#3 1) $\rho = \text{density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \Rightarrow m = \rho V$

$\rho = 1\ g/cm^3 = 10^3\ kg/m^3$

$V = 10\ \text{meters} \times \underbrace{\text{area of oceans}}_{\underbrace{70\%\text{ of Earth's surface}}_{70\% \times 4\pi R_e^2}}$

$R_e = 6.4 \times 10^6\ m$

$V = 3.6 \times 10^{15}\ m^3$

$= 3.6 \times 10^{18}\ J$

mass $= m = \rho V =$ mass of ice to melt

$\Delta E = (3 \times 10^3\ J/kg) \times \text{mass}$

$= 1.1 \times 10^{22}\ J$

$\text{Power} = P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{10\ \text{years}}$

$P = 3.4 \times 10^{13}\ W$

normally "sunlight power" $= 1 \times 10^{16}\ W$

$\%\ \text{change} = \frac{P}{1 \times 10^{16}\ W} \times 100$

$= 0.1$ (roughly)

So, an increase of about $\frac{1}{10}$ of a percent in the sunlight energy hitting the ice caps over ten years would raise sea levels 10m.

#32 $T = \underbrace{0.50\,ms}_{1/2 \text{ square}} = 5.0 \times 10^{-4}\,s$

$$f = \frac{1}{T} = 2.0 \times 10^{3}\ \frac{1}{s} = 2.0 \times 10^{3}\,Hz$$

$$\omega = 2\pi f = 1.3 \times 10^{4}\ rad/s$$

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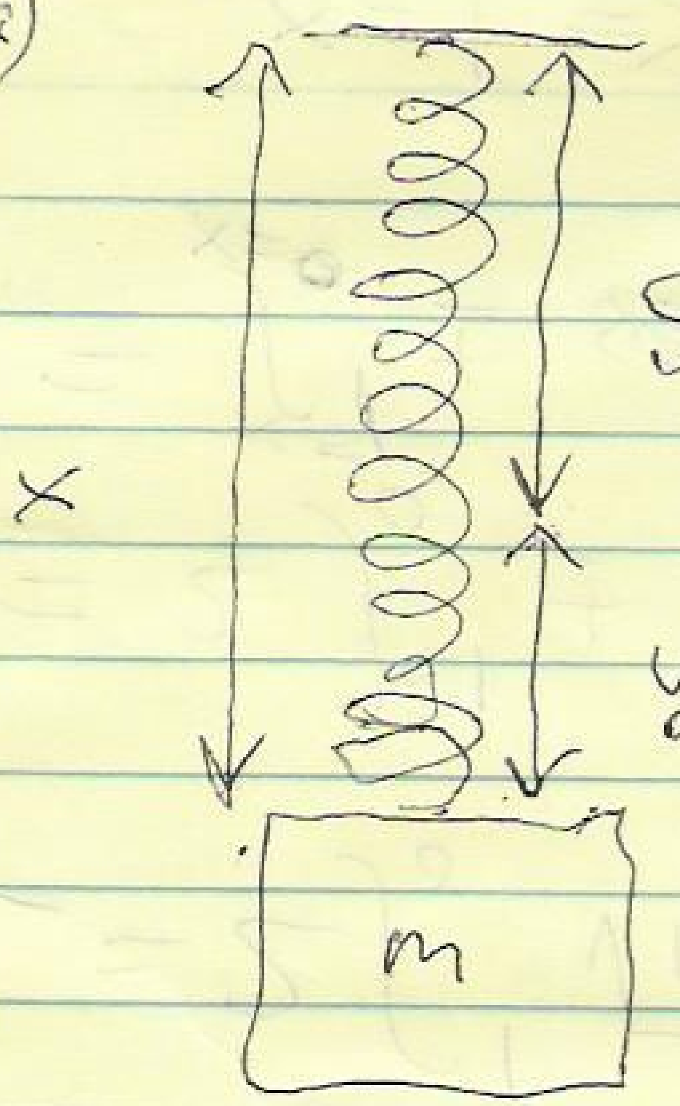
33 a) see next page

b) Given ~~given~~ that the equilibrium position with no hanging mass is $x = S$, but is $x = mg/k + S$ with a hanging mass, I could measure S ~~hanging~~ after hanging the spring and then measure $mg/k + S = x$ after attaching a known mass and waiting friction to eliminate all oscillations. So, knowing m, g, x, and S, I could find k:

$$\frac{mg}{k} + S = x \Rightarrow \frac{mg}{k} = x - S$$

$$\Rightarrow \boxed{k = \frac{mg}{x - S}}$$

#33 a)



