Focus on understanding what I'm saying and doing. If you must take notes, then focus on recording what I say but don't write down.

Classroom expectations. Bring your calculator, book, paper, and pen/pencil. I'm not just going to talk for 50 minutes. I will ask the class questions, and sometimes I will ask you to do a short computation. Pay attention to what I say, not just what I write.

I encourage questions. Just ask; you don't need to raise your hand. However, if you do not have a question, then do not interfere with your fellow students' learning. Do not talk to other students while I am talking. Do not let your phone/laptop/etc. distract your neighbors.

Approximate Schedule of Topics

Date	Day	Topic
25-Aug	R	syllabus; derivatives: approximating small changes
30-Aug	T	product rule; derivatives of polynomials
1-Sep	R	derivatives as slopes
6-Sep	T	higher derivatives and concavity
8-Sep	R	maxima and minima
13-Sep	T	curve sketching
15-Sep	R	Exam I
20-Sep	T	optimization
22-Sep	R	optimization
27-Sep	T	derivatives of logarithms and exponentials
29-Sep	R	chain rule
4-Oct	T	chain rule
6-Oct	R	related rates
11-Oct	T	related rates
13-Oct	R	Exam II
18-Oct	T	from sums to integrals (video lecture)
20-Oct	R	(fall break)
25-Oct	T	integrals of polynomials
27-Oct	R	area between polynomial curves
1-Nov	T	consumer surplus
3-Nov	R	deadweight loss
8-Nov	T	integration by substitution
10-Nov	R	integration by parts
15-Nov	T	Exam III
17-Nov	R	partial derivatives
22-Nov	T	partial derivatives
24-Nov	R	(Thanksgiving break)
29-Nov	T	two-variable maxima and minima
1-Dec	R	iterated integration
6-Dec	T	volumes of solids
8-Dec	R	Final Exam, 11AM

$\begin{pmatrix}13\text{-}1\\13\text{-}2\end{pmatrix}$ Antiderivatives $\int df = f + c$

$\begin{pmatrix}13\text{-}4\\13\text{-}5\end{pmatrix}$ Definite integrals $\int_{a=x}^{b=x} df = \overbrace{f(b) - f(a)}^{f|_a^b}$

(14-1) Areas between curves

(14-2) CS, PS, $\boxed{\text{DWL}}$ ⟸ see lecture notes

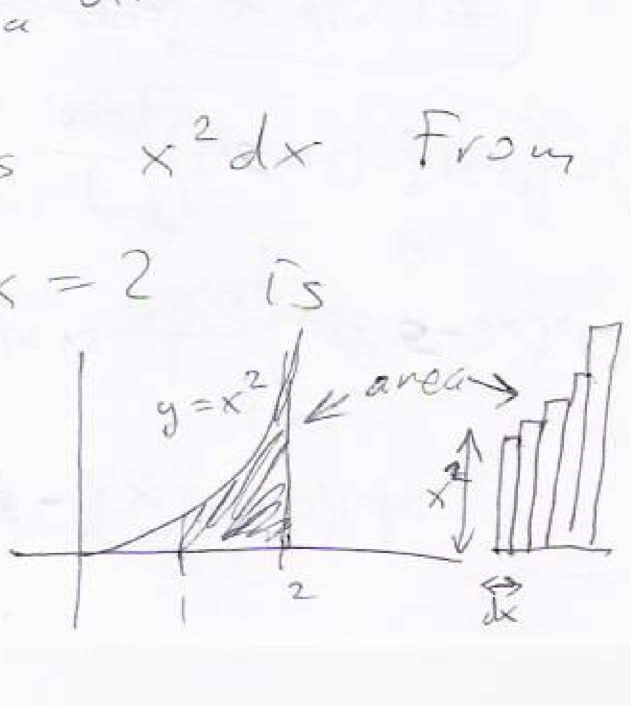
(14-3) Integration by parts

More Applications: Averages / Totals (Story problems from 13-5, 14-3)

Total of all the df's from $x=a$ to $x=b$ is $\int_a^b df$

Total areas $x^2 dx$ from $x=1$ to $x=2$ is

$$\int_1^2 x^2 dx = \text{area}$$



Total changes:

