HW Solutions (2/1/11 assignment)

3. Mass is not conserved:

$$m_U - (m_{Th} + m_{He}) = \underbrace{7.6}_{\text{certain}}\underbrace{2}_{\text{uncertain}} \times 10^{-30}\ \text{kg}$$

Mass has been lost, and been converted into energy: $\Delta E = (-\Delta m)c^2$

Extra info $\rightarrow$

$$\Delta E = (7.62 \times 10^{-30}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})^2$$

$$= 6.86 \times 10^{-3}\ \text{kg} \cdot \text{m}^2/\text{s}^2 \text{ of energy per atom}$$

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8. $$\text{flow} = \frac{\text{volume}}{\text{time}} = \frac{\text{area} \times \text{length}}{\text{time}} = \text{area} \times \text{speed}$$

$\Rightarrow$ area $\times$ speed is conserved (assuming constant density), so when the area doubles, the speeds must halve.

Algebraically: $A_{new} = 2A_{old}$
$v_{new} = v_{old}/2$

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9. There are lots of ways to measure air speed at a particular location. For example, you can release puffs of smoke and directly observe their speed. Since area $\times$ speed is conserved, once you know the speed in one part of the tunnel, you measure (cross-sectional) areas to find the speed anywhere in the tunnel.

10. $V = b\int_0^a y\,dx$ is true for $y=y_1$, $y=y_2$, and $y=y_{total}$.

Let's check that this doesn't contradict the equation $y_{total} - h = (y_1 - h) + (y_2 - h)$ where $h = V/(ab)$:

$$b\int_0^a (y_{total} - h)\,dx = b\int_0^a \left((y_1 - h) + (y_2 - h)\right) dx$$

$$\underbrace{b\int_0^a y_{total}\,dx}_{V} - \underbrace{b\int_0^a h\,dx}_{ah} = \overbrace{b\int_0^a y_1\,dx}^{V} - b\overbrace{\int_0^a h\,dx}^{ah} + \underbrace{b\int_0^a y_2\,dx}_{V} - b\underbrace{\int_0^a h\,dx}_{ah}$$

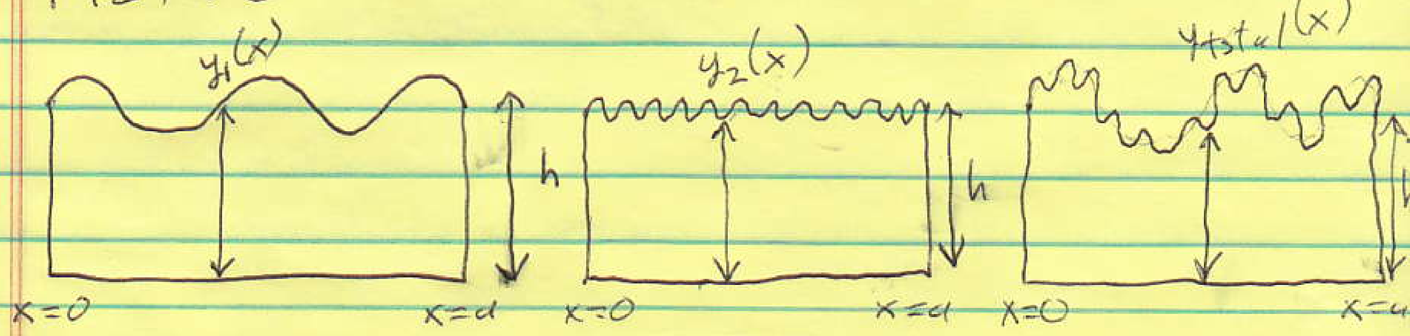
$$V - bah = V - bah + V - bah$$

$$V - ba\left(\frac{V}{ab}\right) = V - ba\left(\frac{V}{ab}\right) + V - ba\left(\frac{V}{ab}\right)$$

$$V - V = V - V + V - V$$

$$0 = 0 \quad \checkmark$$

Picture:



$\text{area} = \int_0^a y\,dx \qquad \text{volume} = b\int_0^a y\,dx$

Convert 7.5×10^{-3} mm/(s^2)

to km/(hr^2)

$$7.5\times10^{-3}\ \frac{mm}{s^2} \times \underbrace{\underbrace{\left(\frac{3600\ s}{hr}\right)}_{1}^{2}}_{1^2 = 1}$$

$$7.5\times10^{-3}\ \frac{\cancel{mm}}{\cancel{s^2}} \times \frac{3600^2\ \cancel{s^2}}{hr^2} \times \frac{10^{-3}\ \cancel{m}}{\cancel{mm}} \times \frac{km}{10^3\ \cancel{m}}$$

$$7.5\times10^{-3} \times \frac{3600^2}{hr^2} \times 10^{-3} \times \frac{km}{10^3}$$

acceleration, velocity, position

$$\longleftarrow \qquad\qquad \longleftarrow$$

$$a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

$$\longrightarrow \qquad\qquad \longrightarrow$$

$$v_0 + \int_0^t a\,dt = v \qquad x_0 + \int_0^t v\,dt = x$$

Constant a (e.g. Free fall $a = 9.8\,m/s^2$)

$$v_0 + at = v \qquad x_0 + \int_0^t (v_0 + at)\,dt = x$$

$$\Downarrow \qquad x_0 + v_0 t + \frac{1}{2}at^2 = x$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