$x = 0$

S = spring equilibrium length

$$U = -mgx + \frac{1}{2} k y^2$$

$$y + S = x$$

$$y = x - S$$

$$U = -mgx + \frac{1}{2} k (x - S)^2$$

$$dU = -mg\,dx + \frac{1}{2} k\, 2(x - S)\, d(x - S)$$

$$d(u^2) = 2u\,du$$

$$dU = -mg\,dx + k(x - S)(dx - 0)$$

S constant

$$\frac{dU}{dx} = -mg + k(x - S)$$

Solve $\frac{dU}{dx} = 0$.

$$\boxed{x = \frac{mg}{k} + S}$$

34)

$$T_{osc} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}}$$

From #33b, $k = \frac{mg}{x-s} = \frac{mg}{h}$, so

$$T_{osc} = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}}$$

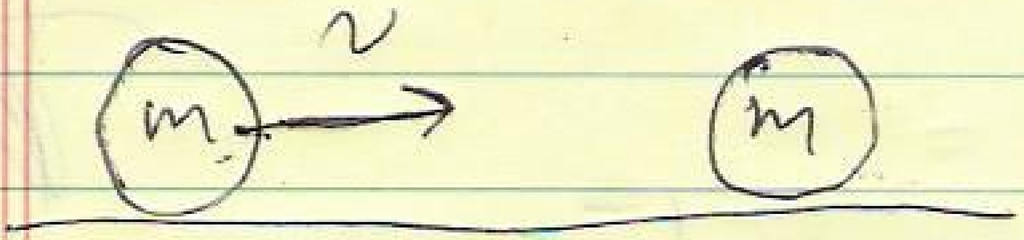
$T_{fall} = ?$

~~In free fall, decreasing height by h~~
~~means $\Delta U = -mgh$, so $\Delta K = mgh$~~
~~(to conserve energy, $\Delta U + \Delta K = 0$).~~
~~so, $\frac{1}{2} m \Delta(v^2) = mgh$, so $\Delta(v^2) = 2gh$.~~

Given free fall from rest at $t=0$, $y=0$,
$y = -\frac{1}{2}gt^2$, so $-h = -\frac{1}{2}gT_{fall}^2$, so
$\frac{2h}{g} = T_{fall}^2$, so $T_{fall} = \sqrt{\frac{2h}{g}}$.

$$\boxed{\frac{T_{osc}}{T_{fall}} = \frac{\sqrt{2h/g}}{2\pi\sqrt{h/g}} = \frac{\sqrt{2}}{2\pi} \approx 0.225}$$

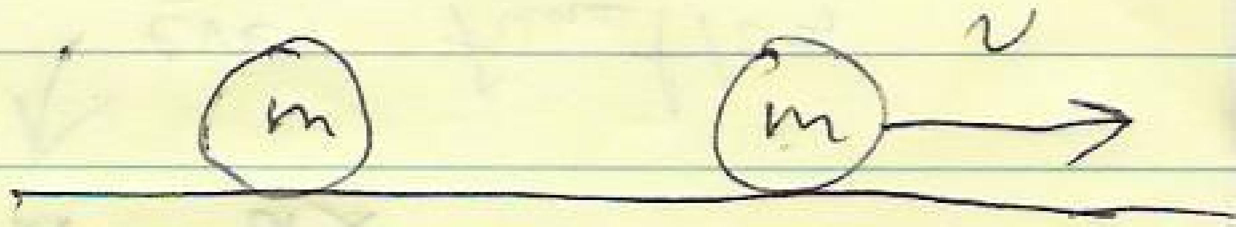
Before



$K = \frac{1}{2}mv^2, \quad 0$

$p = mv, \quad 0$

After



$K = 0 \qquad K = \frac{1}{2}mv^2$

$p = 0, \qquad p = mv$

Why not this?