If $y = x^2$, then the total change Δy from $x=1$ to $x=2$ is $2^2 - 1^2$ (easy)

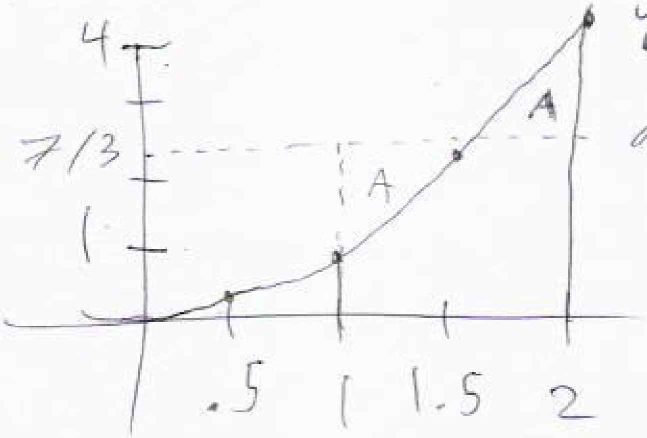
If $dy = x^2 dx$, then the total change of y from $x=1$ to $x=2$ is $\int_{x=1}^{x=2} dy = \int_1^2 x^2 dx$

[Compare to #65 (14-3)
$dS = 350 \ln(t+1)\, dt$]

If $y = 5$ when $x = 1$ and $dy = x^2 dx$, then $y = 5 + \int_1^2 x^2 dx$ when $x = 2$

Averages: average value of x^2 from $x=1$ to $x=2$ is $\dfrac{\int_1^2 x^2 dx}{\int_1^2 dx} = \dfrac{\int_1^2 x^2 dx}{2-1}$

[Compare to #71 (14-3)]



$y = x^2$

average height y from $x=1$ to $x=2$

$$\boxed{\int e^x\,dx = e^x + c}$$

$$\int e^{x^2}\underbrace{x\,dx} = \frac{1}{2}\int e^{x^2}\,d(x^2) = \frac{1}{2}e^{x^2} + c$$

$$x\,dx = \frac{1}{2}\,d(x^2)$$

$$d(x^2) = 2x\,dx$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c} \quad (\text{if } n \neq -1)$$

$$\int \frac{x\,dx}{(x^2+1)^3} = \frac{1}{2}\int \frac{d(x^2+1)}{(x^2+1)^3} = \underbrace{\frac{1}{2}\int (x^2+1)^{-3}\,d(x^2+1)}$$

$$d(x^2+1) = 2x\,dx \qquad \hookrightarrow = \frac{1}{2}\cdot\frac{(x^2+1)^{-2}}{-2} + c$$

$$\frac{1}{2}\,d(x^2+1) = x\,dx$$

$$\boxed{\int \frac{dx}{x} = \ln|x| + c} = \begin{cases} \ln x + c & \text{if } x>0 \\ \ln(-x) + c & \text{if } x<0 \end{cases}$$

$$\int \frac{x\,dx}{x^2+1} = \frac{1}{2}\int \frac{d(x^2+1)}{x^2+1} = \frac{1}{2}\ln|x^2+1| + c$$

$$\boxed{k \text{ constant} \Rightarrow \int k\,df = k\int df}$$

$$\int 5x\,dx = 5\int x\,dx = 5\left(\frac{x^2}{2}\right) + c$$

$$\boxed{\int (df \pm dg) = \int df \pm \int dg}$$

$$\int (x - x^2 + x^3)\,dx = \int x\,dx - \int x^2 dx + \int x^3 dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} + c$$

$$\boxed{\int f\,dg = fg - \int g\,df} \qquad \text{(I.B.P.)}$$

$$\int \underbrace{x}_{f} \underbrace{e^x dx}_{dg} = \underbrace{x}_{f} \underbrace{e^x}_{g} - \int \underbrace{e^x}_{g} \underbrace{dx}_{df} = \underbrace{xe^x - e^x + c}_{(x-1)e^x + c}$$

