2/10 HW Solutions

#3, 5: See p. 865

#7 a) The kinetic energy increased (leaping), so magnetic energy decreased.

b) The kinetic energy decreased (deceleration), so magnetic energy increased

#10 $\frac{\Delta E}{\Delta t} = 200\text{W}$ $\quad \Delta E = mc\,\Delta T$

$m = 60\text{kg}; \quad c = 4.186 \frac{\text{J}}{\text{gram} \cdot {}^\circ\text{C}}; \quad \Delta T = \overbrace{6^\circ\text{C}}^{1 \text{ sig fig}}$

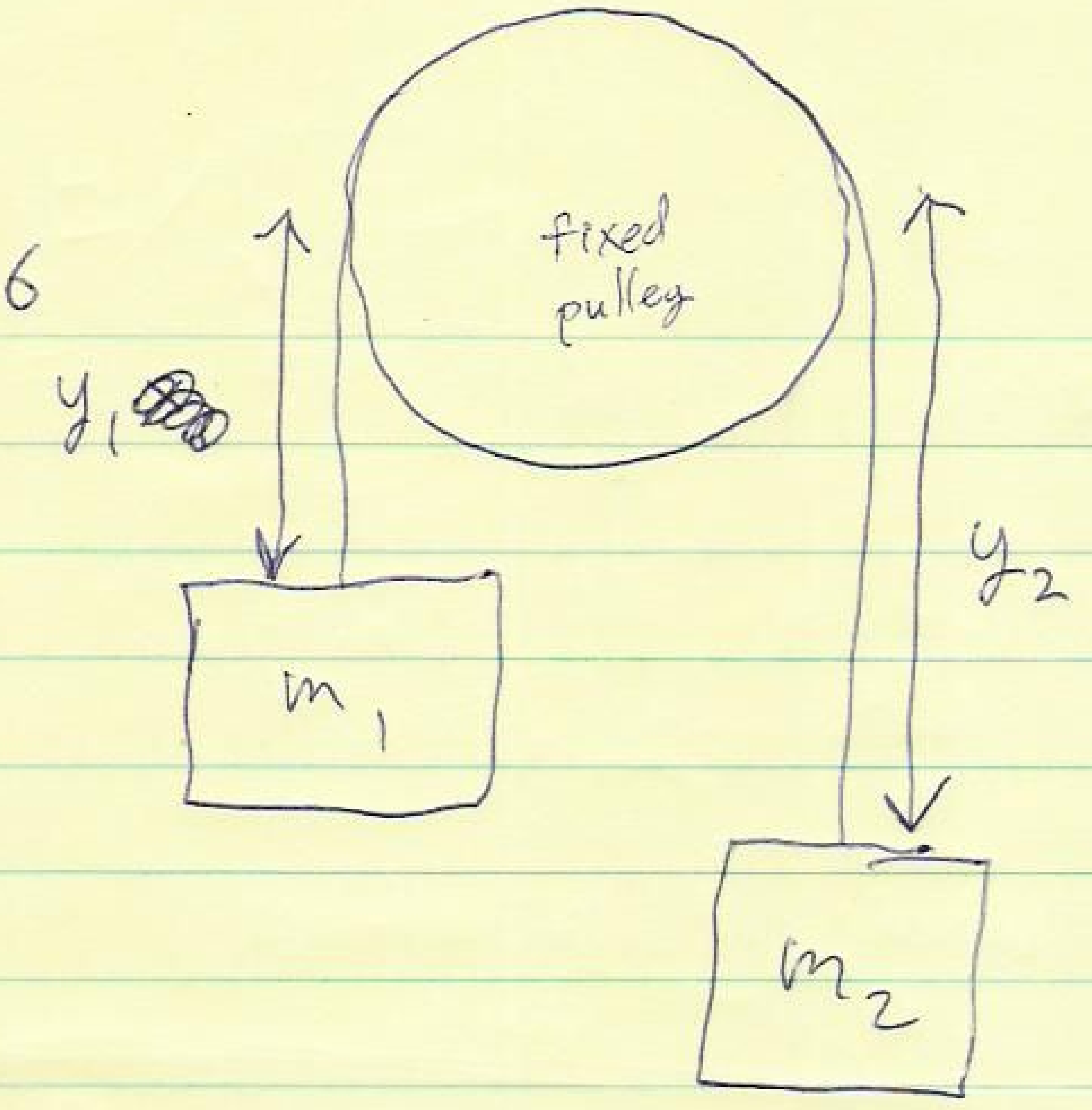
$$\Delta E = (60\text{kg})\left(\frac{4.186\text{ J}}{\text{g} \cdot {}^\circ\text{C}}\right)\left(\frac{10^3\text{g}}{\text{kg}}\right)(6^\circ\text{C}) = 1.5 \times 10^6\text{J}$$

