Energy:

(2.1) Kinetic: $K = \frac{1}{2} m v^2$ $\qquad 1\,J = 1\,kg\,m^2/s^2$

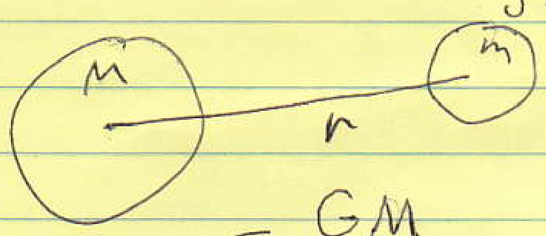
(2.1) Potential: $U$

from gravity near surface of Earth:

$U = mgh$ $\qquad h = \text{height}$ $\qquad g = 9.8\,m/s^2$

from gravity in space: $G = 6.67 \times 10^{-11}\ J \cdot m/kg^2$

(2.3) $U = -\frac{GMm}{r}$




circular orbits: $v = \sqrt{gr}$ $\qquad g = \frac{GM}{r^2}$

(2.1) Heat: $mc\Delta T = \Delta E$

specific heat $\qquad$ change in temperature

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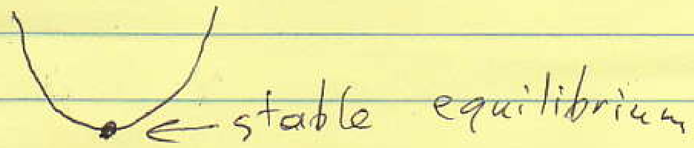
(2.1) Power: $P = \frac{dE}{dt}$

(2.4) Heat is kinetic energy from random microscopic motion

(2.5) Oscillations




IF $U \approx \frac{1}{2} k x^2$ & $E = K + U$
$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$,

then period $= T \approx 2\pi \sqrt{\frac{m}{k}}$

equilibria: where $\frac{dU}{dx} = 0$

Finding $k$ for approximating $U$ with parabola:

$$k = \frac{d^2U}{dx^2}$$ at the stable equilibrium point

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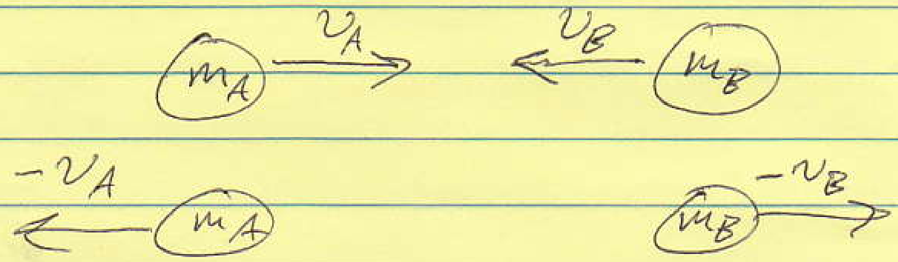
(3.1) Momentum: $p = mv$

Total momentum is conserved in collisions.

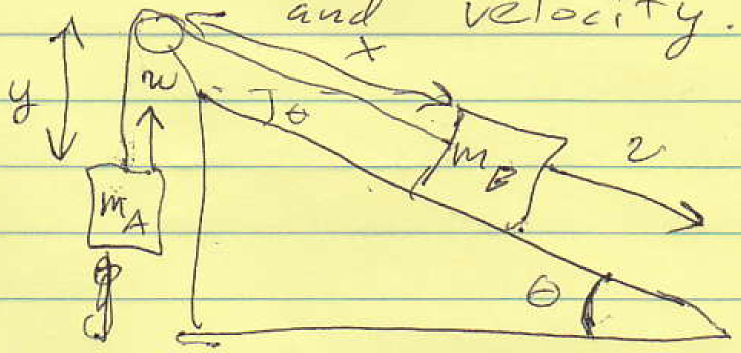
If kinetic energy is also conserved, then, in the <u>center of mass frame</u>, the rebound speeds equal the initial speeds.

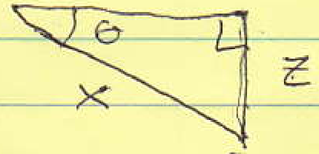



This is for head-on collision of 2 objects

(2.1) Using conservation of energy to find acceleration

Express $K$ and $U$ as formulas depending on only one object's position and velocity.




$U_B = -m_B g z$

$\sin\theta = \frac{z}{x}$

$x \sin\theta = z$

$$K = \frac{1}{2} m_A w^2 + \frac{1}{2} m_B v^2$$

$$U = -m_A g y - m_B g x \sin\theta$$

$K = f(v)$ $\quad U = h(x)$

use $y + x$ is constant...

