)^{-3}/(-3)\right] + c = \frac{-1}{9(e^{3x}+2)^3} + c$

HW: Compute the following integrals:

5) $\int e^{-x^2} x \, dx$ 6) $\int \left[(\sqrt{x}+3)^5 / \sqrt{x} \right] dx$

7) $\int \frac{10x^4 - 2}{x^5 - x + 1} \, dx$ 8) $\int \frac{(e^x + 1)\,dx}{(e^x + x)^4}$

Summarizing rules again:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq 1)$$

$$\int x^{-1} dx = \int \frac{dx}{x} = \ln|x| + c$$

$$\int e^x \, dx = e^x + c$$

$$\int (df \pm dg) = \int df \pm \int dg$$

$$\int k \, df = k \int df$$

(n & k must be constant above)

(c is an arbitrary constant)