$K = \frac{K}{2} + \frac{K}{2}$

Answer: conservation of momentum.

$p = mw + mw$

$= \frac{mv}{\sqrt{2}} + \frac{mv}{\sqrt{2}}$

$= mv\left(\frac{2}{\sqrt{2}}\right) \approx \underbrace{1.4mv}$

change from $mv + 0$

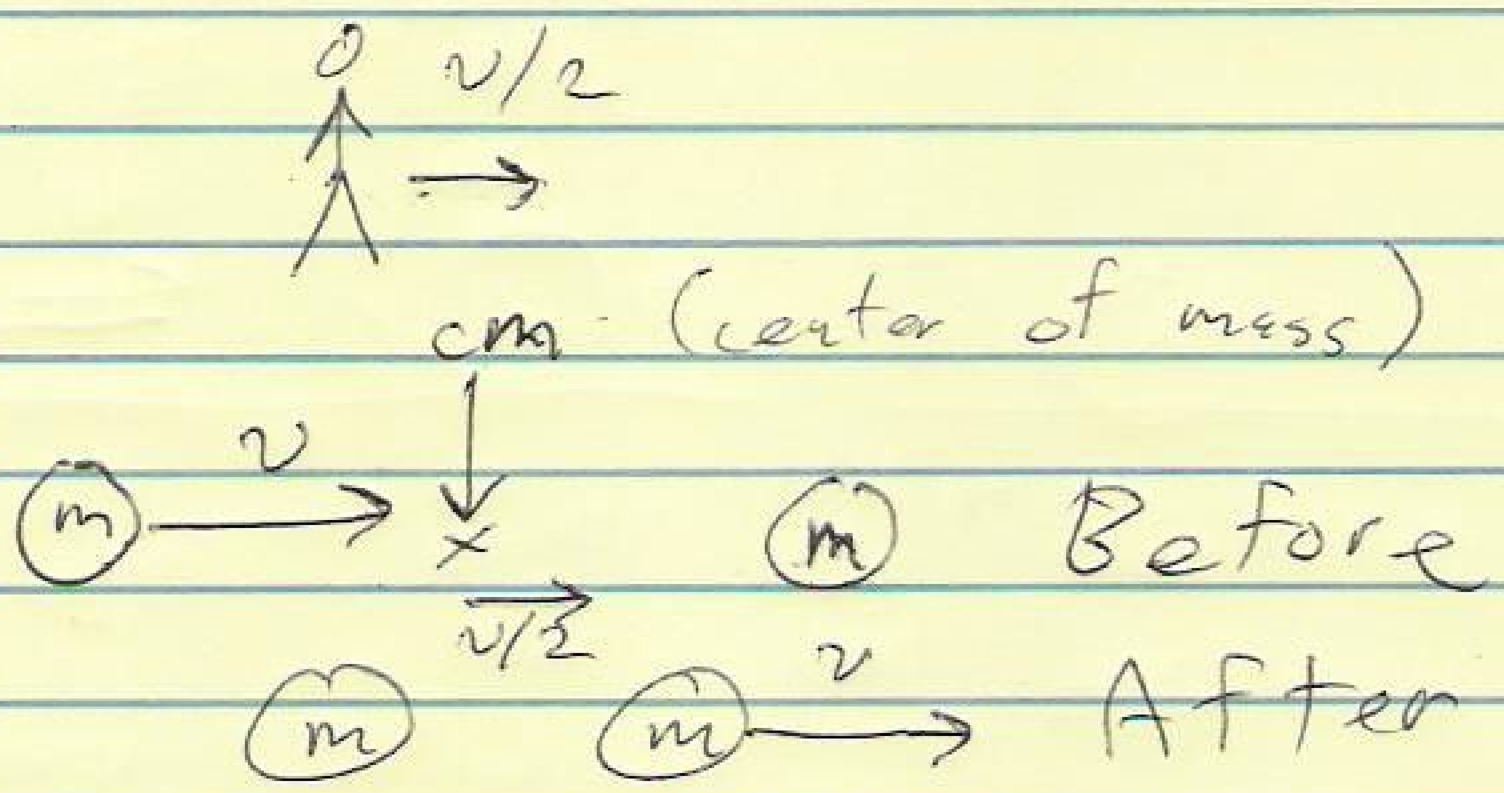
$\frac{K}{2} = \frac{1}{2}mw^2$

$\frac{1}{2}mv^2 = K = mw^2$

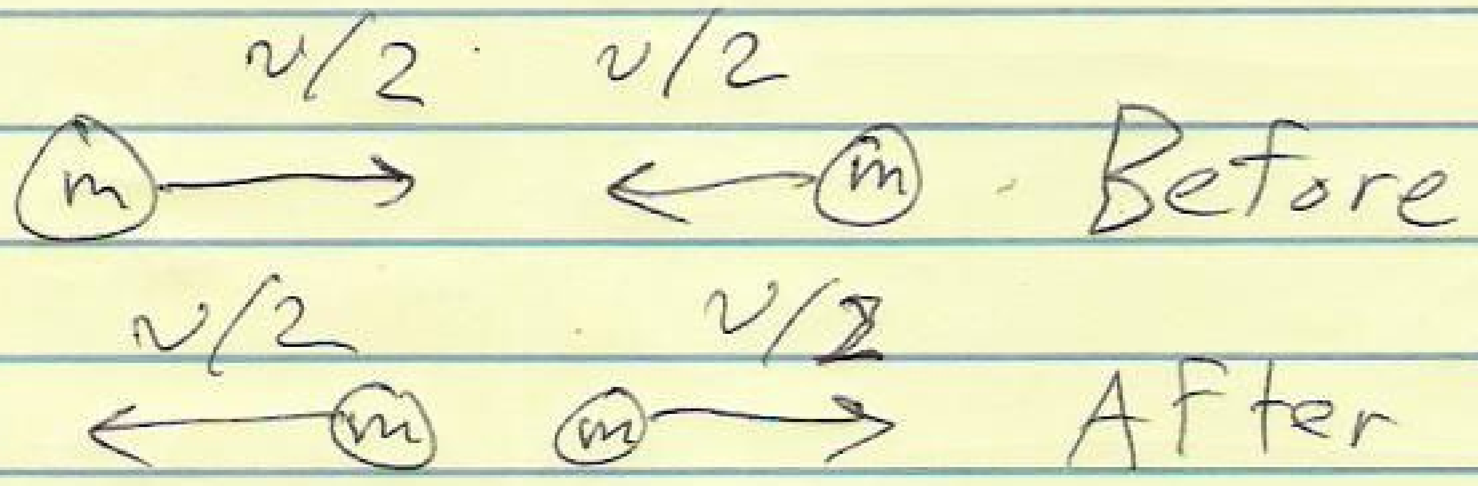
$\frac{1}{2}v^2 = w^2$