Then use $0 = \frac{dE}{dt} = \frac{dK}{dt} + \frac{dU}{dt}$.

Then solve for $\frac{dv}{dt}$.

Solutions to HW assigned 3/3. (Chapter 3, section 1) Page 213:

#1 $K = \frac{1}{2}mv^2$ & $p = mv$

$p = mv \Rightarrow v = \frac{p}{m} \Rightarrow K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{m^2} \cdot \frac{m}{2}$

$$\Rightarrow \boxed{K = \frac{p^2}{2m}}$$

#2 Ignoring friction between water and the boat, and sound, other frictions, etc, there is nowhere for the total momentum of the boat and the rowers to be transferred to; so, to keep the center of their mass from moving, the total momentum of the rowers (and the boat's zero momentum) must be $\boxed{\text{zero}}$.

#3 a) For the bullet, $m = 10g = 10^{-2}kg$ and $K = 90J$, so $\frac{1}{2}(10^{-2}kg)v^2 = 90\,kg\,m^2/s^2$, so $v^2 = 4500\,m^2/s^2$, so $v = \sqrt{4500}\,m/s \approx \boxed{130\,m/s}$

3b) For the bullet, $p = m \cdot v = (10^{-2}kg)(130m/s)$

$$\boxed{p = 1.3\ kg \cdot m/s}$$

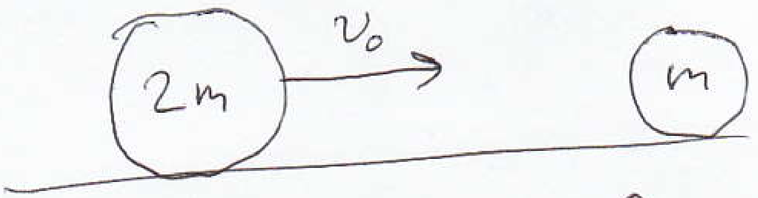
3c) By conservation of momentum, since the initial total momentum of gun and bullet was 0, ~~before and after~~ it is 0 after firing too. So, the gun recoils with the same momentum as the flying bullet, $\boxed{1.3\,\mathrm{kg\cdot m/s}}$, but in the $\boxed{\text{opposite direction}}$, so that the momenta cancel.

3d) From #1, $K = \dfrac{p^2}{2m} = \dfrac{(1.3\,\mathrm{kg\cdot m/s})^2}{2(4\,\mathrm{kg})}$

$= \boxed{0.21\,\mathrm{J}}$, hardly enough to kill someone holding the gun.

#4 Solution p. 865

#5




"Table" frame (Before)

$$v_{cm} = \frac{2m v_0 + m\cdot 0}{2m+m} = \frac{2}{3} v_0$$

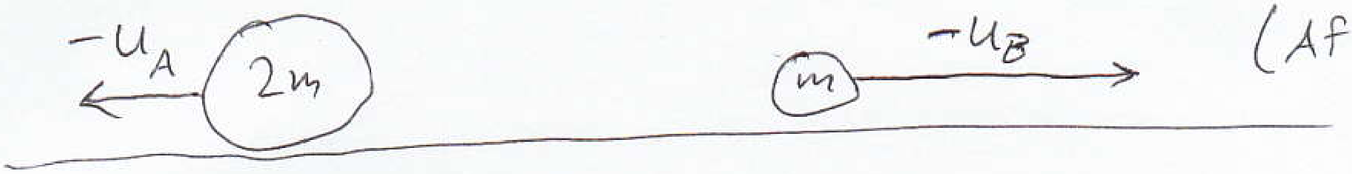
subtract $v_{cm}$ to go to center of mass frame:

$$u_A = v_0 - v_{cm} = \frac{1}{3} v_0$$
$$u_B = 0 - v_{cm} = -\frac{2}{3} v_0$$


c.m. frame
(Before)

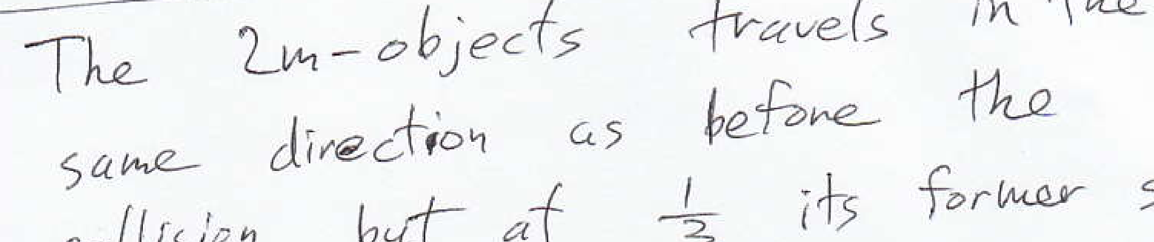
Conservation of momentum & kinetic energy implies recoils of equal speed in the c.m. frame:

c.m. frame
(After)



