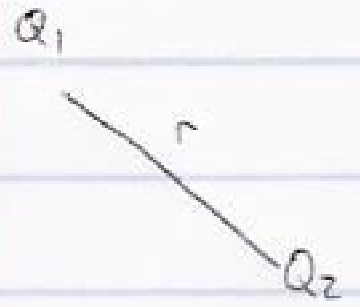
Chapter 21

$F = \frac{K Q_1 Q_2}{r^2}$ = Coloumbs law



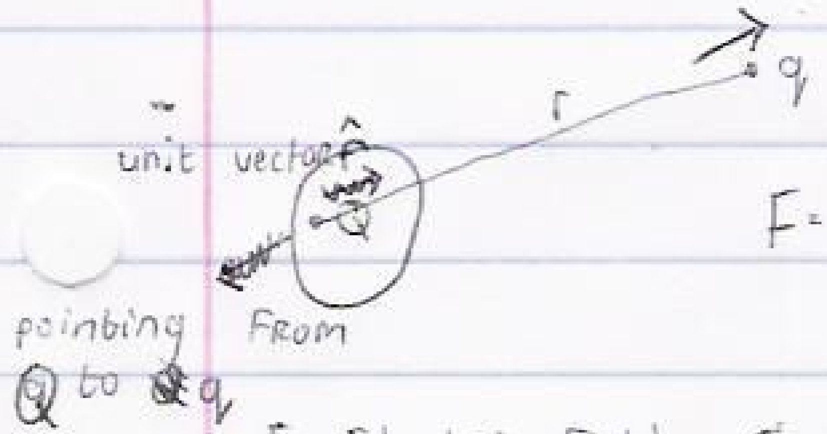
- $K = 8.99 \times 10^{9}\ N.m^2/c^2$ OR $K = \frac{1}{4\pi\epsilon_0} \Leftrightarrow \epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12}$ $\frac{C^2}{N.m^2}$

ϵ_0 = "permittivity of free space"

- $e = 2.718...$

also e (italics) = magnitude charge of an electron = 1.60×10^{-18} C

- electrons are negatively charged ∴ $-e$
- protons have charge $+e$
- Neutrons have charge of 0.