$\frac{v}{\sqrt{2}} = w$

$\approx 0.7v$



You see:



Conservation of energy according to all observers implies $p = mv$ is conserved, too.

In general, there's the center-of-mass frame.

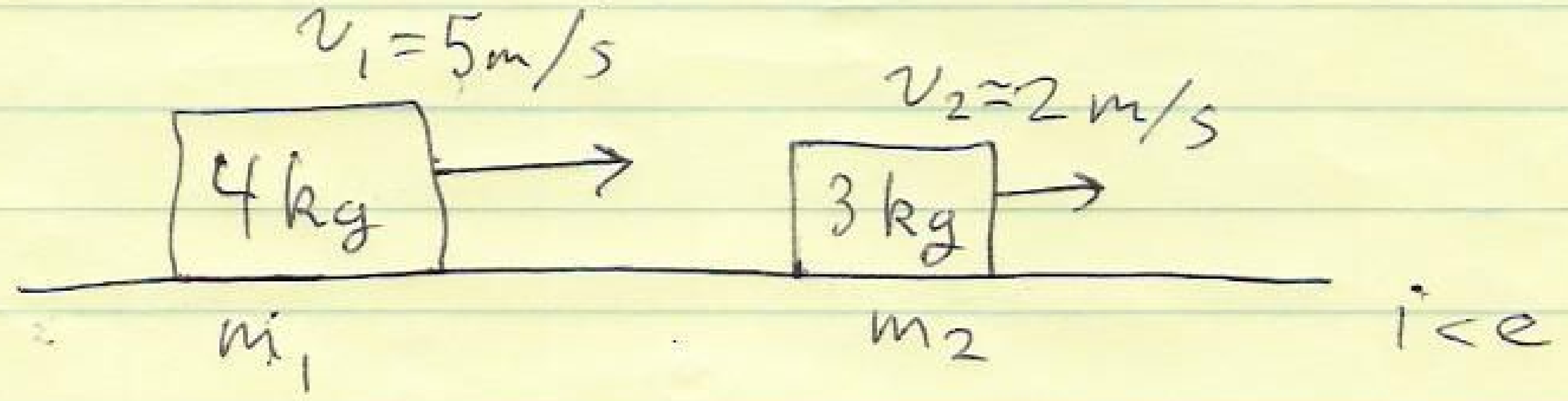
Given masses m_1, m_2, m_3 at positions x_1, x_2, x_3, the c.m. is at $X_{cm} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3}$$