$$\Delta t = \frac{\Delta E}{200\text{W}} = \frac{1.5 \times 10^6\text{J}}{2 \times 10^2\text{ J/s}} = 7.5 \times 10^3\text{ s}$$

$$\Delta t = 7.5 \times 10^3\text{ s}\left(\frac{1\text{ min}}{60\text{ s}}\right) = \overbrace{10^2 \text{ minutes}}^{1 \text{ sig fig}}$$

(about 2 hours)

#16



$$y_1 + y_2 = \text{constant}$$

$$U = (-m_1 y_1 - m_2 y_2) g$$

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$v_1 = \frac{dy_1}{dt} \qquad v_2 = \frac{dy_2}{dt}$$

$$\text{constant} = E = K + U = (-m_1 y_1 - m_2 y_2) g + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$0 = dE = dK + dU = (-m_1 dy_1 - m_2 dy_2) g + m_1 v_1 dv_1 + m_2 v_2 dv_2$$

$$0 = d(y_1 + y_2) = dy_1 + dy_2 \Rightarrow \left(\frac{dy_2}{dt} = -\frac{dy_1}{dt}\right) \Rightarrow v_2 = -v_1$$

$$dv_2 = -dv_1$$

$$0 = (-m_1 dy_1 - m_2(-dy_1)) g + m_1 v_1 dv_1 + m_2(-v_1)(-dv_1)$$

$$0 = g(m_2 - m_1) dy_1 + (m_1 + m_2) v_1 dv_1$$

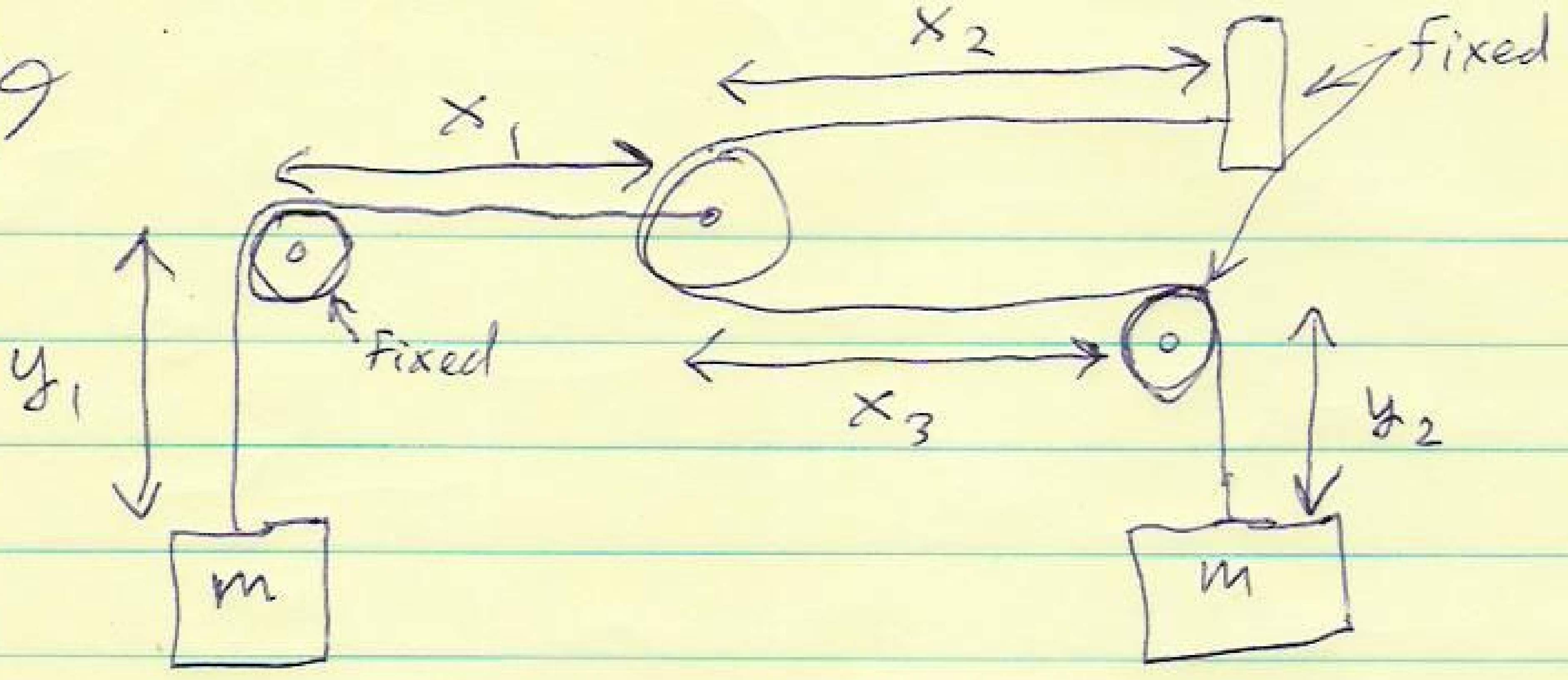
$$0 = g(m_2 - m_1) \frac{dy_1}{dt} + (m_1 + m_2) v_1 \frac{dv_1}{dt}$$

$$0 = g(m_2 - m_1) v_1 + (m_1 + m_2) v_1 a_1$$

