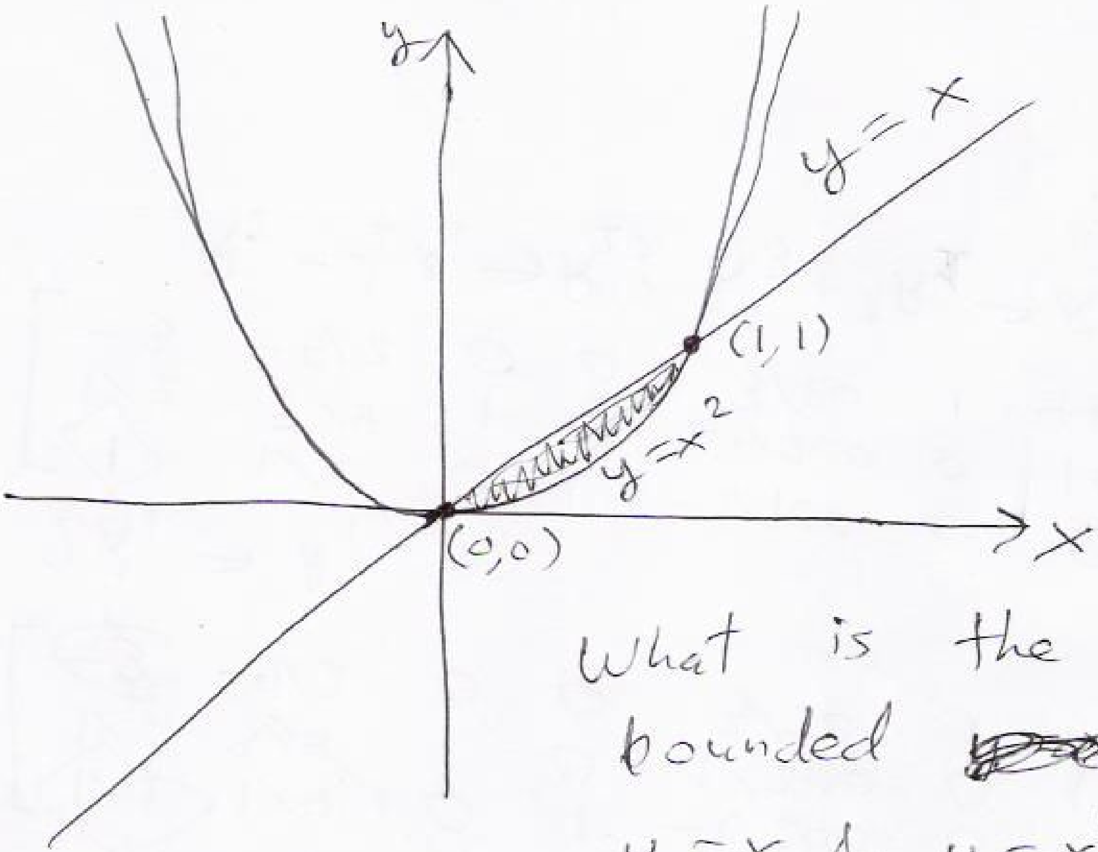
Today: Areas between curves (14-1)

Nov. 3 (Thur) Optional:
re-take Test II
(questions will differ;
set of possible topics will be same)

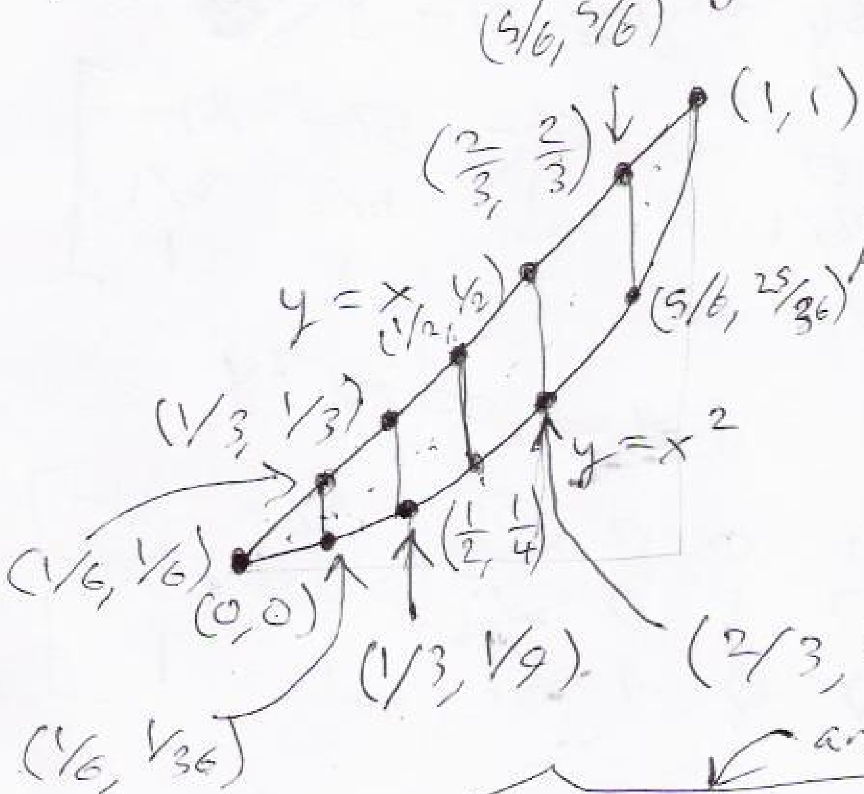
Review session Nov. 1 or Nov. 2.
(Email me favorite times.)