To go to the c.m. Frame:

$$v_1 \longrightarrow v_1 - v_{cm}$$
$$v_2 \longrightarrow v_2 - v_{cm}$$
$$v_3 \longrightarrow v_3 - v_{cm}$$

Example collision:



What are the speeds are the collision?

Conservation of Energy

$$E = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 \quad \text{can't change}$$

Conservation of momentum

$$p = m_1 v_1 + m_2 v_2 \quad \text{can't change}$$

$$E = \left(\tfrac{1}{2}\, 4 \cdot 5^2 + \tfrac{1}{2}\, 3 \cdot 2^2\right) J = 56\,J$$

$$p = (4 \cdot 5 + 3 \cdot 2)\,\text{kg m/s} = \cancel{\text{kg}\cdot\text{m/s}}\ 26 \ldots$$

(ice frame) w_1, w_2 are the speeds after.

$$56\,J = E = \frac{1}{2} m_1 w_1^2 + \frac{1}{2} m_2 w_2^2$$

$$26\ kg\cdot m/s = p = m_1 w_1 + m_2 w_2$$

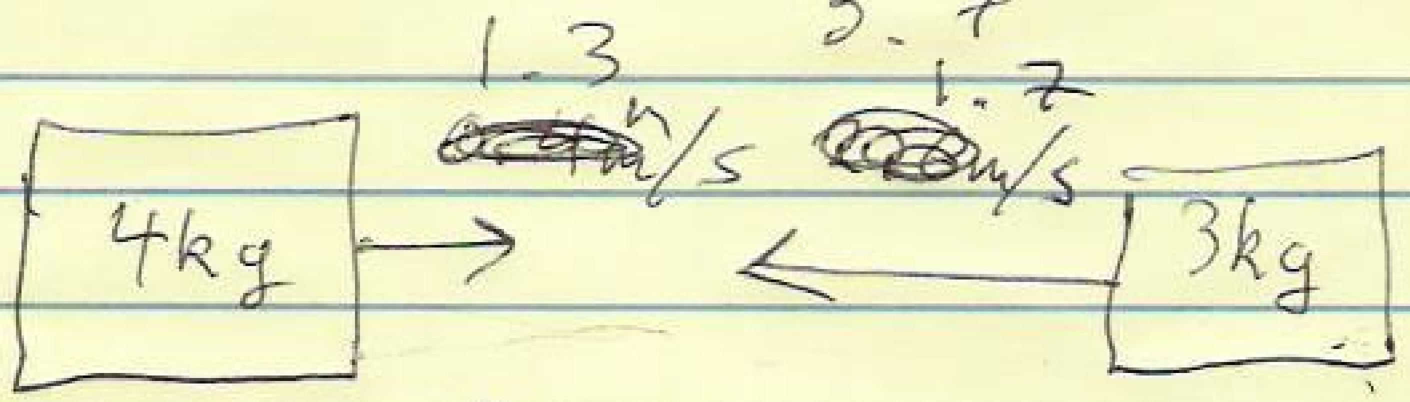
In the c.m. frame, $p = 0$.
This makes the equations nicer.

