You may turn in HW tomorrow before 5PM tomorrow @ LBV 321.

But you still need to sign in today.

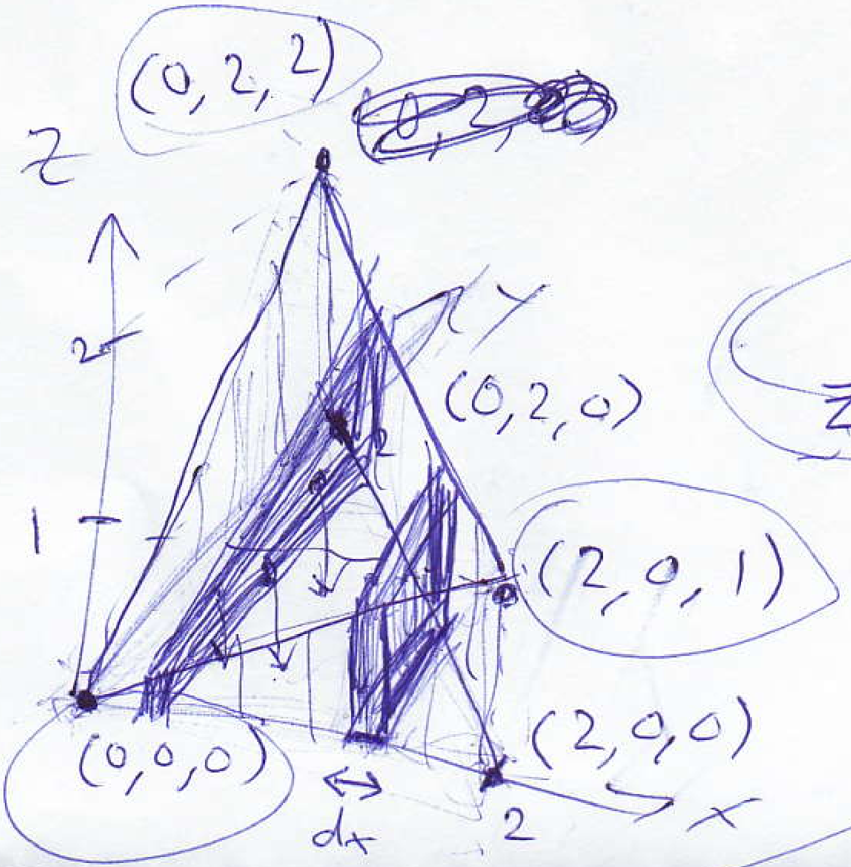
Topics since last test

$\approx$ half of final

Sections 15-(2, 3, 6, 7) + notes from today & 12/2 on planes

Older topics $\approx$ other half (Ch. 10-15)

Final is 11AM–2PM, here, Thursday, 12/8.
Bring calculator & 2 sheets (4 pages) of notes.



top side

$$z = \frac{1}{2}x + y$$

Last time: HW #1 Find an equation for the plane through those 3 points: (0,0,0), (0,2,2), (2,0,1).

#2 Find the volume of the region under that triangle and above the xy plane.

Hint: either use pyramid geometry or slice it (similar to example 7, 15-7, p. 860)

For a plane you have 2 slopes:

$$\Delta z = a \Delta x + b \Delta y$$

a & b are the same for all pairs of points on a plane

For a line, $\Delta y = m \Delta x$

m same for all pairs of points on a line.

Our plane (the top face):

3 points given:

(x, y, z)
$(0, 0, 0)$
$(0, 2, 2)$
$(2, 0, 1)$

start	end	Δx	Δy	Δz
0,0,0	0,2,2	0	2	2
0,0,0	2,0,1	2	0	1

$$2 = a \cdot 0 + b \cdot 2$$

$$\Delta z = a \Delta x + b \Delta y$$

$$1 = a \cdot 2 + b \cdot 0$$

$$\begin{cases} 2 = 2b \\ 1 = 2a \end{cases} \Rightarrow \begin{cases} b = 1 \\ a = 1/2 \end{cases}$$

