Today: Partial Derivatives
(15 - 2)

Tuesday: Exams returned

4 more class days after today

Final exam Dec. 8, 11AM, here.

$$z = x^2 + 5xy + 2y^2$$

$$dz = 2x\,dx + 5\,d(xy) + 2(2y\,dy)$$

$$dz = 2x\,dx + 5((dx)\cdot y + x\cdot dy) + 4y\,dy$$

$$dz = \underbrace{(2x+5y)}_{\frac{\partial z}{\partial x}}dx + \underbrace{(5x+4y)}_{\frac{\partial z}{\partial y}}dy$$

Partial derivatives

For small Δx, Δy, setting $dx = \Delta x$, $dy = \Delta y$

$$\Delta z \approx dz = (2x+5y)\,dx + (5x+4y)\,dy$$

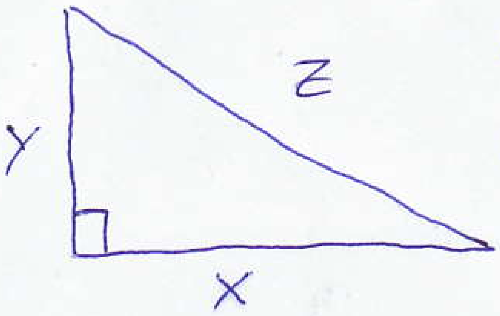
x	y	z	$dx = \Delta x$	$dy = \Delta y$	Δz	$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}\underbrace{dy}_{0}$
1	1	8				
1.1	1	8.71	.1	0	.71	.7

$$dz = (2(1)+5(1))(.1)+(5(1)+4(1))(0)$$

x	y	z	$dx=\Delta x$	$dy=\Delta y$	Δz	$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$
1	1	8				
1.2	0.9	8.46	.2	−.1	.46	.5

$$dz = (2(1)+5(1))(.2)+(5(1)+4(1))(-.1)$$

HW. #1



$$z = \sqrt{x^2+y^2}$$
$$z = (x^2+y^2)^{1/2}$$

$$x=3,\ y=4 \Rightarrow z=\sqrt{9+16}=\sqrt{25}=5$$

If x changes to 3.15 and y changes to 4.07, use differentials to estimate the new z.

#2 If $z = e^{-(x^2+y^2)/2}$,

find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$.

Application from economics:

Cobb-Douglas Functions.

2 inputs: L = labor in man-hours

K = capital (equipment, land, fuel, ...)

1 output: Y = production in \$

Model: $Y = AK^{\alpha}L^{\beta}$

A, α, β constants

A = total factor productivity

α = ~~marginal elasticity of~~ capital-elasticity of production

β = labor-elasticity of production.

$y = f(x) \Rightarrow$ x-elasticity of y is

$$\frac{dy/y}{dx/x}.$$

$$\alpha = \frac{dY/Y}{dK/K} \quad \text{when } dL = 0$$

$$\beta = \frac{dY/Y}{dL/L} \quad \text{when } dK = 0.$$

$$dY = A\, d(K^\alpha L^\beta)$$

$$dY = A\, d(K^\alpha) L^\beta + K^\alpha d(L^\beta)$$

$$dY = A \alpha K^{\alpha-1}(dK) L^\beta + K^\alpha \beta L^{\beta-1} dL$$

$$d(x^n) = n x^{n-1} dx$$

If $dL = 0$, then

$$dY = A\alpha K^{\alpha-1}(dK)L^\beta$$

$$\frac{dY}{dK} = A\alpha K^\alpha L^\beta / K^1 = \alpha Y/K$$

$$\frac{dY/Y}{dK/K} = \alpha \quad (\text{when } dL = 0)$$

$$\frac{\Delta Y}{Y} \approx 0.6 \frac{100}{1700} + 0.4 \frac{-100}{1250}$$

$$\frac{\Delta Y}{Y} \approx 0.6(5.88\%) + 0.4(-8\%)$$

$$\frac{\Delta Y}{Y} \approx -0.26\%$$

Exact change found

~~Slightly better estimate~~

using $d(\ln x) = \frac{dx}{x}$

$$d(\ln Y) = 0.6\, d(\ln K) + 0.4\, d(\ln L)$$
$$\Delta(\ln Y) \approx 0.6\, \Delta(\ln K) + 0.4\, \Delta(\ln L)$$
$$Y = A K^{\alpha} L^{\beta} = 20 K^{0.6} L^{0.4}$$

$$\ln Y = \ln(20 K^{0.6} L^{0.4})$$
$$\ln Y = \ln 20 + \ln(K^{0.6}) + \ln(L^{0.4})$$
$$\ln Y = \ln 20 + 0.6 \ln K + 0.4 \ln L$$
$$\Delta(\ln Y) = 0.6\, \Delta(\ln K) + 0.4\, \Delta(\ln L)$$

In general, if $z = a + bx + cy$, then $\Delta z = b\Delta x + c\Delta y$.

Just like $y = mx + b \Rightarrow \Delta y = m\Delta x$.

K	$\ln K$	L	$\ln L$
1700	7.43838	1250	7.13089
1800	7.49554	1150	7.04751

$\Delta(\ln K) = 0.057158$ ~~5.7158%~~

$\Delta(\ln L) = -0.083381$

$\Delta(\ln Y) = 0.6\,\Delta(\ln K) + 0.4\,\Delta(\ln L)$

$\Delta(\ln Y) = 0.00094240$

$\ln(Y+\Delta Y) - \ln Y = \ln\left(\frac{Y+\Delta Y}{Y}\right)$

$\frac{Y+\Delta Y}{Y} = e^{0.00094240} = 1.00094284$

$\frac{Y}{Y} + \frac{\Delta Y}{Y} = 1 + \frac{\Delta Y}{Y} \Rightarrow \frac{\Delta Y}{Y} = 0.00094284$

$\Delta Y/Y = 0.094\%$

HW #3: For US economy, the model is more like

$Y = AK^{0.26}L^{0.74}$.

A constant.

~~If we set our units such that~~

~~$A=1$,~~ Use differentials to estimate $\Delta Y/Y$ if $\Delta K/K = 1\%$ and $\Delta L/L = 2\%$.

Use logorithms to compute $\Delta Y/Y$ exactly.

Hint: $$\Delta(\ln x) = \ln(x+\Delta x) - \ln x$$
$$= \ln\left(\frac{x+\Delta x}{x}\right)$$
$$= \ln\left(1+\frac{\Delta x}{x}\right)$$