Ch. 7 & 8:

Work, energy, power.

Recall from Ch. 2:

$v^2 - v_0^2 = 2a(x - x_0)$ if a constant

$\Delta(v^2) = 2a\Delta x$

$\frac{1}{2} m \Delta(v^2) = m a \Delta x \qquad F = ma$

$$\underbrace{\Delta\left(\tfrac{1}{2} m v^2\right)}_{\text{change in kinetic energy}} = \underbrace{F \Delta x}_{\text{work}}$$

slightly modify

Still true if a not constant:

$$\Delta\left(\tfrac{1}{2} m v^2\right) = \int_{v_0}^{v} d\left(\tfrac{1}{2} m v^2\right) = \int_{v_0}^{v} \tfrac{1}{2} m (2v\,dv)$$

$$\frac{d(v^2)}{dv} = 2v$$

$$\Delta\left(\tfrac{1}{2} m v^2\right) = \int_{v_0}^{v} m v\,dv$$

$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$

$$\Delta\left(\tfrac{1}{2}mv^2\right) = \int_{v_0}^{v} m\,\frac{dx}{dt}\,dv = \int_{x_0}^{x} m\,\frac{dv}{dt}\,dx$$

$d\,\square$ = tiny change in $\square$

$\int d\,\square$ = sum of tiny changes in $\square$

= total change in $\square$

$= \Delta\,\square$

$$\Delta\left(\tfrac{1}{2}mv^2\right) = \int_{x_0}^{x} ma\,dx = \int_{x_0}^{x} F\,dx$$

not quite $F\Delta x$.

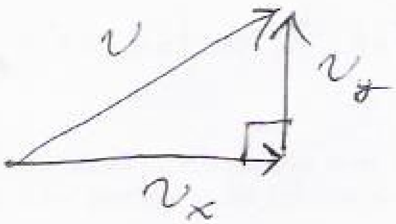
If F constant,

$$\int_{x_0}^{x} F\,dx = F(x - x_0) = F\,\Delta x$$

For tiny changes:

$$d\left(\tfrac{1}{2}mv^2\right) = F\,dx$$

2D version



$$v_x^2 + v_y^2 = v^2$$

$$\Delta\left(\frac{1}{2} m v_x^2\right) = \int_{x_0}^{x} F_x \, dx$$

$$\Delta\left(\frac{1}{2} m v_y^2\right) = \int_{y_0}^{y} F_y \, dy$$

$$\frac{1}{2} m \left(v_y^2 - v_{y0}^2\right) = \int_{y_0}^{y} F_y \, dy$$

$$\frac{1}{2} m \left(v_x^2 - v_{x0}^2\right) = \int_{x_0}^{x} F_x \, dx$$

$$\underbrace{\Delta\left(\frac{1}{2} m v^2\right)}_{K = \text{kinetic energy}} = \frac{1}{2} m \left(v^2 - v_0^2\right) = \underbrace{\int_{(x_0, y_0)}^{(x, y)} \left(F_x \, dx + F_y \, dy\right)}_{\text{work done by force } F}$$

$$\vec{r} = (x, y)$$

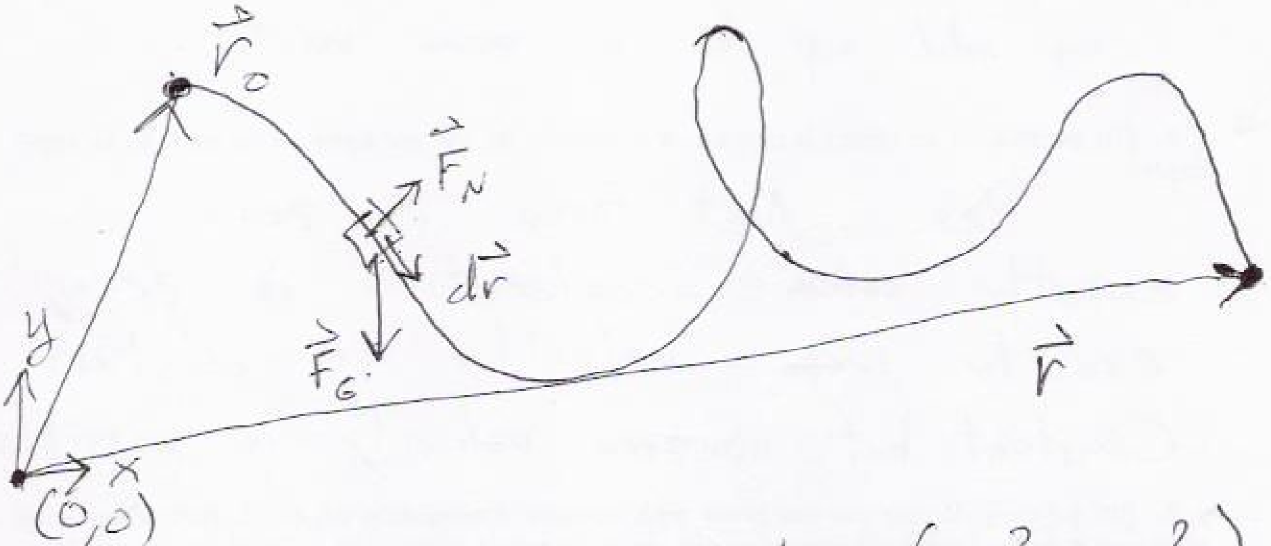
$$d\vec{r} = (dx, dy)$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

