Today: integration by parts (14-3)

Thursday: • review session in class

• I'll return optional tests "2B"

~~Next: Nov~~

Tuesday: Test III (Ch. 13 & 14)

↑ Nov. 15

$$d(x^n) = nx^{n-1}dx \longrightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$d(e^x) = e^x dx \longrightarrow \int e^x dx = e^x + c$$

$$d(\ln|x|) = \frac{dx}{x} \longrightarrow \int \frac{dx}{x} = \ln|x| + c$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\int \ln x \, dx = ?$$

use "integration by parts"

$$d(fg) = df \cdot g + f \cdot dg = g \cdot df + f \cdot dg$$

$$fg = \int df \cdot g + \int f \cdot dg = \int g \, df + \int f \, dg$$

$$f \cdot g - \int g\,df = \int f\,dg$$

$$\boxed{\int f\,dg = fg - \int g\,df}$$ I.B.P.

$$\int \underbrace{\ln x}_{f}\,\underbrace{dx}_{dg} = (\ln x)x - \int x\left(\frac{dx}{x}\right)$$

$$df = \frac{dx}{x} \qquad g = x$$

$$\int \ln x\,dx = (\ln x)x - \int dx$$

$$\int \ln x\,dx = x\ln x - x + c$$

Check:

$$d(x\ln x - x + c)$$

$$= (dx)\ln x + x\,d(\ln x) - dx + 0$$

$$= \ln x\,dx + x\,dx/x - dx$$

$$= \ln x\,dx + dx - dx$$

$$= \ln x\,dx \quad \checkmark$$

like ^{14}C

A radioactive isotope is decaying with a half life of 5700 years. What is the average time it takes a particle of this isotope to decay, among the particles in the first 20000 years?

X = amount at time t

X_0 = amount at time 0

$X = X_0 e^{-kt}$ (exponential decay)

$\frac{1}{2} X_0 = X_0 e^{-k(5700)}$

$\frac{1}{2} = e^{-5700k}$

$2 = e^{5700k}$

$\ln 2 = 5700k$

$(\ln 2)/5700 = k \approx 1.216 \cdot 10^{-4} = 0.0001216$

average decay time

$$= \frac{\text{total decay times of particles}}{\text{number of particles}}$$

$$= \frac{\text{total of (decay times of little chunks } dX) \cdot (-dX)}{\text{total amount decayed}}$$

$$= \frac{\int_{t=0}^{t=20000} t(-dX)}{X_0 - X_0 e^{-k \cdot 20000}}$$

$$dX = d(X_0 e^{-kt}) = X_0 e^{-kt} d(-kt)$$

X_0, k constant

$$dX = \underbrace{-kX_0 e^{-kt} dt}_{\text{negative}}$$

We want $\underbrace{-dX = kX_0 e^{-kt} dt}_{\text{amount that decays}}$

avg. decay time is:

$$\frac{\int_0^{200000} t k X_0 e^{-kt}\, dt}{X_0\left(1 - e^{-20000k}\right)}$$

$$= \frac{k X_0 \int_0^{20000} t e^{-kt}\, dt}{X_0\left(1 - e^{-20000k}\right)}$$

$$= \frac{k \int_0^{20000} t e^{-kt}\, dt}{1 - e^{-20000k}} \approx \boxed{6,297 \text{ years.}}$$

$k \approx .0001216\ldots$

$$1 - e^{\overbrace{-20000k}^{-2.43\ldots}} \approx 0.912147\ldots$$

see calculations that follow

$$\int_0^{20000} t e^{-kt}\, dt = ?$$

$$\int_a^b f\, dg = fg\Big|_a^b - \int_a^b g\, df$$

$$\int \underbrace{e^{-kt}}_{f} \underbrace{t\,dt}_{dg} = e^{-kt}\, t^2/2 - \int \frac{t^2}{2}(-ke^{-kt})dt$$

$$= \frac{t^2}{2} e^{-kt} + (k/2)\int t^2 e^{-kt} dt$$

$$g = \frac{t^2}{2} \left(= \int t' dt\right)$$

Worse.

$$df = d(e^{-kt}) = e^{-kt} d(-kt)$$

$$df = -ke^{-kt} dt$$

$$\int \underbrace{t}_{f} \underbrace{e^{-kt} dt}_{dg} = t\left(-\frac{1}{k}e^{-kt}\right) - \int\left(-\frac{1}{k}e^{-kt}\right) dt$$

$$dg \rightarrow g = \int e^{-kt} dt$$

$$f \downarrow$$

$$df = dt$$

$$\int e^{x} dx = e^{x} + c$$

$$d(-kt) = -k\,dt$$

$$-\frac{1}{k} d(-kt) = dt$$

$$g = \int e^{-kt}\left(-\frac{1}{k} d(-kt)\right)$$

$$g = -\frac{1}{k}\int e^{-kt} d(-kt)$$

$$g = -\frac{1}{k} e^{-kt}$$

$$\int te^{-kt}\,dt = -\frac{t}{k}e^{-kt}$$

$$+\frac{1}{k}\underbrace{\int \underbrace{e^{-kt}\,dt}}_{dg}$$

$$g + c$$

$$\int te^{-kt}\,dt = -\frac{t}{k}e^{-kt} + \frac{1}{k}\left(-\frac{1}{k}e^{-kt}\right) + c$$

$$\int_0^{20000} te^{-kt}\,dt = -\frac{t}{k}e^{-kt}\Big|_0^{20000}$$

$$+ -\frac{1}{k^2}e^{-kt}\Big|_0^{20000}$$

$$= \frac{-1}{k}\left(t + \frac{1}{k}\right)e^{-kt}\Big|_0^{20000}$$

$$= \left(-2.0389\ldots \cdot 10^7\right)$$

$$- \left(-6.7623\ldots \cdot 10^7\right)$$

$$\int_0^{20000} te^{-kt}\,dt = 4.7233\ldots \cdot 10^7$$

HW (14-3) #65, 67, 71

$$I = \int_1^e \frac{\ln x \, dx}{x} = ?$$

$$I = \int_1^e \frac{1}{x} \ln x \, dx$$

$$\int f \, dg = fg - \int g \, df$$

$$\int \underbrace{\ln x}_{f} \underbrace{\frac{dx}{x}}_{dg} = (\ln x)(\ln x) - \int \ln x \frac{dx}{x}$$

$$dg \longrightarrow g = \int \frac{dx}{x} = \ln|x| = \ln x \text{ when } x > 0$$

$$f \downarrow \quad df = dx/x$$

$$2 \int \ln x \frac{dx}{x} = \ln^2 x$$

$$\int \ln x \frac{dx}{x} = \frac{1}{2} \ln^2 x + c$$

$$I = \frac{1}{2} \ln^2 x \Big|_1^e = \frac{1}{2}\Big(\underbrace{(\ln e)^2}_{1^2} - \underbrace{(\ln 1)^2}_{0^2}\Big) = \boxed{\frac{1}{2}}$$