If you retake Test II;
I'll forget whichever score,
original or re-take was lower.

I encourgage students (you) to
come to my office or email me
with questions.

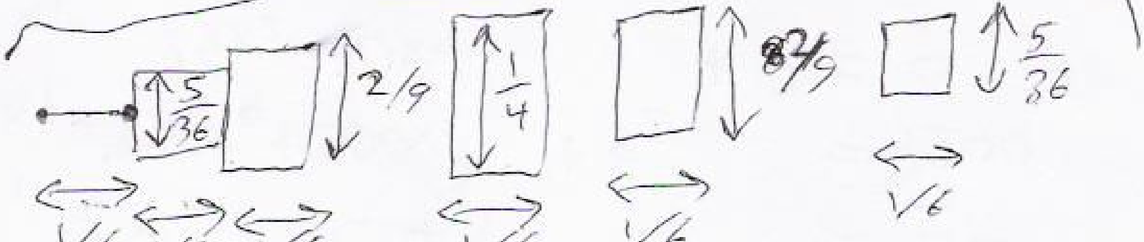


What is the area bounded by $y = x$ & $y = x^2$?



$$A = \Delta A_1 + \Delta A_2 + \Delta A_3 + \Delta A_4 + \Delta A_5 + \Delta A_6$$

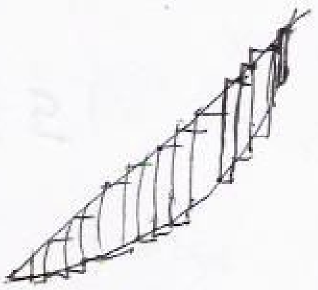
area $\approx \frac{35}{216} \approx 0.162037\ldots$



each $\Delta A \approx h \, \Delta x = (x - x^2)\Delta x$

height 1/6 = width

The smaller Δx, the better the estimate.



$$dA = (x - x^2)\,dx$$

$$dA = x\,dx - x^2\,dx$$

$$\int dA = \int x\,dx - \int x^2\,dx$$

$$d\left(\tfrac{1}{2}x^2\right) = \tfrac{1}{\cancel{2}}(\cancel{2}x\,dx)$$

$$d\left(\tfrac{1}{3}x^3\right) = \tfrac{1}{\cancel{3}}\cancel{3}x^2\,dx$$

$$A = \int dA = \frac{1}{2}x^2 - \frac{1}{3}x^3 + c$$

We want total ΔA from $A = 0$ when $x = 0$ to $A = ?$ when $x = 1$.