A rock is dropped from 50m height. What is its speed half way down?

$$v^2 - v_0^2 = 2a(x - x_0)$$

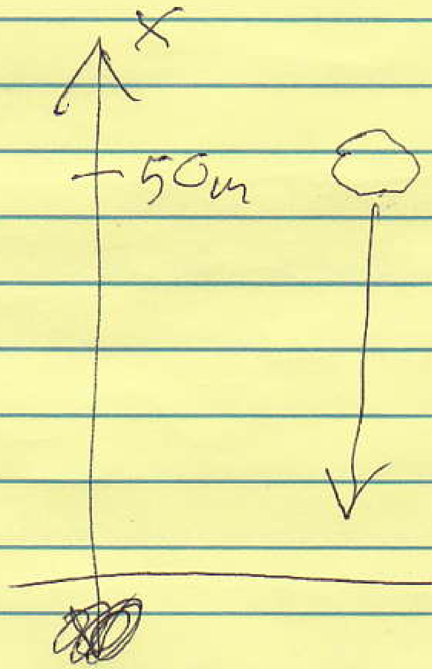
$v_0 = 0$

$x_0 = 50m$

$x = 25m$

$-9.8 m/s^2$

How long does it take for the rock to fall to the ground?

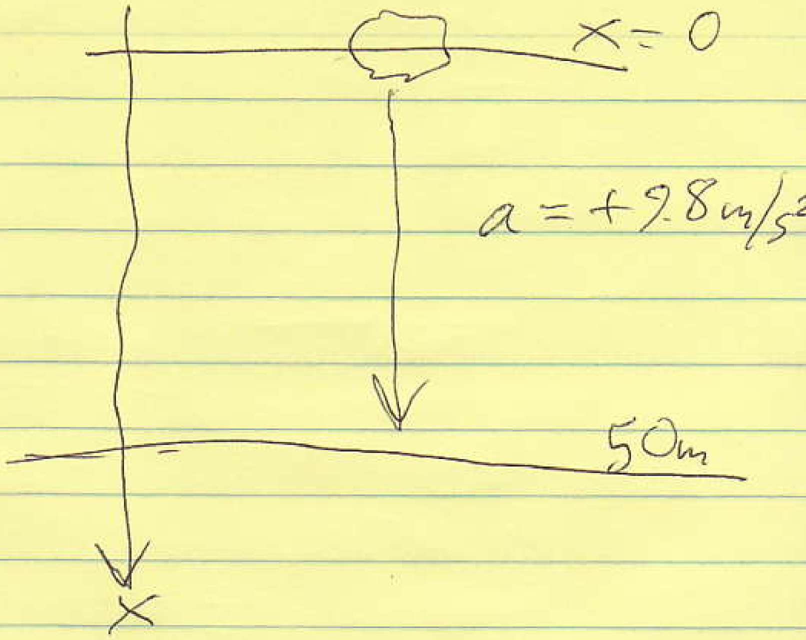


$a = -9.8 m/s^2$

$$x_0 + v_0 t + \frac{1}{2} a t^2 = \cancel{x}$$

$50m$ 0 $-9.8 m/s^2$ 0

Alternative



conservation of mass

If density is constant,

then, since $\text{density} = \frac{\text{mass}}{\text{volume}}$,

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{\text{constant}}{\text{constant}} \text{ is constant}$$

$$\text{volume} = (\text{cross-sectional area}) \times \text{length}$$

$$\frac{\text{volume}}{\text{time}} = \text{area} \times \frac{\text{length}}{\text{time}} = \underbrace{\text{area} \times \text{speed}}_{\text{conserved}}$$

$$1\,L = 1000\,cm^3 = 10^3\,cm^3 = (10\,cm)^3$$

Scaling

E.g. doubling volume correspond to multiplying diameter by $\sqrt[3]{2}$

$$V \propto L^3$$

$$\sqrt[3]{V} \propto L$$

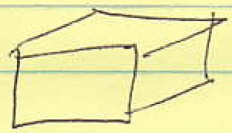
$$L \propto \sqrt[3]{V} \qquad L = k\sqrt[3]{V}$$

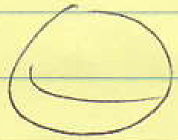
k depends on shape

$$2 = \frac{V_{new}}{V_{old}} \Rightarrow \sqrt[3]{2} = \frac{\sqrt[3]{V_{new}}}{\sqrt[3]{V_{old}}} = \frac{k\sqrt[3]{V_{new}}}{k\sqrt[3]{V_{old}}} = \frac{L_{new}}{L_{old}}$$

Order of magnitude estimates

- Estimate length, and multiply to get areas, volumes.
- Oversimplify the shapes





- Sometimes it helps to use density. Often density is close to that of H_2O: $1\ g/cm^3 = \underline{1000}\ kg/m^3$.