Quiz If $\vec{v}$ is a vector, then which of these equals kinetic energy?

A) $\frac{1}{2} m \vec{v} \cdot \vec{v}$

B) $\frac{1}{2} m |\vec{v}|$

C) $\frac{1}{2} m |\vec{v}|^2$

D) A and B

E) A and C

F) B and C

($\vec{v}$ is for velocity)

Connection between U & F

Gravity near earth: $\vec{y}$

1-dimensional: $a = -g = -9.8\, m/s^2$ (in y direction)

$$F = \frac{dp}{dt} = m\frac{dv}{dt} = ma = -mg$$

3D: $\vec{a} = -9.8 m/s^2 \, \hat{y}$

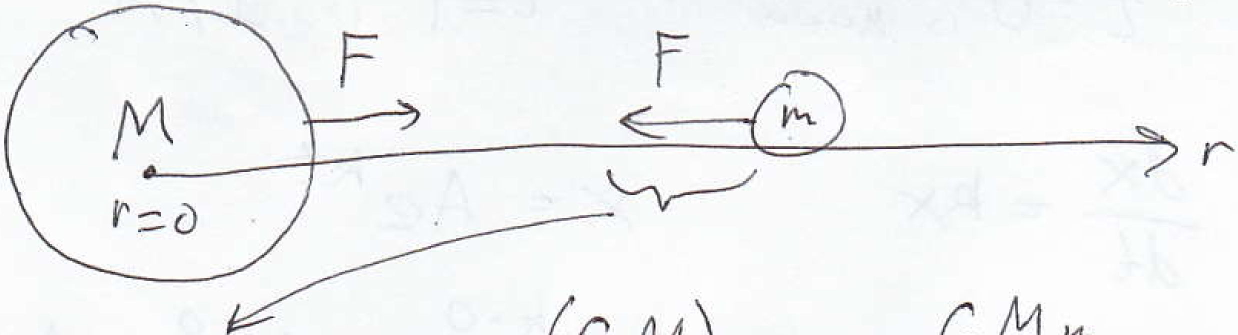
$\hat{w}$ is a vector of magnitude one

$$\hat{w} = \frac{\vec{w}}{|\vec{w}|}$$

1D: $F = -mg \qquad U = mgy$

$$dU = mg\, dy = -F\, dy$$

$$F = -\frac{dU}{dy}$$



$$F = -mg = -m\left(\frac{GM}{r^2}\right) = -\frac{GMm}{r^2}$$

(in the r-direction)

$$U = -\frac{GMm}{r}$$

Check that $F = -\dfrac{dU}{dr}$

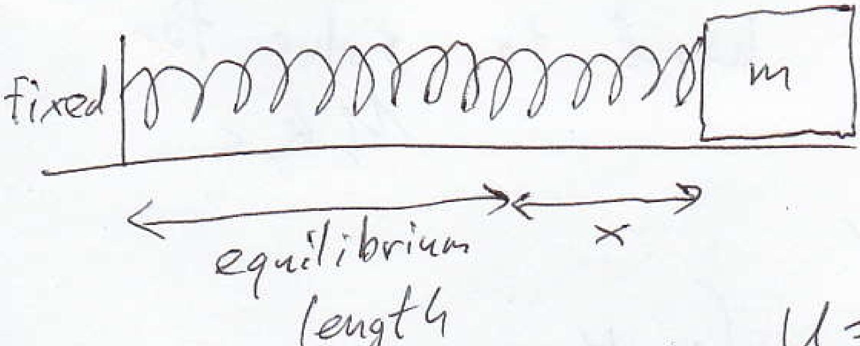
In general, for a force F in the x-direction, if F is caused by an interaction with potential energy U, then $F = -\dfrac{dU}{dx}$.

$$F = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dx}v$$

$$= \frac{mv\,dv}{dx} = \frac{d(\frac{1}{2}mv^2)}{dx} = \frac{dK}{dx} = -\frac{dU}{dx}$$

If $E = K + U$ is conserved.