$$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$$

$$\Delta\left(\frac{1}{2}mv^2\right) = \Delta K = \int_{\vec{r}_0}^{\vec{r}} |\vec{F}|\,|d\vec{r}|\cos\theta$$

Frictionless roller coaster:



$$\Delta K = K - K_0 = \frac{1}{2}m\left(v^2 - v_0^2\right)$$

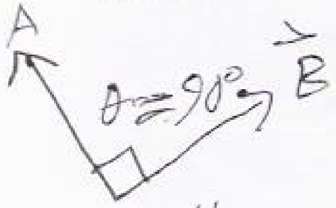
$F_{Gx} = 0$ gravity only force

$F_{Gy} = -mg$ not perpendicular to path of roller coaster car.

$d\vec{r} \perp \vec{F}_N$

$\cos 90° = 0$

$\vec{A} \perp \vec{B} \Rightarrow$



$\Rightarrow \vec{A}\cdot\vec{B} = 0$

The normal force "does no work."

$$\Delta K = \int_{\vec{r}_0}^{\vec{r}} (\vec{F}_G + \vec{F}_N) \cdot d\vec{r}$$

$$\vec{F}_N \perp d\vec{r} \Rightarrow \vec{F}_N \cdot d\vec{r} = 0$$

$\vec{F} \perp \vec{v} \Rightarrow \vec{F}$ only changes direction, not magnitude of $\vec{v}$.

$$\Delta K = \int_{\vec{r}_0}^{\vec{r}} \vec{F}_G \cdot d\vec{r} = \int_{x_0}^{x} F_{Gx}\, dx + \int_{y_0}^{y} F_{Gy}\, dy$$

$F_{Gx} = 0 \qquad F_{Gy} = -mg$

constant

$$\Delta K = -mg\,\Delta y$$

If the roller coaster's final height is 50m lower than its initial height, ~~and the~~

then $\frac{1}{2} m \Delta v^2 = -mg\,\Delta y$,

so $\Delta v^2 = 2g(50\text{m}) = 980\ \text{m}^2/\text{s}^2$

If $v_0 = 2$ m/s, then $v^2 - v_0^2 = 980\ \text{m}^2/\text{s}^2$,

so $v^2 = (2\text{m/s})^2 + 980\ \text{m}^2/\text{s}^2 = 984\ \text{m}^2/\text{s}^2$

