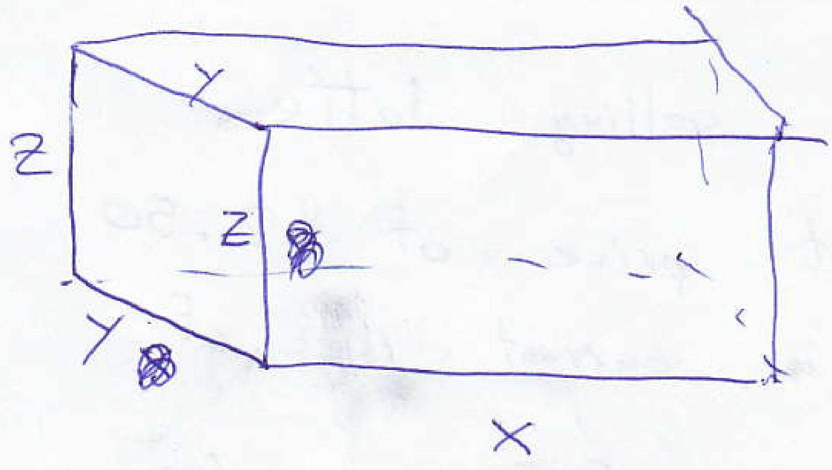
2-variable optimization (15-3)



$$108 \geq \text{girth} + \text{length} = 2y + 2z + x$$

Maximize $V = xyz$

$$108 = 2y + 2z + x$$

$$108 - 2y - 2z = x$$

$$V = (108 - 2y - 2z)yz$$

To find max(V), solve

$$\frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z} \quad \text{for } y, z.$$

$$dV = \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

Solve $\begin{cases} 108z - 4yz - 2z^2 = 0 \\ 108y - 2y^2 - 4yz = 0 \end{cases}$

$$\to 108z - 2z^2 = 4yz$$

$$27 - z/2 = y$$

$$108\left(27 - \frac{z}{2}\right) - 2\underbrace{\left(27 - \frac{z}{2}\right)^2}_{27^2 - 2(27)(z/2) + \left(\frac{z}{2}\right)^2}$$

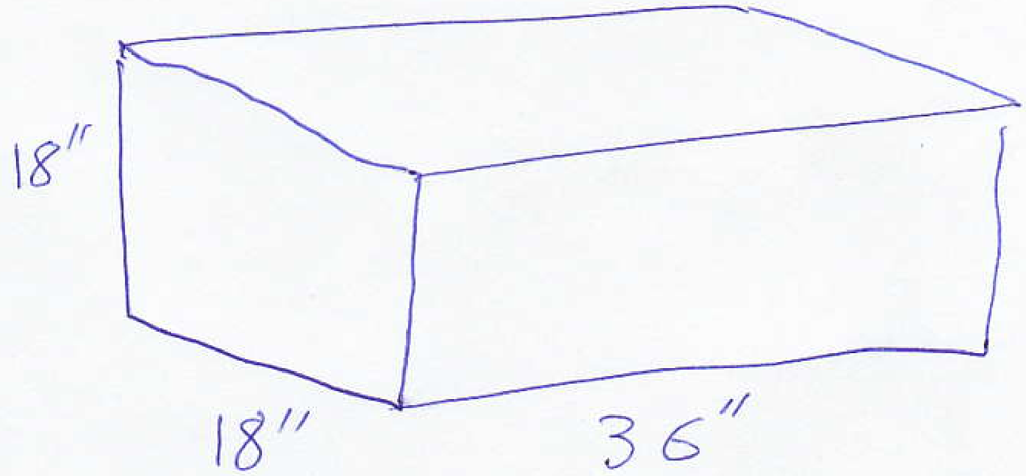
$$-4\left(27 - \frac{z}{2}\right)z = 0$$

$$\left[\begin{array}{l} 108 \cdot 27 - 2 \cdot 27^2 \cancel{-4 \cdot 27} \\ -54z + 54z - 108z \\ -\frac{z^2}{2} + 2z^2 \end{array}\right]$$

$$\to = \underbrace{\overset{1458}{\cancel{1330}}}_{c} \underbrace{-108}_{b} z + \underbrace{\frac{3}{2}}_{a} z^2$$