HW #1 If a mass is sliding along, connected to a spring, with negligible effects from gravity & friction, then what is the force on the mass, given

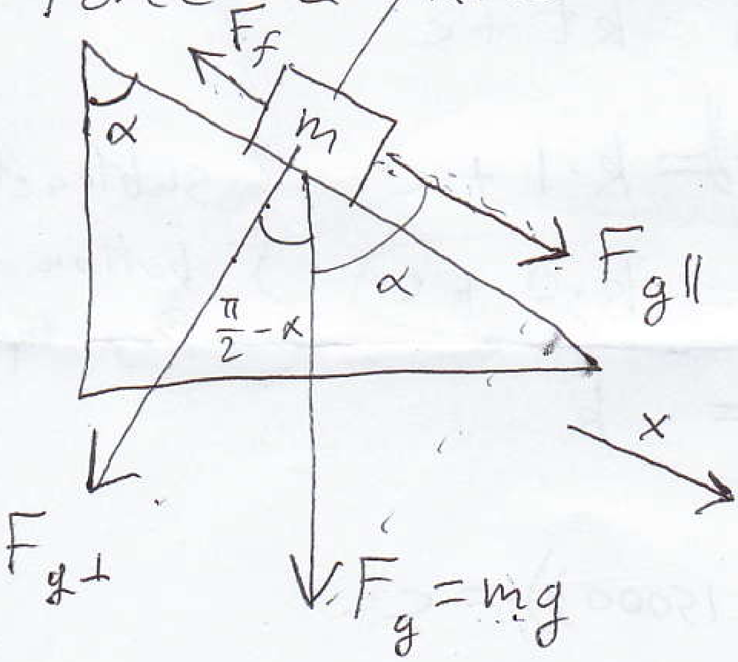


$$U = \frac{1}{2}kx^2,$$

$k = 3.1\ J/(m^2)$, $x = 15cm$?

If the mass is $m = 2.4kg$, then what is the acceleration?

Force & kinetic energy



$$\vec{F}_g = \vec{F}_{g\parallel} + \vec{F}_{g\perp}$$

$$F_{g\parallel} = F_g \cos\alpha$$

$$F_{g\perp} = F_g \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$F_{g\perp} = F_g \sin\alpha$$

$$\vec{F}_n + \vec{F}_{g\perp} = \vec{0}$$

Assume the block is sliding down.

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad ma = \frac{dp}{dt} = F_{g\parallel} - F_f$$

$$\frac{dK}{dx} \overset{\text{see 2 pages ago}}{=} \cdots = ma = F_{g\parallel} - F_f$$

In general, $\frac{dK}{dx}$ equals the net Force in the x-direction.

$$dK = (F_{g\parallel} - F_f)\,dx$$

$$\Delta K = K_f - K_i = \int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} \underbrace{(F_{g\parallel} - F_f)}_{\text{constant}}\,dx$$

$$= (F_{g\parallel} - F_f)\int_{x_i}^{x_f} dx = (F_{g\parallel} - F_f)\,\Delta x$$

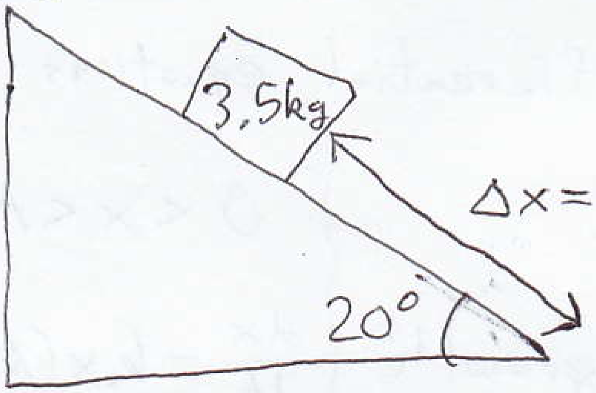
ΔK is also called net work W.

Often we talk about work done by each force

energy converted from potential to kinetic $\leftarrow \int_{x_i}^{x_f} F_{g\parallel}\,dx =$ work done by gravity

energy converted to heat $\leftarrow \int_{x_i}^{x_f} (-F_f)\,dx =$ work done by friction

HW#2



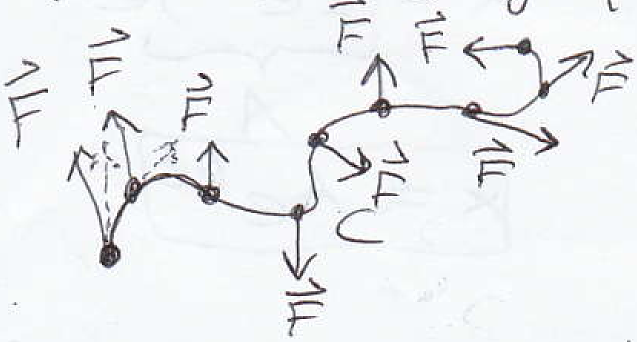
Assume the block starts sliding from rest.