$v = \sqrt{984}\ m/s = 31\ m/s \simeq 60\ mph$

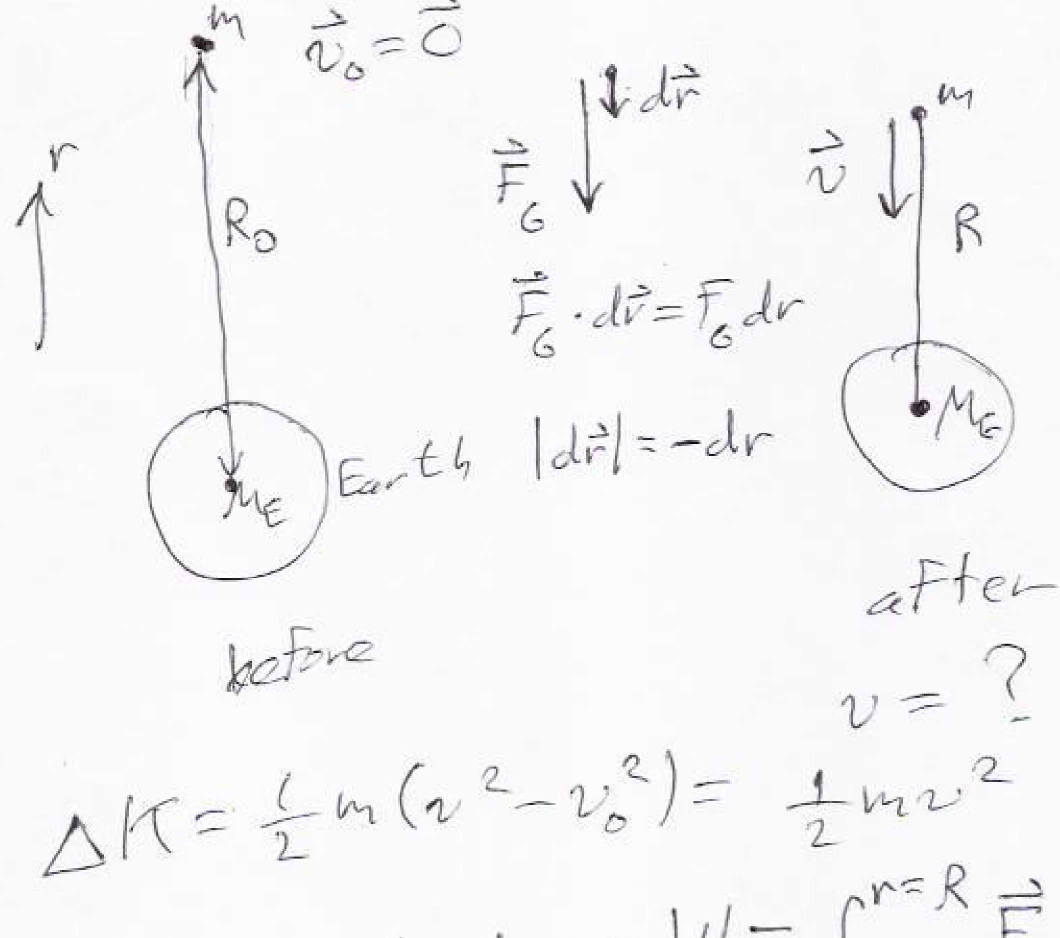
$\Delta v = 29\ m/s$

If $v_0 = 10\ m/s$, then

$v^2 = (10\ m/s)^2 + 980\ m^2/s^2 = 1080\ m^2/s^2$

$v = 33\ m/s \Rightarrow \Delta v = 23\ m/s$

Gravity in space:



$\vec{F}_G \cdot d\vec{r} = F_G\, dr$

$|d\vec{r}| = -dr$

$\Delta K = \frac{1}{2} m (v^2 - v_0^2) = \frac{1}{2} m v^2$

$\Delta K = \text{work done} = W = \int_{r=R_0}^{r=R} \vec{F}_G \cdot d\vec{r}$

$W = \int_{R_0}^{R} \frac{G M_E m}{r^2}\, dr = G M_E m \int_{R_0}^{R} r^{-2} (-dr)$

$$W = -GM_E m\left(-r^{-1}\right)\Big|_{R_0}^{R}$$

$$W = -GM_E m\left(\left(-R^{-1}\right) - \left(-R_0^{-1}\right)\right)$$

$$W = GM_E m\left(\frac{1}{R} - \frac{1}{R_0}\right)$$

If you go from 5.0×10^{7} m to 3.0×10^{7} m, then

$$\frac{1}{2}\cancel{m}v^2 = GM_E\cancel{m}\left(\frac{1}{3.0\times10^{7}\,m} - \frac{1}{5.0\times10^{7}\,m}\right)$$

$$v = \sqrt{2GM_E\left(\frac{1}{3.0\cdot10^{7}\,m} - \frac{1}{5.0\cdot10^{7}\,m}\right)}$$