$$\Delta A = \Delta\left(\frac{1}{2}x^2 - \frac{1}{3}x^3 + c\right)$$

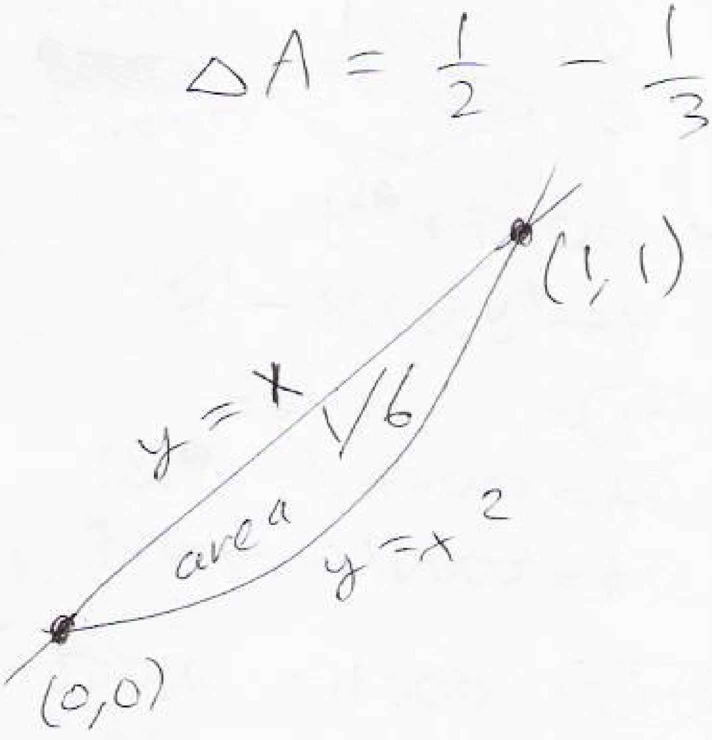
from $x = 0$ to $x = 1$

$$\Delta A = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3 + c\right)\Big|_0^1$$

$$\Delta A = \left[\frac{1}{2}(1^2) - \frac{1}{3}(1^3) + c\right] - \left[\frac{1}{2}(0^2) - \frac{1}{3}(0^3) + c\right]$$

$$\Delta A = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} = 0.1666666\ldots$$

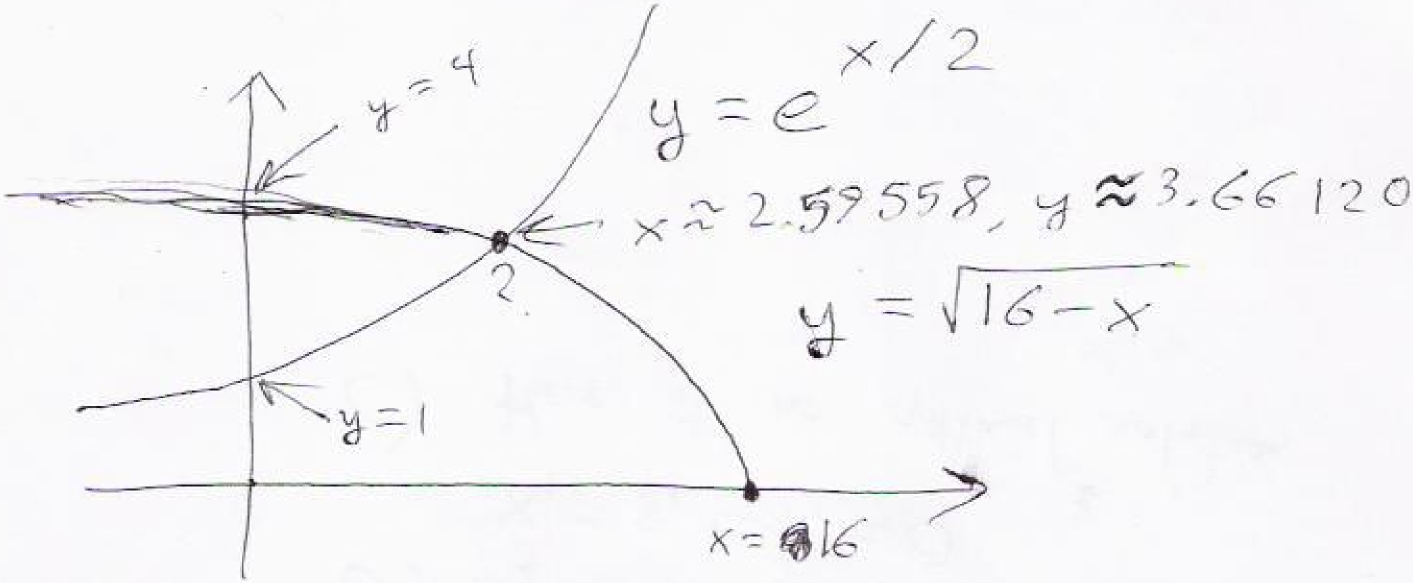
$= 36/216$



Definite integral (notation):