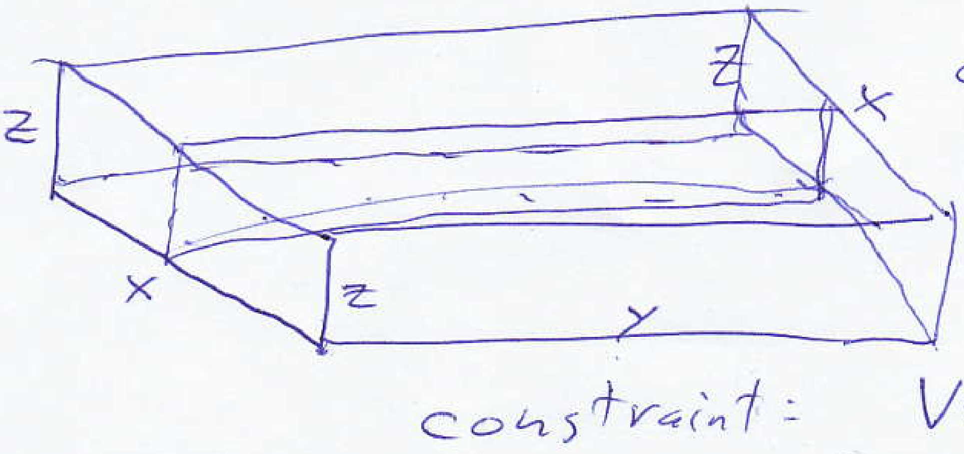
$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 18, 54$$

z	$y = 27 - \frac{z}{2}$	$x = 108 - 2y - 2z$
18	18	36
54	0 throw out	0



$\max(V) = 11{,}664 \text{ in}^3 = 6.75 \text{ ft}^3$

Example 4 page 820



open top & partition

constraint: $V = xyz = 48$

Minimize area $= 3yz + 2xz + xy = A$

$$V = 108yz - 2x^2z - 2yz^2$$

$$dV = 108\,d(yz) - 2\,d(y^2z) - 2\,d(yz^2)$$

$$d(yz) = dy \cdot z + y\,dz$$

$$d(y^2z) = \underbrace{d(y^2)}_{2y\,dy} \cdot z + y^2\,dz$$

$$d(yz^2) = dy \cdot z^2 + y \cdot \underbrace{d(z^2)}_{2z\,dz}$$

$$dV = 108[dy \cdot z + y \cdot dz]$$

$$-2[2y\,dy \cdot z + y^2\,dz]$$

$$-2[dy \cdot z^2 + y \cdot 2z\,dz]$$

$$dV = \underbrace{(108z - 4yz - 2z^2)}_{\partial V/\partial y}\,dy$$

$$+ \underbrace{(108y - 2y^2 - 4yz)}_{\partial V/\partial z}\,dz$$

$$z = \frac{48}{xy} \Rightarrow A = \frac{144}{x} + \frac{96}{y} + xy$$

Find x, y where $\frac{\partial A}{\partial x} = \frac{\partial A}{\partial y} = 0$.

Find $\min(A)$

$$\min(A) = 72 \Leftarrow \begin{cases} x = 6 \\ y = 4 \\ z = 2 \end{cases}$$

HW #31, 33, 35 (15-3)

Maximizing profit with two goods.

x = quantity of good A sold
y = quantity of good B sold
p = price at which good A sold
q = price at which good B sold

Constraints: $\boxed{x = f(p,q)}$ (demand for A)
$\boxed{y = g(p,q)}$

R = Revenue $= px + qy = pf(p,q) + qg(p,q)$
$C = h(p,q)$ = cost P = profit $= R - C$

$\text{Cost} = 60x + 80y =$

plug in

$x = f(p,q) = 260 - 3p + q$

$y = g(p,q) = 180 + p - 2q$