If it takes 0.93 s for the block to slide down the ramp, then compute

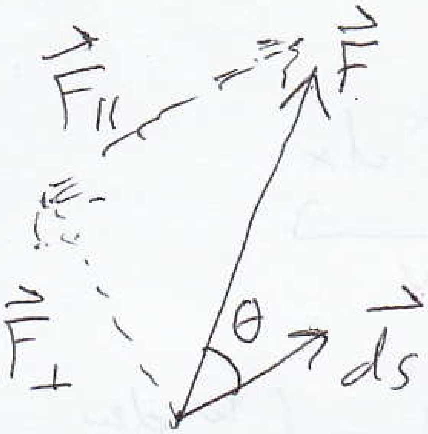
a) the loss of potential energy
b) the gain of kinetic energy
c) the work done by friction
d) the work done by gravity
e) the final velocity at the bottom of the ramp.

3D version of work done by Force $\vec{F}$ along path C

ds = length



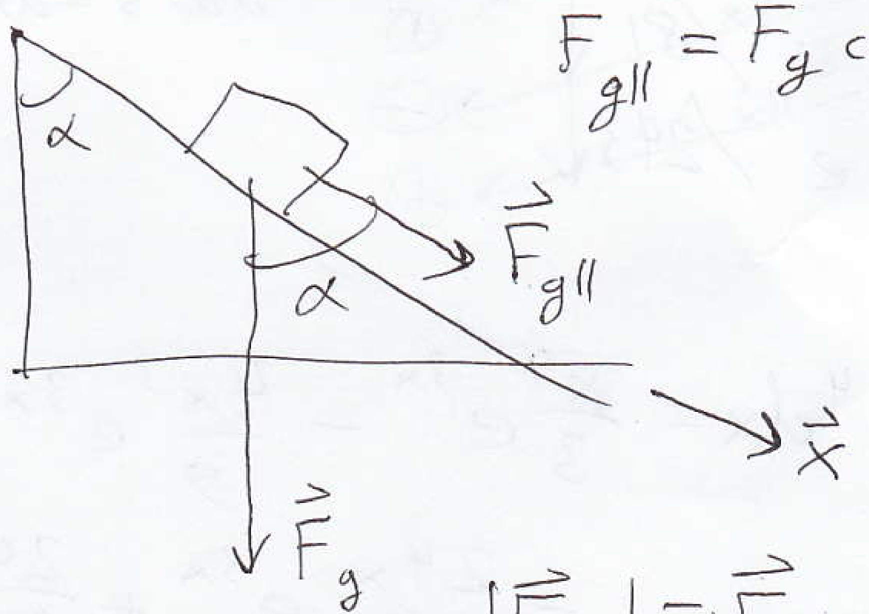
$$\int_C F_{\parallel}\, ds = \int_C \vec{F} \cdot \vec{ds}$$



$$\vec{F}\cdot\vec{ds} = \pm|\vec{F}_{\parallel}||\vec{ds}|$$
$$= |\vec{F}||\vec{ds}|\cos\theta$$

Compare to ramp:

$$F_{g\parallel} = F_g \cos\alpha$$



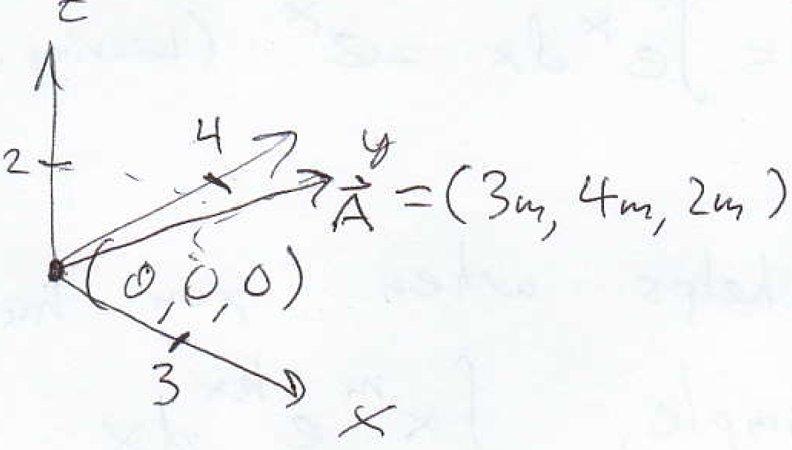
$$|\vec{F}_{g\parallel}| = \pm\vec{F}_g\cdot\hat{x}$$

