Correction to 2/24 notes:

$$T = 2\pi/\omega.$$

For example, if $\omega = 1.9$ rad/s, then in 1 second, the angle changes by 1.9 radians.

So, in $\frac{2\pi}{1.9}$ seconds, the angle changes by $\left(\frac{2\pi}{1.9}\right) \cdot 1.9 = 2\pi$ radians, which is 1 cycle.

Frequency f is $1/T = \omega/2\pi$.

For example, if $T = 0.025\,s$, then

$$f = \frac{1}{0.025\,s} = \frac{40}{s} = 40 \text{ cycles per second.}$$
$$= 40 \text{ Hz}$$

Summary:

period	$T = 1/f = 2\pi/\omega$
frequency	$f = 1/T = \omega/2\pi$
angular frequency	$\omega = \frac{2\pi}{T} = 2\pi f$

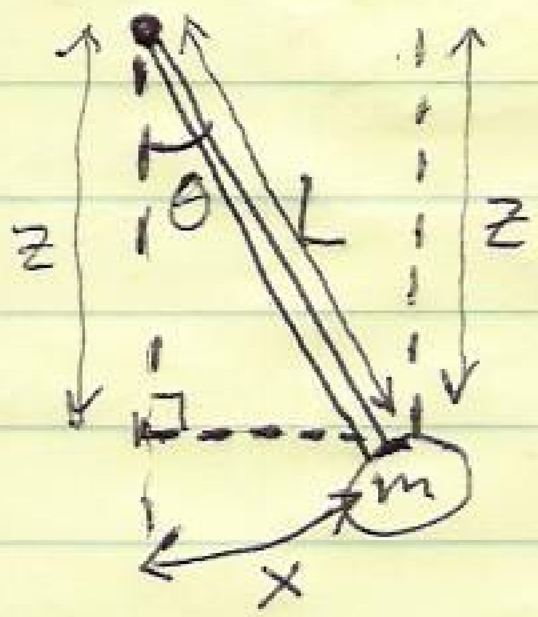
Oscillation:

$$\omega = \sqrt{\frac{k}{m}} \quad \left(\text{if} \quad E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2\right)$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow f = \frac{\omega}{2\pi} = \frac{\sqrt{k/m}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Pendulum:



$$\left.\begin{array}{l} \theta = \dfrac{x}{L} \\[2ex] \dfrac{z}{L} = \cos\theta \end{array}\right\} \Rightarrow z = L\cos\frac{x}{L}$$

$$U = -mgz = -mgL\cos\frac{x}{L}$$

$$dU = -mgL\,d\left(\cos\frac{x}{L}\right) = -mgL\left(-\sin\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$= -mgL\left(-\sin\frac{x}{L}\right)\frac{dx}{L} = mg\left(\sin\frac{x}{L}\right)dx$$

$\boxed{\text{Here I used } d(\cos u) = (-\sin u)\,du.}$

$$d^2U = d\left(mg\sin\frac{x}{L}\right)dx = mg\left(\cos\frac{x}{L}\right)d\left(\frac{x}{L}\right)dx$$

$$= mg\left(\cos\frac{x}{L}\right)\frac{dx}{L}dx = \frac{mg}{L}\left(\cos\left(\frac{x}{L}\right)\right)dx^2$$

$\boxed{\text{Here I used } d(\sin u) = \cos u\,du.}$

$$d^2U/dx^2 = \frac{mg}{L}\cos\frac{x}{L}$$

At the equilibrium position, $x=0$,

$$\frac{d^2U}{dx^2} = \frac{mg}{L}\cos\frac{0}{L} = \frac{mg}{L}\cos 0 = \boxed{\frac{mg}{L}}$$

So, for x small, $U \approx \frac{1}{2}kx^2 + c$

where c is a constant

(the constant is U at $x=0$) and

$$k = \boxed{\frac{mg}{L}}.$$

Period of oscillation if x stays small:

$$T \approx 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}} = 2\pi\sqrt{\frac{L}{g}}$$

For example, if $L = 1.0\,m$, then

$$T \approx 2\pi\sqrt{\frac{1.0\,m}{9.8\,m/s^2}} = 1.0\,s.$$

Observe that $T = 2\pi\sqrt{\frac{L}{g}}$ does not depend on the mass of the pendulum bob, or on the initial position & velocity of the bob.