$$0 = g(m_2 - m_1) + (m_1 + m_2) a_1 \Rightarrow g(m_1 - m_2) = (m_1 + m_2) a_1$$

$$\Rightarrow \boxed{a_1 = \frac{(m_1 - m_2) g}{m_1 + m_2}} \qquad \frac{dv_2}{dt} = -\frac{dv_1}{dt} \Rightarrow \boxed{a_2 = -a_1 = g\left(\frac{m_2 - m_1}{m_1 + m_2}\right)}$$

#19



$v_1 = \frac{dy_1}{dt}$

$v_2 = \frac{dy_2}{dt}$

$U = -mgy_1 - mgy_2 \qquad K = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$\begin{cases} x_1 + y_1 = \text{constant}; & x_2 + x_3 + y_2 = \text{constant} \\ x_1 + x_3 = \text{constant}; & x_1 + x_2 = \text{constant} \end{cases}$ (rope lengths) (horizontal distances between fixed pulleys)

$\Rightarrow -dy_1 = dx_1 \qquad \Rightarrow dy_2 = -dx_2 - dx_3$

$\begin{cases} dx_1 + dy_1 = 0; & dx_2 + dx_3 + dy_2 = 0; \\ dx_1 + dx_3 = 0; & dx_1 + dx_2 = 0; \end{cases}$

$\Rightarrow dx_3 = -dx_1 \qquad \Rightarrow dx_2 = -dx_1$

$dy_2 = -dx_2 - dx_3 = -(-dx_1) - (-dx_1) = 2dx_1 = -2dy_1$

$\Rightarrow dy_2 = -2dy_1$

You may be able to figure out $dy_2 = -dy_1$ by thinking about the pulleys & ropes & skipping the algebra!