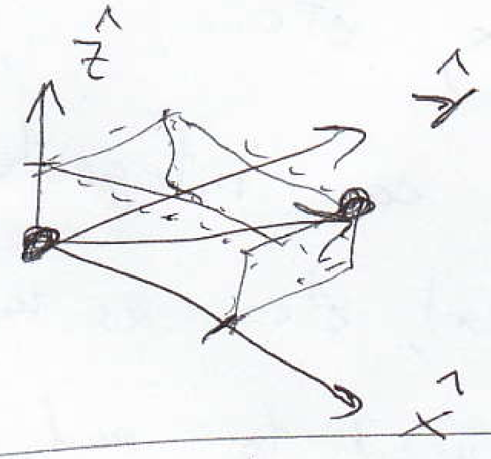
HW #3.

$\vec{A} = (3m, 4m, 2m)$

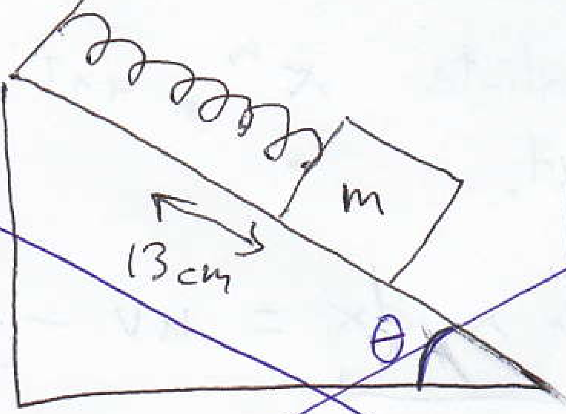
$\vec{B} = (6m, -1m, 3m)$.

Find the angle between $\vec{A}$ and $\vec{B}$.





HW #4



$U_{spring} = \frac{1}{2}kx^2$

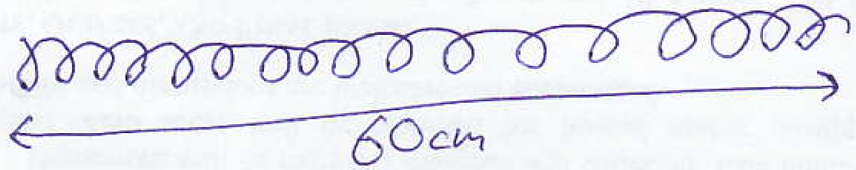
$m = 1.7kg$

$\theta = 25°$

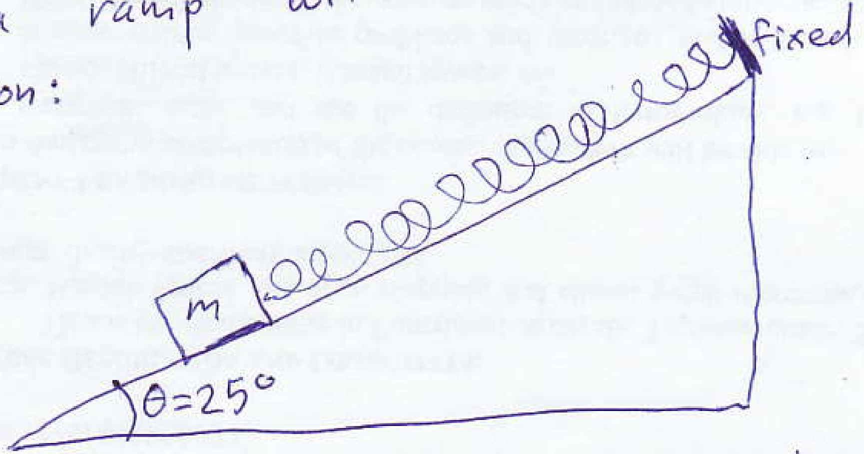
Suppose we release the mass from rest, and in 0.50s, it slides up a distance 13cm along the ramp, ~~then~~ starts sliding down. If $\mu_k = 0.20$, then find the spring's constant k.

Easier version of #4:

When resting horizontally, a spring has equilibrium length of 60cm.



Now suppose we attach a mass of $m = 1.7\,kg$ to the spring and place them on a ramp with a $\theta = 25^\circ$ angle of elevation:



Pulling on the block, we stretch the spring to a length of 68 cm. We release the block and observe the spring contract to 55cm before starting to length again. If the coefficient of (sliding) kinetic friction is $\mu_k = 0.20$, then what is the spring's constant k?

$\left(U_{spring} = \frac{1}{2} k x^2\right)$

Give answer in J/m^2.