$$6.67\cdot10^{-11}\ N\,m^2/kg^2$$

$$M_E = 5.97\cdot10^{24}\ kg$$

$$v = \sqrt{1.06186\ldots\cdot10^{7}\ Nm/kg}$$

$$N\cdot m/kg = \left(kg\cdot m/s^2\right)\cdot m/kg$$

$$= m^2/s^2$$

$$v = 3.3\cdot10^{3}\ m/s \approx 7000\,mph$$

~~Next time:~~

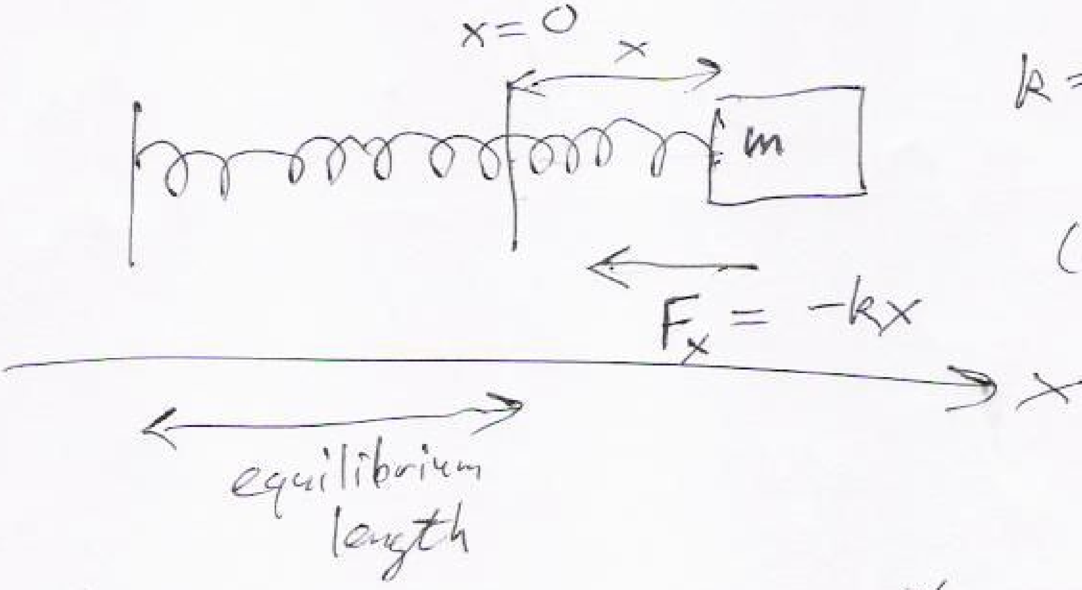
Power: $d(\text{Work})/d(\text{time}) = \dfrac{dW}{dt}$

average power: $\dfrac{\Delta W}{\Delta t}$

$$dW = \vec{F} \cdot d\vec{r} \Rightarrow P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

$$P = \vec{F} \cdot \vec{v} = F v \cos\theta$$

Springs



k = spring constant (indep. of m)

$$W = \int_{x_0}^{x} \underbrace{-kx}_{F_x}\, dx = -k \int_{x_0}^{x} x\, dx = \frac{k}{2}(x_0^2 - x^2)$$

$$= \Delta K = \frac{1}{2} m (v^2 - v_0^2)$$

$$P = dW/dt = F_x v_x = -kxv$$

Potential energy:

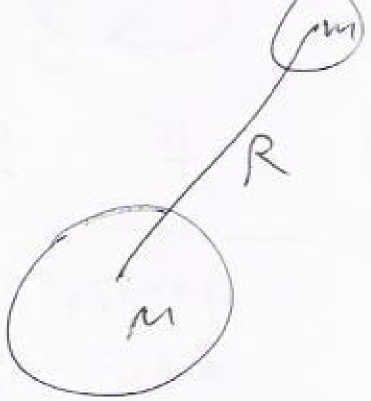
If the work done by a force F only depends initial & final positions, then we define change in potential energy $\Delta U_F = -W_F$

Work done by F

Gravity near earth's surface:

$$\Delta U_G = mg\,\Delta y$$

Gravity in space:



$$\Delta U_G = -GMm\,\Delta\left(\frac{1}{R}\right)$$

Spring: $\Delta U_s = \frac{1}{2}k\,\Delta(x^2)$

Friction creates heat, not potential energy, from kinetic energy.