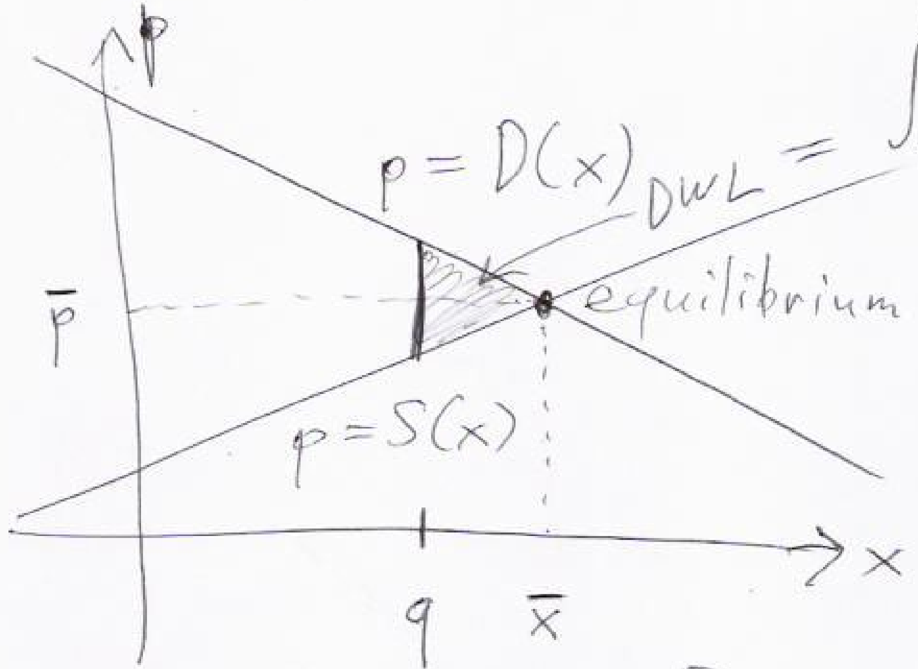
$df = dx \quad g = \int dg = \int e^x dx = e^x$ (pick $c = 0$)

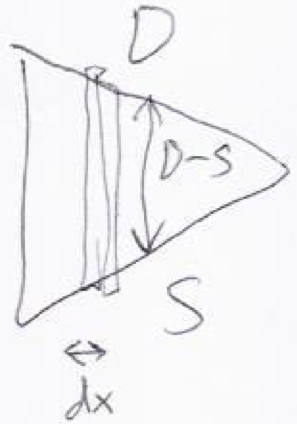
$$\int \underbrace{e^x}_{f} \underbrace{x\,dx}_{dg} \quad \text{won't work} \ldots$$

$$= fg - \int g\,df = e^x x^2/2 - \int \frac{x^2}{2} e^x dx$$

Deadweight loss:

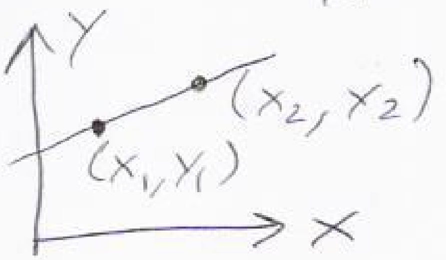
$$DWL = \int_{x=q}^{x=\bar{x}} (D - S)\,dx$$





Simplest case: linear demand, linear supply

Given two data points for each, you can find the line for each



$$y - y_1 = m(x - x_1)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Equilibrium: Solve $D(x) = S(x)$ for x to get equilibrium quantity $\bar{x}$.
Plug in $x = \bar{x}$ to get $\bar{p} = D(\bar{x}) = S(\bar{x})$.

If there's a tax, say, 4%, then solve $D(x) = 1.04\, S(x)$ to get after-tax equilibrium quantity q.