$$\Delta z = \frac{1}{2} \Delta x + 1 \Delta y.$$

start = (0,0,0) end = (x, y, z) any point in plane

$$z - 0 = \frac{1}{2}(x-0) + 1(y-0)$$

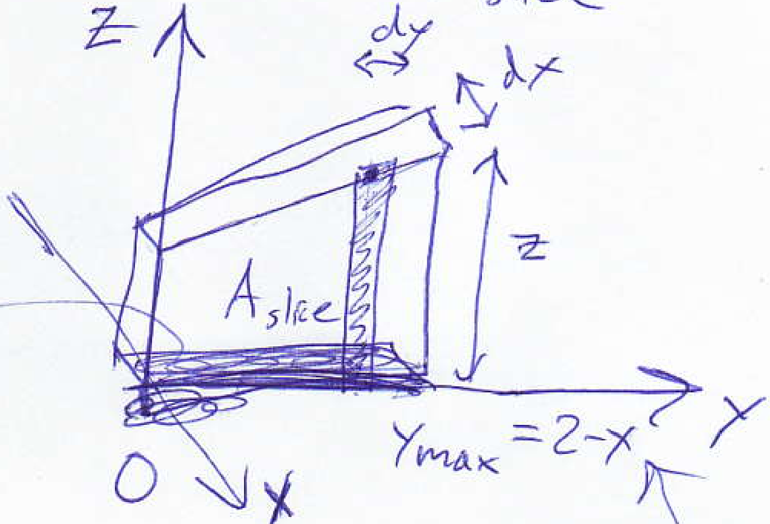
$$z = \frac{1}{2} x + y$$

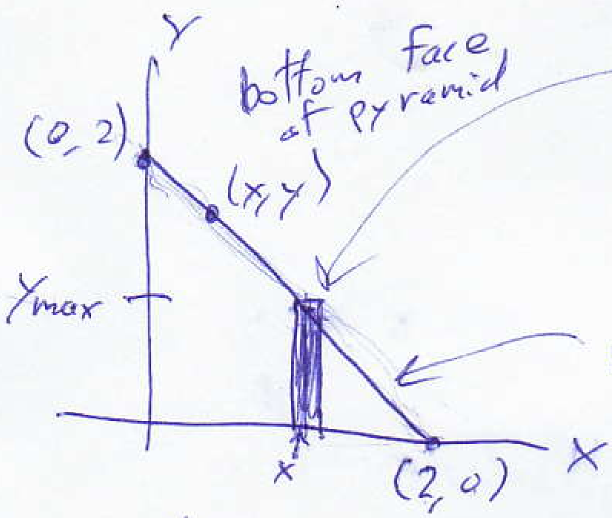
To find volume, slice, then slice again.

First, slice perpendicular to x-axis.

$dV =$ volume of slice $\approx dx \cdot A_{slice}$

$$V = \int_{x=0}^{x=2} dV$$



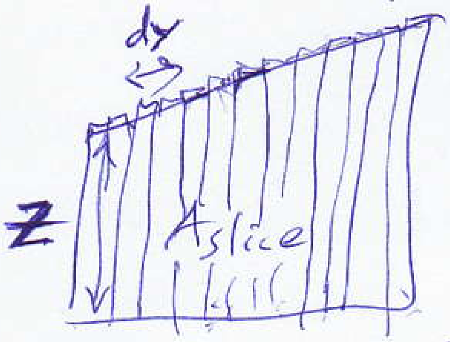


$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0-2}{2-0} = -1$$

$$\frac{y-2}{x-0} = \frac{\Delta y}{\Delta x} = -1$$

$$y - 2 = -1(x - 0)$$

$$y = 2 - x$$



$$A_{slice} = \int_{y=0}^{y=2-x} z\, dy = \int_{y=0}^{y=2-x} \left(\tfrac{1}{2}x + y\right) dy$$

$$V = \int_{x=0}^{x=2} A_{slice} \cdot dx = \int_{x=0}^{x=2} \left(\int_{y=0}^{y=2-x} \left(\tfrac{1}{2}x + y\right) dy \right) dx$$