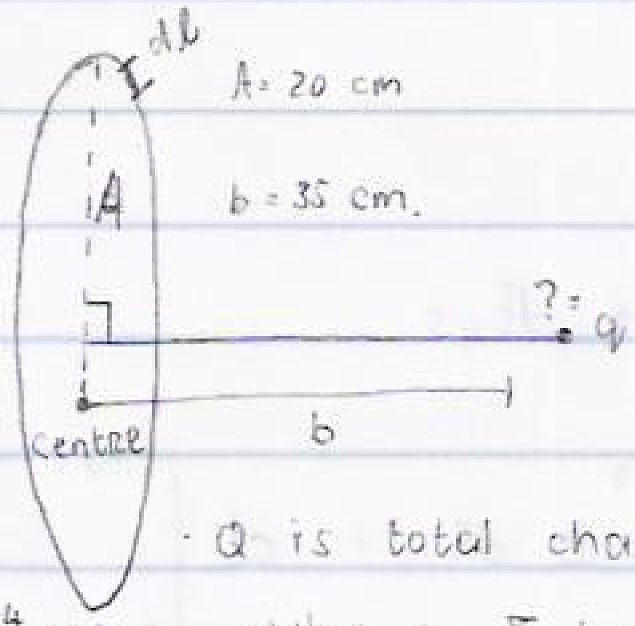
$F = \frac{KQq}{r^2}$

$\vec{F} = \frac{KQq\hat{r}}{r^2}$

- E = Electric Field strength $E = \frac{F}{q} = \frac{KQ}{r^2}$

$\vec{E} = \frac{KQ\hat{r}}{r^2}$

} Vector Form



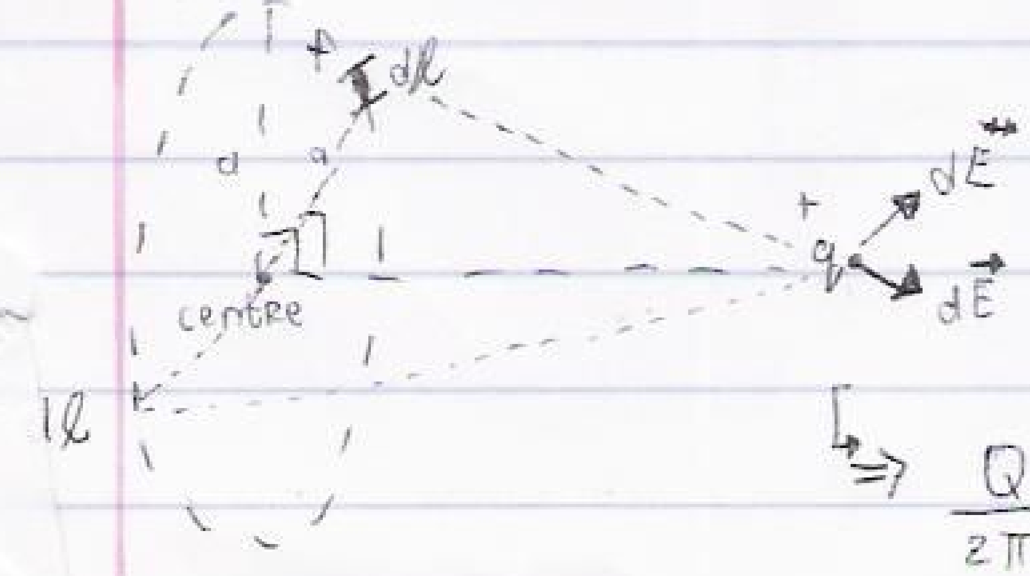
small test charge. (positive ∴ repells)

- Q is total charge on the ring / equally distributed.

$5.00 \times 10^{-4} = Q$

- What is E here ?

constant = $\frac{\text{length}}{\text{charge}}$



$\frac{2\pi a}{Q} = \frac{\text{length}}{\text{charge}}$; $\frac{Q}{2\pi a} = \frac{\text{charge}}{\text{length}} = \frac{dQ}{dl}$

$\Rightarrow \frac{Q}{2\pi a} dl = dQ$

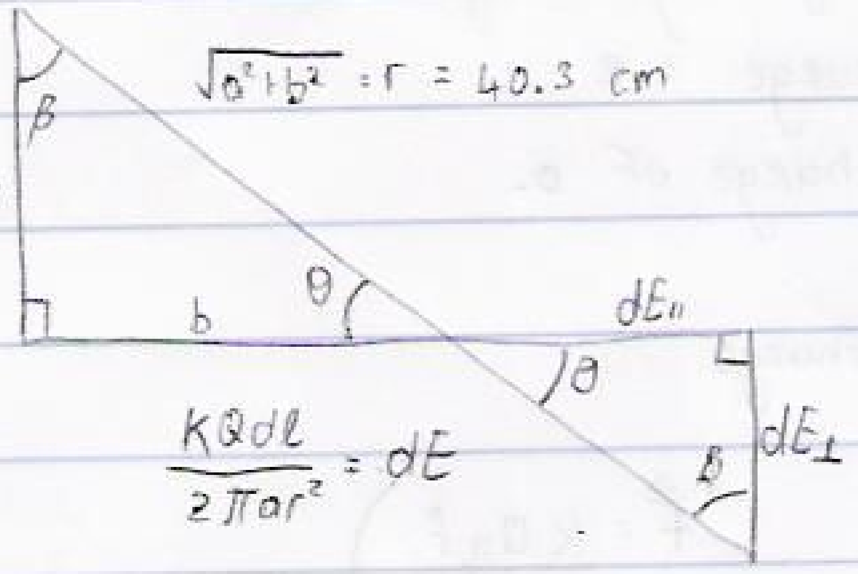
no ARROW

magnitude $\neq$ $dE = |dE| = \frac{k\,dQ}{r^2} = \frac{k}{r^2} \cdot \frac{Q}{2\pi a} \cdot d\ell$