$\Downarrow$

$v_2 = -2v_1$

$\Downarrow$

$dv_2 = -2dv_1$

$dU = -mg\,dy_1 - mg\,dy_2 = -mg\,dy_1 + 2mg\,dy_1 = mg\,dy_1$

$dK = mv_1\,dv_1 + mv_2\,dv_2 = mv_1\,dv_1 + m(-2v_1)(-2dv_1)$

$dK = 5mv_1\,dv_1$

constant $= E = K + U \Rightarrow 0 = dK + dU$

$0 = mg\,dy_1 + 5mv_1\,dv_1 \Rightarrow 0 = mg\frac{dy_1}{dt} + 5mv_1\frac{dv_1}{dt}$

$\Rightarrow 0 = mg\,v_1 + 5mv_1\,a_1 \Rightarrow 0 = g + 5a_1$

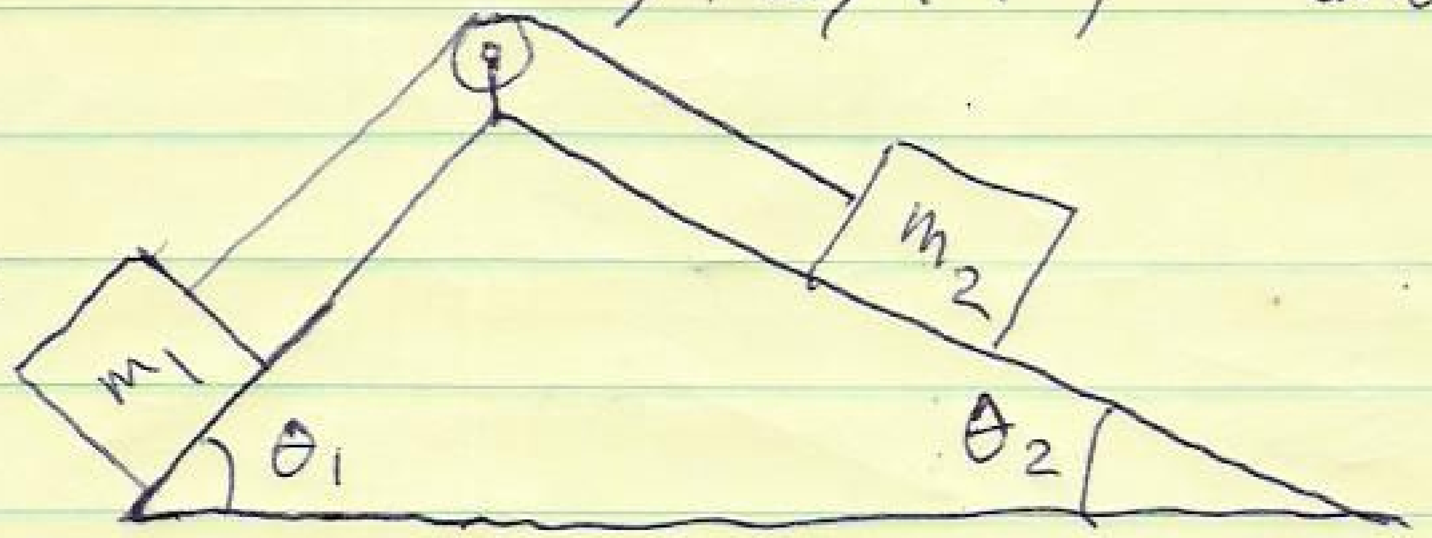
$\Rightarrow a_1 = -g/5 \Rightarrow$ $\boxed{\text{upward acceleration of left mass is } g/5}$

Reading due 2/22: 2.3

HW 2/17 (due 2/22):

