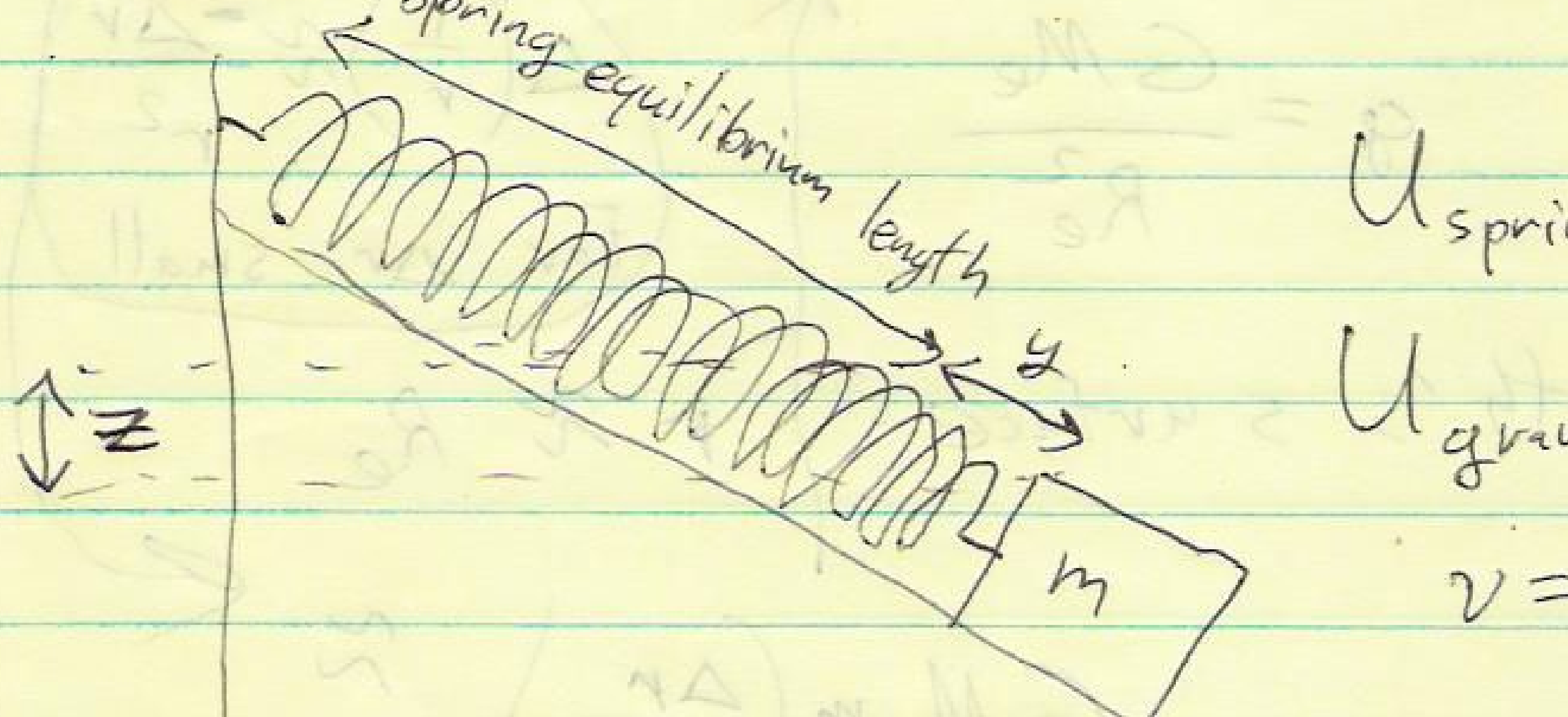
A mass of 5.0 kg slides along a ramp with a 35° angle of elevation. It is connected to a spring that goes to the top of the ramp. The spring has spring constant $k_0 = 2.0\ kg/s^2$.

↑ Same as J/m^2

What is the period T of oscillation of the sliding mass?



$$U_{spring} = \frac{1}{2} k_0 y^2$$

$$U_{gravity} = -mgz$$

$$v = \frac{dy}{dt} \qquad K = \frac{1}{2} m v^2$$

$$U = U_{spring} + U_{gravity}$$

$$\sin\theta = \frac{y}{z} \qquad z = \frac{y}{\sin\theta}$$

$$U = \frac{1}{2} k_0 y^2 - mg\left(\frac{y}{\sin\theta}\right) = \frac{k_0 y^2}{2} - \frac{mgy}{\sin\theta}$$

~~Find the equilibrium pos~~

$$\frac{dU}{dy} = k_0 y - \frac{mg}{\sin\theta} \qquad \frac{d^2U}{dy^2} = k_0 \text{ (everywhere)}$$

$$\Rightarrow \left(\frac{d^2U}{dy^2} \text{ at the equilibrium } y\text{-position} \right) = k_0$$

$$\Rightarrow U = \frac{1}{2} k x^2 + \underbrace{c}_{\text{some constant}}$$

where $k = k_0$ and x is displacement from equilibrium.

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{k_0}} = 2\pi \sqrt{\frac{5.0\,\text{kg}}{2.0\,\text{kg}/\text{s}^2}}$$

$$T = 8.9\,\text{s}$$

Observe that T does not depend on the angle of the ramp, the initial position or initial velocity of the mass, nor the gravitational field strength. Only the spring's constant and the mass affect T in this example.