$$\int_0^1 (x - x^2)\,dx = \text{area}$$

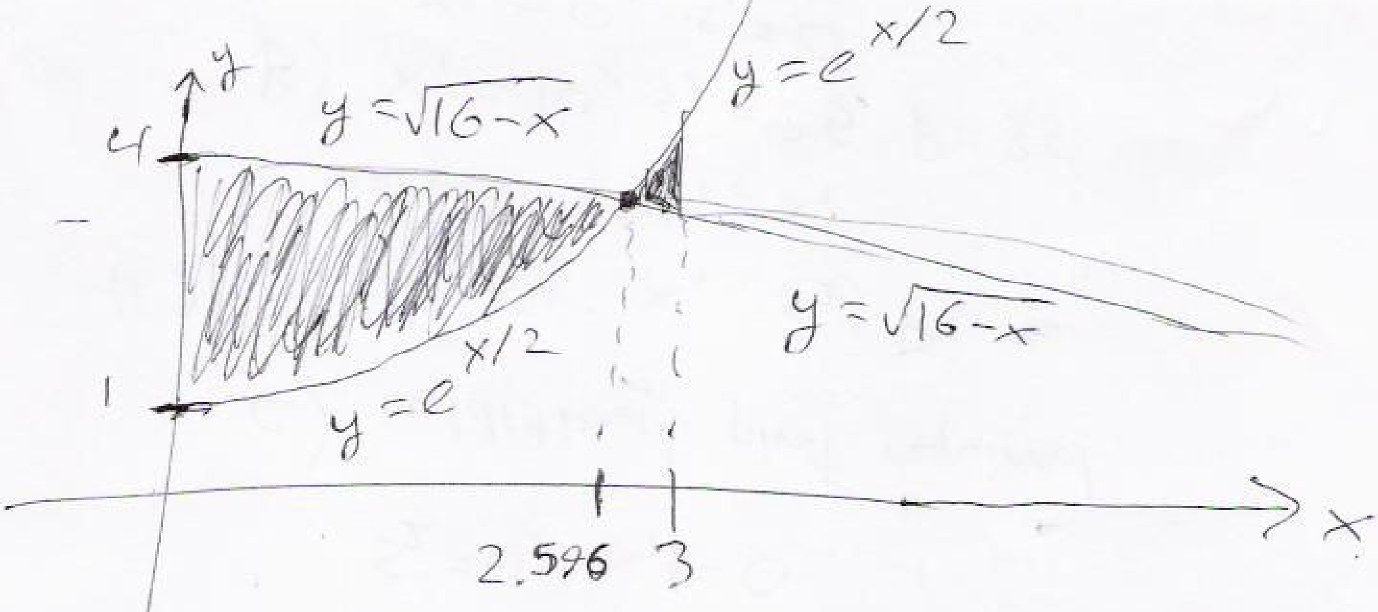
$$= \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \left[\frac{1}{2}(1^2) - \frac{1}{3}(1^3)\right] - \left[\frac{1}{2}(0^2) - \frac{1}{3}(0^3)\right] = 1/6$$



What is the area between

$y=e^{x/2}$ & $y=\sqrt{16-x}$

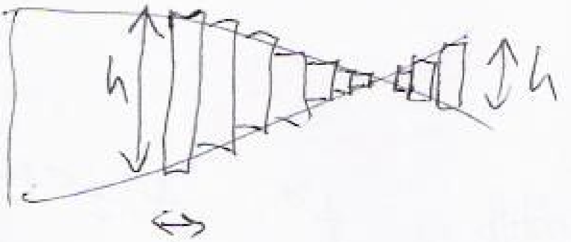
from $x=0$ to $x=3$?



height = top y − bottom y

$$A = A_{left} + A_{right}$$

$$= \int_0^{2.596} h \, dx + \int_{2.596}^{3} h \, dx$$



$dx = \Delta x$

$$A = \int_0^{2.596} \left(\sqrt{16-x} - e^{x/2}\right) dx + \int_{2.596}^{3} \left(e^{x/2} - \sqrt{16-x}\right) dx$$

$$\int \sqrt{16-x} \, dx = \int (16-x)^{1/2} \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad \int x^{1/2} \, dx = \frac{x^{3/2}}{3/2} + c$$

$$d(16-x) = 0 - dx = -dx$$

$$-d(16-x) = dx$$

$$\int \sqrt{16-x} \, dx = \int (16-x)^{1/2} \left(-d(16-x)\right)$$

x	0	2.596	3
$-\frac{2}{3}(16-x)^{3/2}$	$-(42+\frac{2}{3})$	-32.716	-31.248
$2e^{x/2}$	2	7.324	8.963

$$A = [-32.716-(-42.666)] - [7.324-2]$$
$$+ [8.963-7.324] - [-31.248-(-32.716)]$$
$$= 4.797$$

HW: (14-1) #13, 19, 25, 70