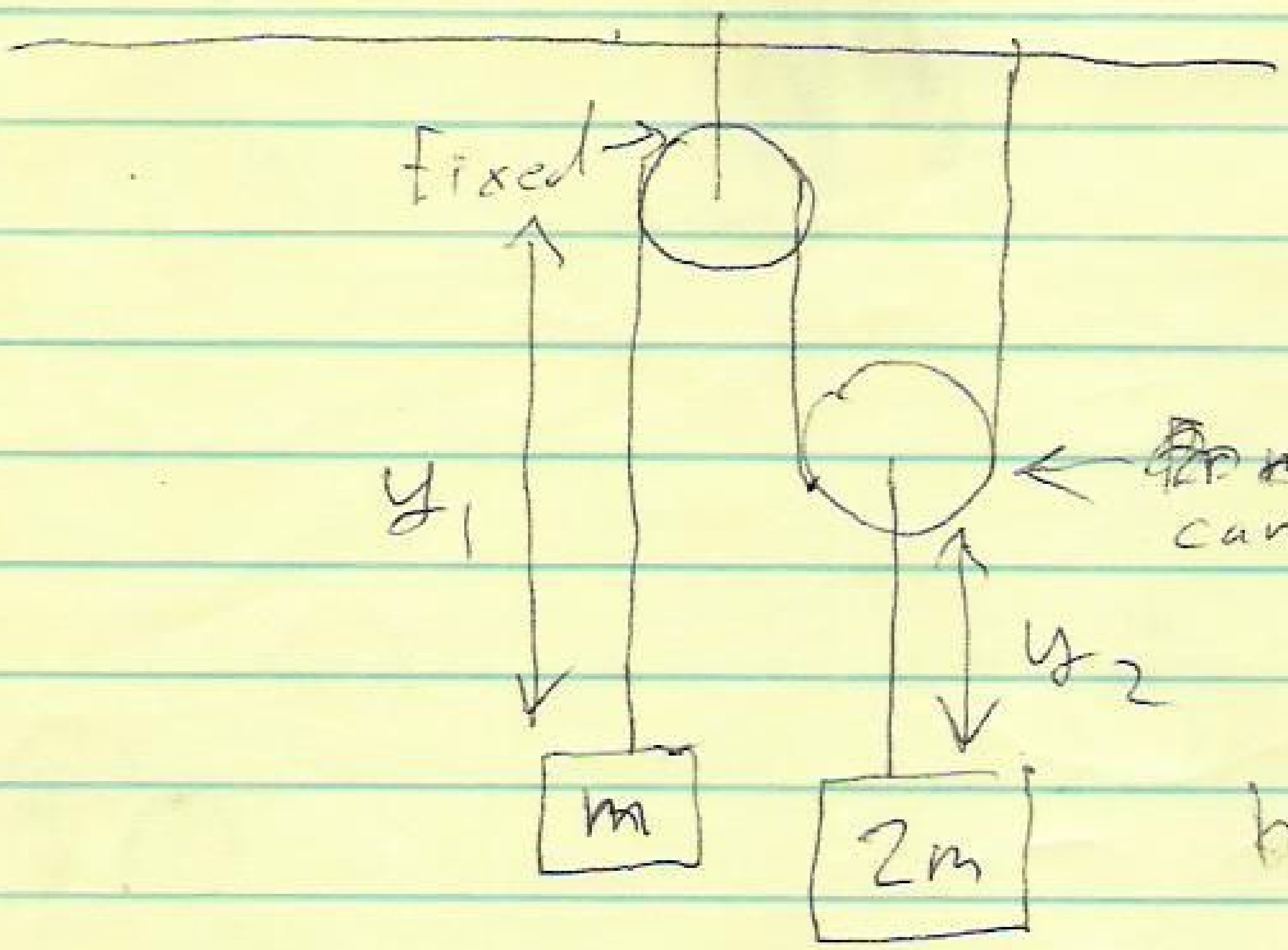
Ch. 2 #6, 12, 17, and "#42":

#42:



Find a formula for the acceleration of $\boxed{m_2}$ along the ramp.