- Add up all the $d\vec{E}_{\parallel}$'s

The $d\vec{E}_{\perp}$s cancel

- Find the magnitude of $dE_{\parallel} = |d\vec{E}_{\parallel}| = ?$



$\frac{kQd\ell}{2\pi a r^2} = dE$

$\frac{b}{\sqrt{a^2+b^2}} = \cos\theta = \frac{dE_{\parallel}}{dE}$ $\left(\frac{\text{adj}}{\text{hyp}}\right)$

$\Downarrow$

$\frac{b\,dE}{\sqrt{a^2+b^2}} = dE_{\parallel}$

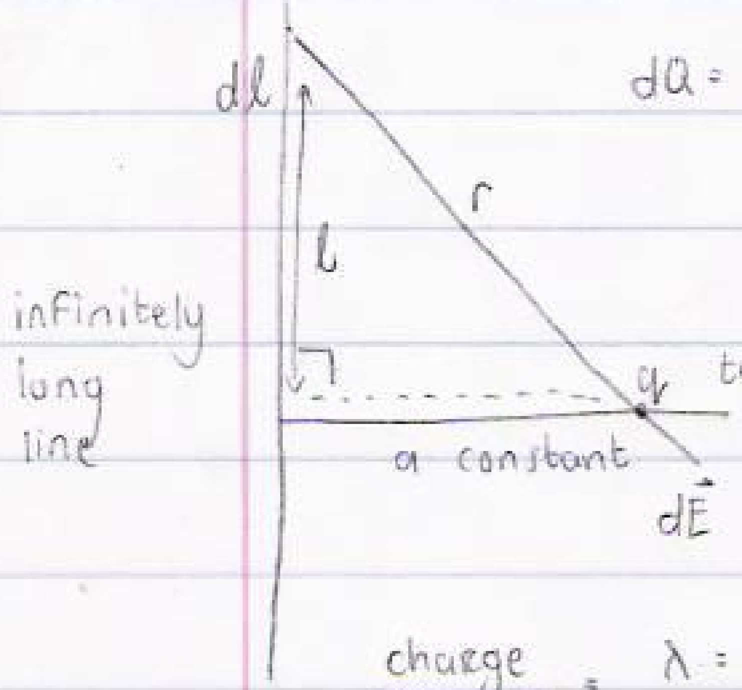
$dE_{\parallel} = \frac{kQ(d\ell)b}{2\pi a r^2\sqrt{a^2+b^2}}$ Add up all the $dE_{\parallel}$s

$E_{\parallel} = \int_0^{2\pi a} \underbrace{\frac{kQb}{2\pi a r^2\sqrt{a^2+b^2}}}_{\text{constant on the ring}}$

$\int_0^{2\pi a} c\,d\ell = c\ell \Big|_0^{2\pi a} = c\cdot 2\pi a - c\cdot 0 = c\cdot 2\pi a.$

$= \frac{kQb \cdot 2\pi a}{2\pi a r^2\sqrt{a^2+b^2}}$

$= \frac{kQb}{r^2\sqrt{a^2+b^2}} = \frac{kQb}{r^3}$



dQ = charge = $\lambda \cdot dl$

$\frac{\text{charge}}{\text{length}} = \lambda$ = constant.

⇓

charge = λ = length.

$$dE = \frac{k\,dQ}{r^2} = \frac{k\,\lambda\,dl}{r^2} = \frac{k\lambda\,dl}{a^2+l^2}$$

$$E = \int_{-\infty}^{\infty} dE_{\parallel} = \int_{-\infty}^{\infty} \frac{k\lambda a\,dl}{(a^2+l^2)^{3/2}}$$

$$= k\lambda a \int_{-\infty}^{\infty} \frac{dl}{(a^2+l^2)^{3/2}}$$

$$= k\lambda a \int_{-\pi/2}^{\pi/2} \frac{a\sec^2\theta\,d\theta}{a^3\sec^3\theta} = \frac{k\lambda}{a}\int_{-\pi/2}^{\pi/2} \frac{d\theta}{\sec\theta} = \frac{k\lambda}{a}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{k\lambda}{a}\sin\theta \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{k\lambda}{a}(1-(-1)) = \frac{2k\lambda}{a}.$$