(ice frame) $$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{p}{7kg} = \frac{26\ kg\ m/s}{7\ kg}$$

$$v_{cm} = \frac{26}{7}\ m/s \approx 3.7\ m/s$$

In the c.m. frame:

$$v_1 = 5m/s - 3.7\ m/s = 1.3\ m/s$$
$$v_2 = 2m/s - 3.7\ m/s = -1.7\ m/s$$



$$p = \left[(4)(1.3) + (-1.7)(3)\right]\ kg\ m/s$$

$$5.2 - 5.1 \approx 0$$

E in c.m. frame:

$$E = \frac{1}{2}(4\,kg)(1.3\,m/s)^2 + \frac{1}{2}(3\,kg)(-1.7\,m/s)^2$$
$$\approx 7.7\,J$$

Conservation of energy & momentum in the c.m. frame:

u_1, u_2 speeds after the collision in the c.m frame.

ice frame speeds w_1, w_2; momentum p

$$\begin{cases} w_1 = u_1 + v_{cm} \\ w_2 = u_2 + v_{cm} \end{cases}$$

v_{cm} is conserved

$p = (m_1 + m_2) v_{cm}$

$$7.7\,J = E = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$0 = p = m_1 u_1 + m_2 u_2 \qquad u_2 = -\frac{m_1 u_1}{m_2}$$

$$-m_1 u_1 = m_2 u_2$$

$$7.7\,J = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 \left(-\frac{m_1 u_1}{m_2}\right)^2$$

$$15.4\,J = m_1 u_1^2 + m_2 \left(\frac{m_1^2 u_1^2}{m_2^2}\right)$$

$$15.4\,J = m_1 u_1^2 + \frac{m_1^2}{m_2} u_1^2 = \underbrace{\underbrace{\left(m_1 + \frac{m_1^2}{m_2}\right)}_{4kg + \frac{(4kg)^2}{3kg}}}_{9.3\,kg} u_1^2$$

$$\frac{15.4\,J}{9.3\,kg} = u_1^2$$

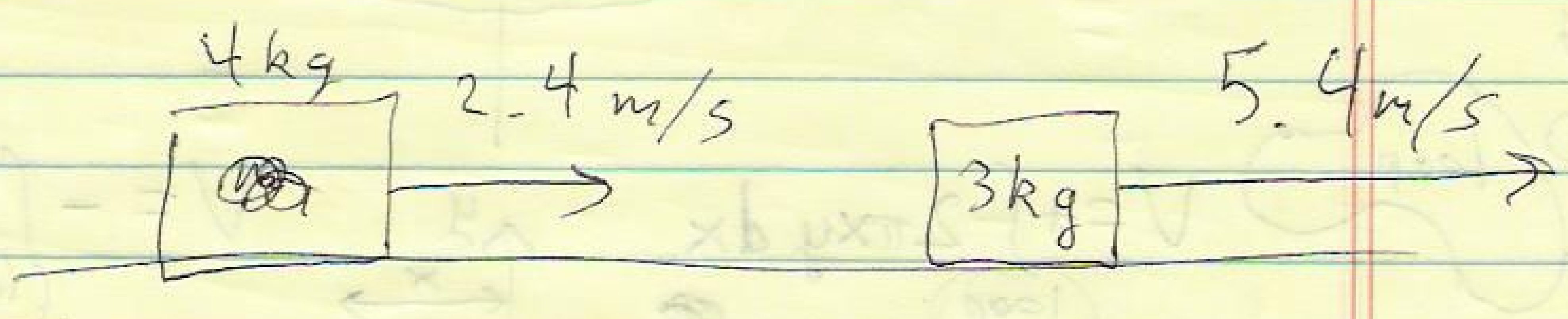
$$-\sqrt{\frac{15.4\,\cancel{kg}\,m^2/s^2}{9.3\,\cancel{kg}}} = u_1$$ ← (see cm frame picture: the 4kg block must recoil to left)

$$-1.3\,m/s = u_1 \qquad u_2 = -\frac{m_1 u_1}{m_2}$$

$$2.4\,m/s = u_1 + \underbrace{v_{cm}}_{3.7\,m/s} = w_1 \qquad u_2 = 1.7\,m/s$$

$$w_2 = u_2 + v_{cm} = 5.4\,m/s$$

After (ice frame):