Assume no friction, a massless rope, and a massless pulley, and consider only the time period before either block reaches the bottom of its ramp.



balanced:

$a = 0$

$$dy_2 = -\frac{1}{2}\,dy_1$$

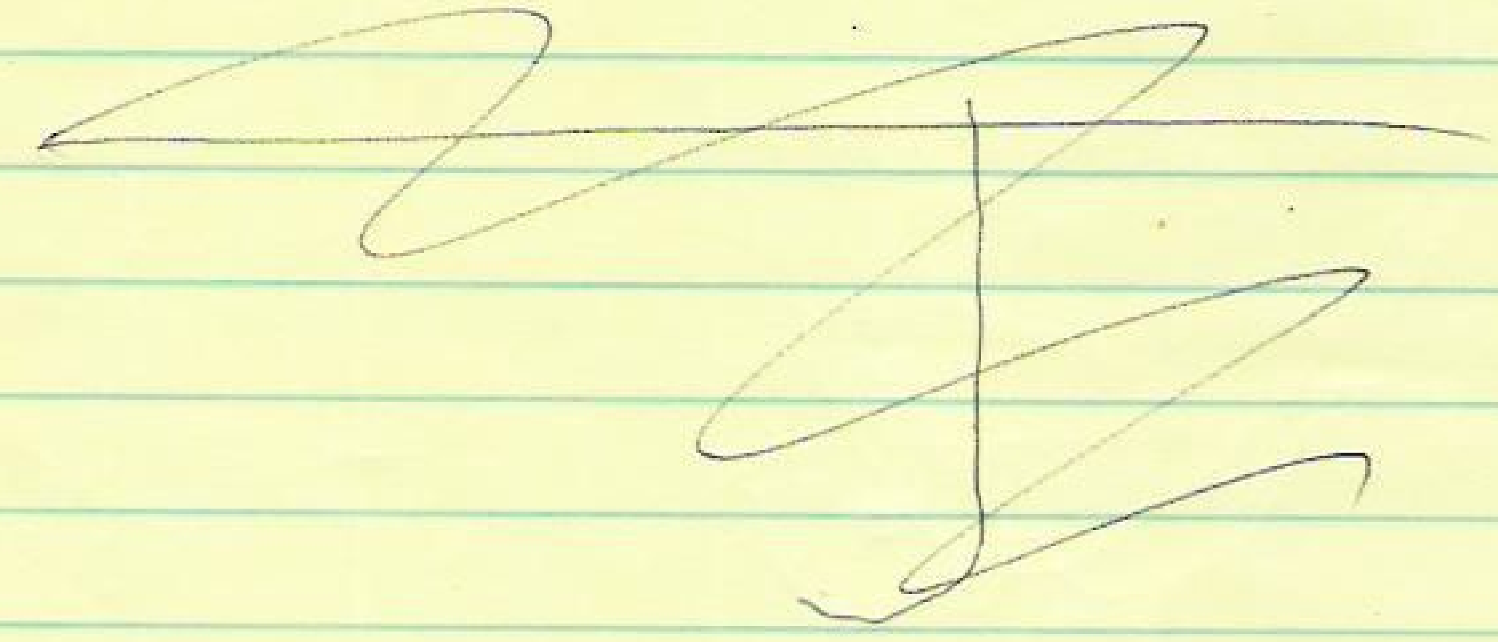
$$U = -mgy_1 - 2mgy_2$$