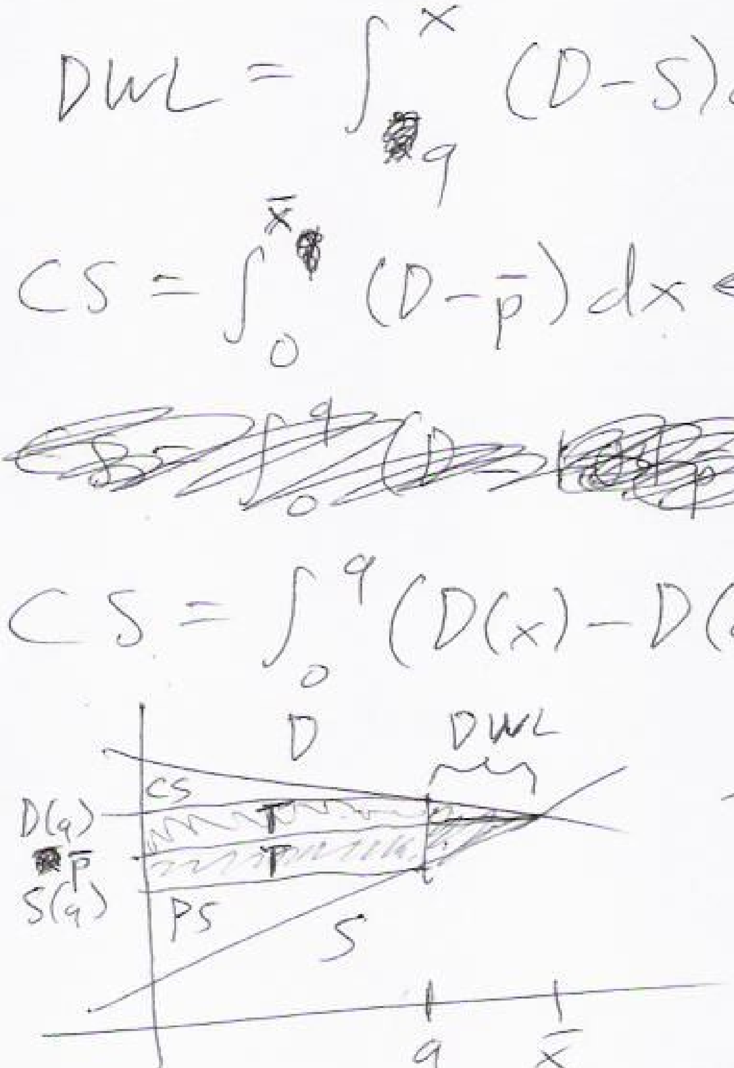
$$DWL = \int_q^{\bar{x}} (D - S)\, dx$$

$$CS = \int_0^{\bar{x}} (D - \bar{p})\, dx \leftarrow \text{pre-tax}$$

$$PS = \int_0^{\bar{x}} (\bar{p} - S)\, dx$$

$$CS = \int_0^q (D(x) - D(q))\, dx \leftarrow \text{post-tax}$$

$$PS = \int_0^q (S(q) - S(x))\, dx$$



Tax revenue

$$(D(q) - S(q))\bar{p} = 0.04\bar{p}\, S(q)$$

(13-5 #82)

$\frac{dA}{dt} = A'(t) = -0.9e^{-0.1t}$

$dA = A'(t)\,dt$

$\underbrace{dA}_{\text{small changes in area}} = -0.9e^{-0.1t}\underbrace{dt}_{\text{small changes in time}}$

Find total change ΔA
from $t=0$ to $t=5$

$\Delta A = \int_{t=0}^{t=5} dA = \int_0^5 -0.9e^{-0.1t}\,dt$

$\Delta A = (-.9)\cdot\int_0^5 e^{-0.1t}\,dt$

$-.1\,dt = d(-0.1t)$ $\qquad \int e^x\,dx = e^x + c$

$dt = -10\,d(-0.1t)$

$\Delta A = (-.9)\int_0^5 e^{-0.1t}(-10\,d(-.1t))$

$\Delta A = 9\int_0^5 e^{-.1t}\,d(-.1t)$

$$\Delta A = 9\left(e^{-.1t}\right)\Big|_0^5 = 9(\underbrace{e^{-.5}}_{\frac{1}{\sqrt{e}}} - \underbrace{e^0}_{1})$$

$$\simeq -3.54 \text{ cm}^2$$

For the second question,

$$\Delta A = \int_{5=t}^{10=t} dA = 9\left(e^{-.1t}\right)\Big|_5^{10}$$

$$= 9(e^{-1} - e^{-.5})$$

$$\simeq -2.15 \text{ cm}^2$$