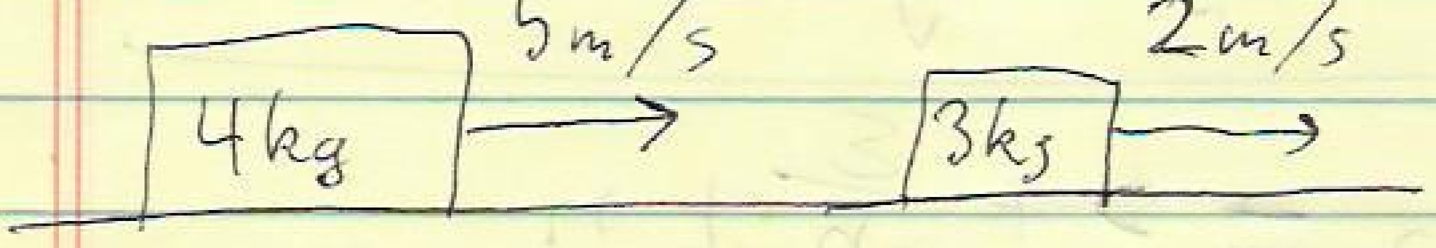


$\boxed{\text{HW \#1–6}}$

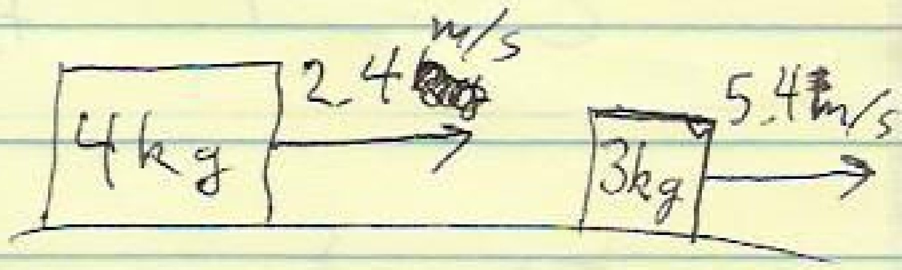
~~#1, 2, 3, 4, 5, 6, 7~~

Ice Frame:

Before

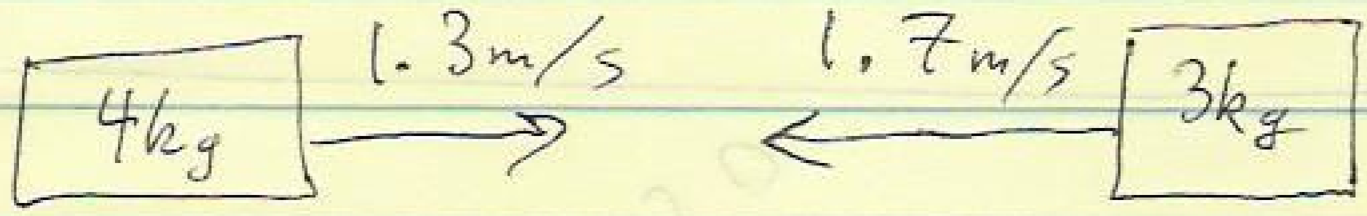


After

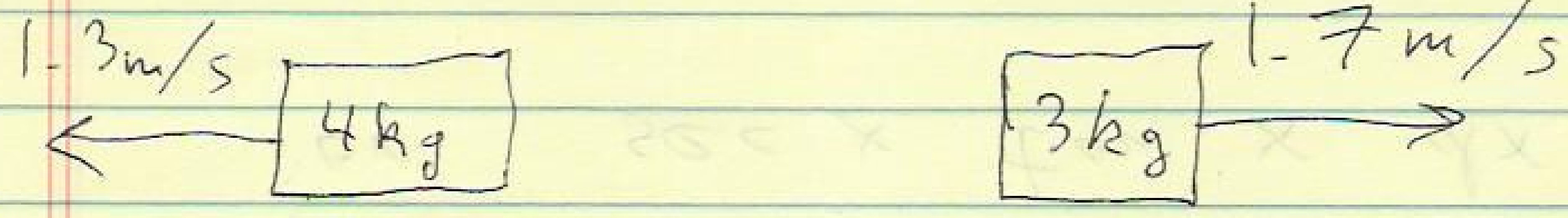


c.m. frame:

Before:



After:



Note how much simpler things look in the c.m frame.