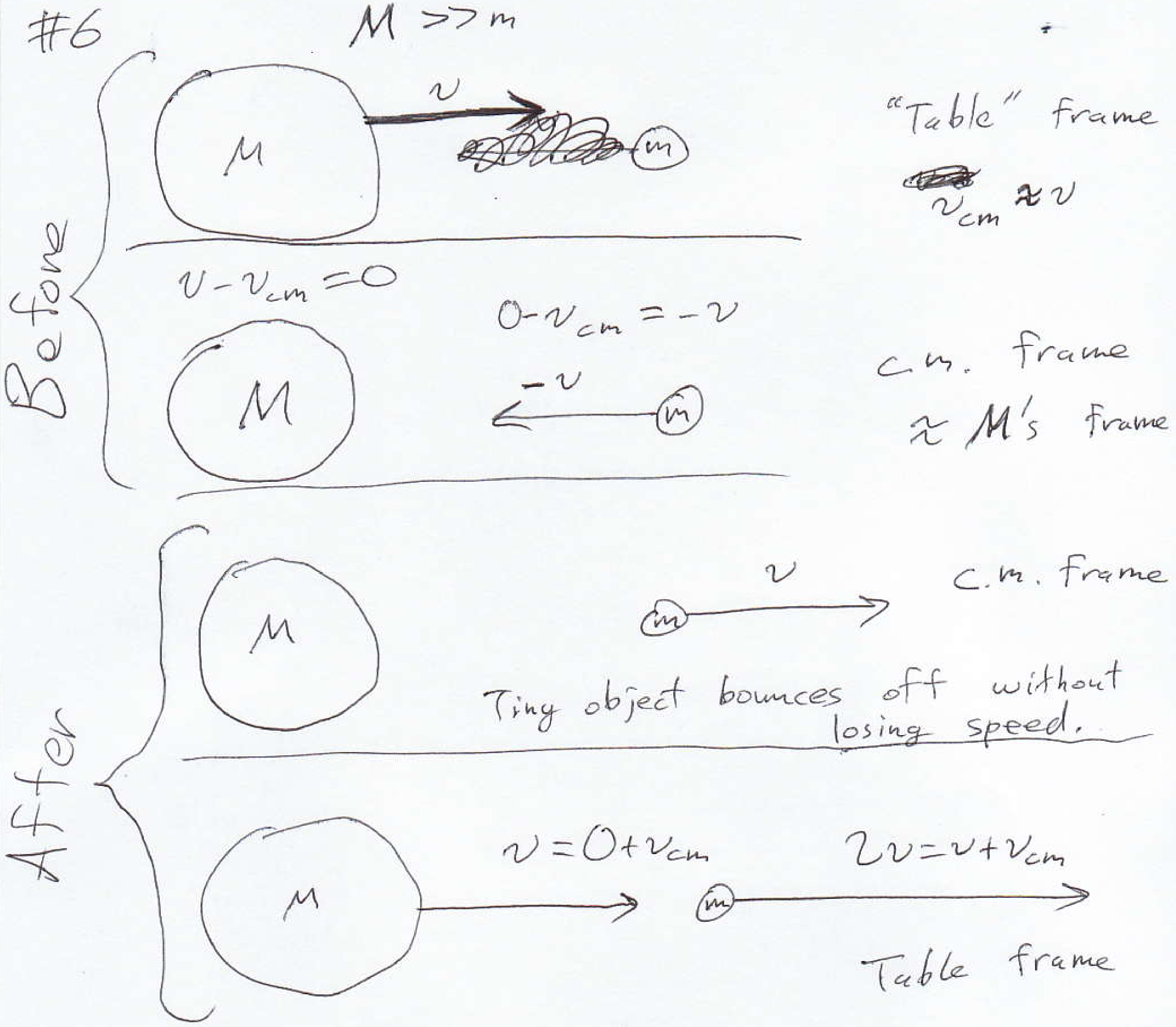

Back to table frame:

$$v_f = -u_A + v_{cm} = \boxed{\frac{1}{3} v_0}$$




The 2m-objects travels in the same direction as before the collision, but at $\frac{1}{3}$ its former speed.

#6


$M >> m$
Before
$v$
$M$
$m$
"Table" frame
$v_{cm} \approx v$
$v - v_{cm} = 0$
$0 - v_{cm} = -v$
$-v$
c.m. frame
$\approx M$'s frame
After
$v$
c.m. frame
Tiny object bounces off without losing speed.
$v = 0 + v_{cm}$
$2v = v + v_{cm}$
Table frame