Evaluate inner integral with x held constant:

$$\int_0^{2-x} \underbrace{\left(\frac{1}{2}x + y\right)dy}$$

$$d\left(\frac{1}{2}xy + \frac{y^2}{2}\right) = \frac{1}{2}x\,dy + \frac{2y\,dy}{2}$$

x constant

$$\int_0^{2-x} \left(\frac{1}{2}x + y\right)dy = \left(\frac{1}{2}xy + \frac{y^2}{2}\right)\Big|_{0=y}^{2-x=y}$$

$$= \left(\frac{1}{2}x(2-x) + \frac{(2-x)^2}{2}\right) - (0+0)$$

$$A_{slice} = \frac{1}{2}(2x - x^2) + \frac{4-4x+x^2}{2}$$

$$A_{slice} = 2 - x$$

$$V = \int_{x=0}^{x=2} \underbrace{(2-x)\,dx} = \left(2x - \frac{x^2}{2}\right)\Big|_0^2$$

$$d\left(2x - \frac{x^2}{2}\right) = 2dx - \frac{2x\,dx}{2}$$

$$V = \left(2\cdot 2 - \frac{2^2}{2}\right) - (0-0) = 4 - \frac{4}{2} = 2$$

2 ways to find partial derivatives:

$$z = f(x,y)$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$df = f_x \, dx + f_y \, dy$$

Compare to $y = g(x)$

$$dg = dy = g'(x)\, dx$$

$$dg/dx = dy/dx = g'$$

$$z = x^2 y^3$$

$$dz = d(x^2)\, y^3 + x^2 d(y^3)$$

$$dz = 2x\,dx \cdot y^3 + x^2 \cdot 3y^2 dy$$

$$= \underbrace{(2xy^3)}_{\partial z/\partial x} dx + \underbrace{(3x^2y^2)}_{\partial z/\partial y} dy$$

Alternative: hold one variable constant & differentiate with respect to the other variable.

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hold y constant, so $dy = 0$

$$dz = d(x^2y^3) = y^3 d(x^2) = y^3(2x\,dx)$$

$$\partial z/\partial x = dz/dx = 2xy^3$$

$$dz/dx = \partial z/\partial x \text{ when } dy = 0$$

Now hold x constant, so $dx = 0$:

$$dz = d(x^2y^3) = x^2 d(y^3) = x^2 \cdot 3y^2 dy$$

$$\partial z/\partial y = dz/dy = 3x^2y^2$$

$$dz/dy = \partial z/\partial y \text{ when } dx = 0$$

To compute $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$, $\frac{\partial^2 z}{\partial y^2}$, there are again 2 ways:

$$dz^2 = d(dz) = \frac{\partial^2 z}{\partial x^2} dx^2 + \frac{\partial^2 z}{\partial x \partial y}(2dx\,dy) + \frac{\partial^2 z}{\partial y^2} dy^2$$

$$dz = 2xy^3 dx + 3x^2y^2 dy$$

You hold dx, dy const, so $d(dx) = d(dy) = 0$

$$d^2z = d(dz) = d(2xy^3)dx + d(3x^2y^2)dy$$

$$= [dx \cdot y^3 + x \cdot 3y^2 dy]2dx$$

$$+ [2x\,dx \cdot y^2 + x^2 \cdot 2y\,dy]3dy$$

$$= \underbrace{2y^3}_{\frac{\partial^2 z}{\partial x^2}} dx^2 + \underbrace{6xy^2}_{\frac{\partial^2 z}{\partial x \partial y}}(2dxdy) + \underbrace{6x^2y}_{\frac{\partial^2 z}{\partial y^2}} dy^2$$

Alternative: take derivatives of $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ while holding 1 variable constant

$$\frac{\partial^2 z}{\partial x^2} = \frac{d(\partial z / \partial x)}{dx} \text{ with } y \text{ constant}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{d(\partial z / \partial y)}{dx} \text{ with } y \text{ constant}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{d(\partial z / \partial y)}{dy} \text{ with } x \text{ constant}$$

Same

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{d(\partial z / \partial x)}{dy} \text{ with } x \text{ constant}$$