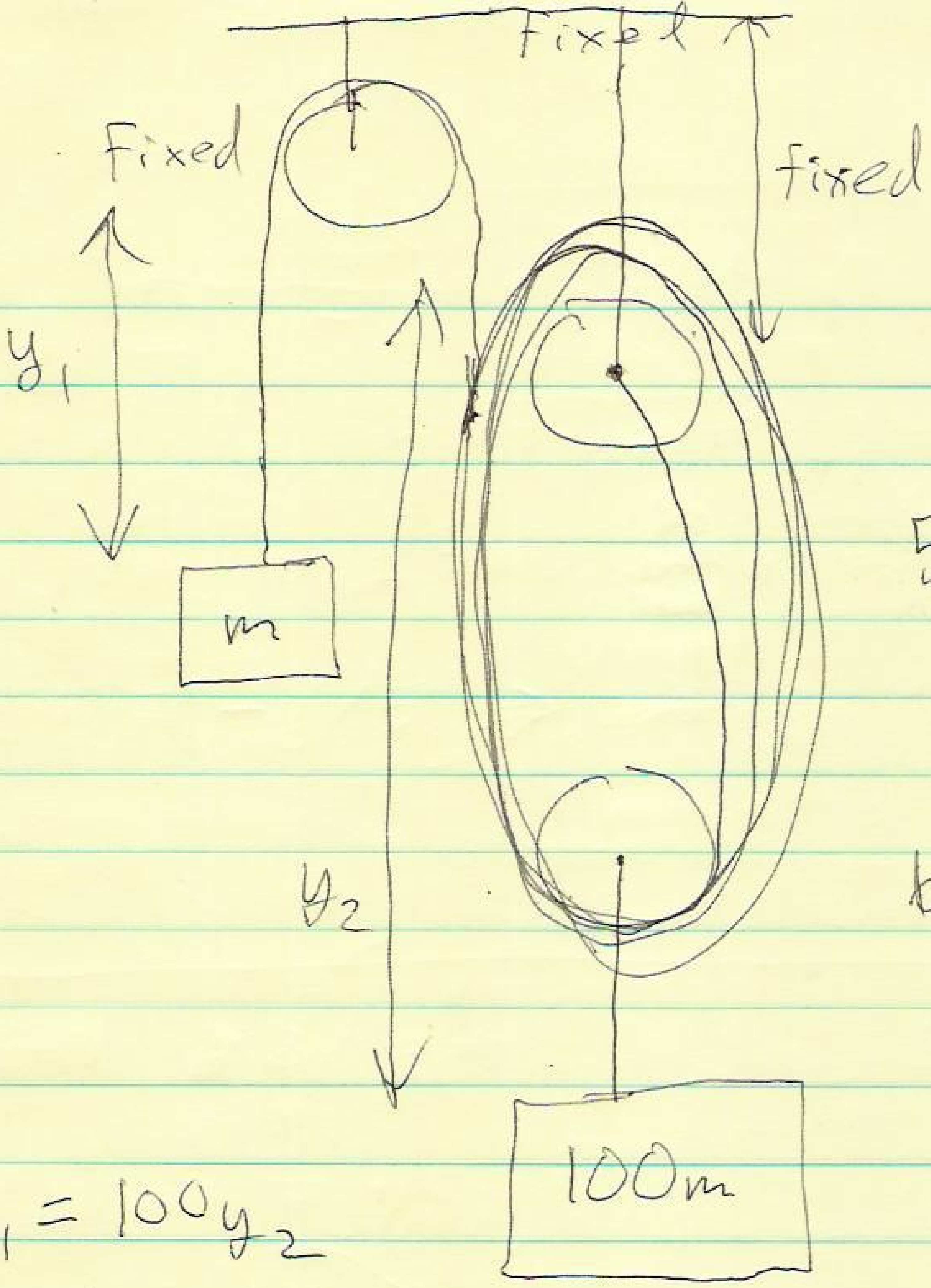
$$dU = -mg\,dy_1 - 2mg\,dy_2$$

$$dU = -mg\,(\underbrace{dy_1 + 2dy_2}_{0}) = 0$$

$dU = 0 \Rightarrow dK = 0$

$a = 0$





50 loops

balanced

$dU = 0 \Rightarrow dK = 0$

$a = 0$

$y_1 = 100 y_2$

$y_1/100 = y_2$