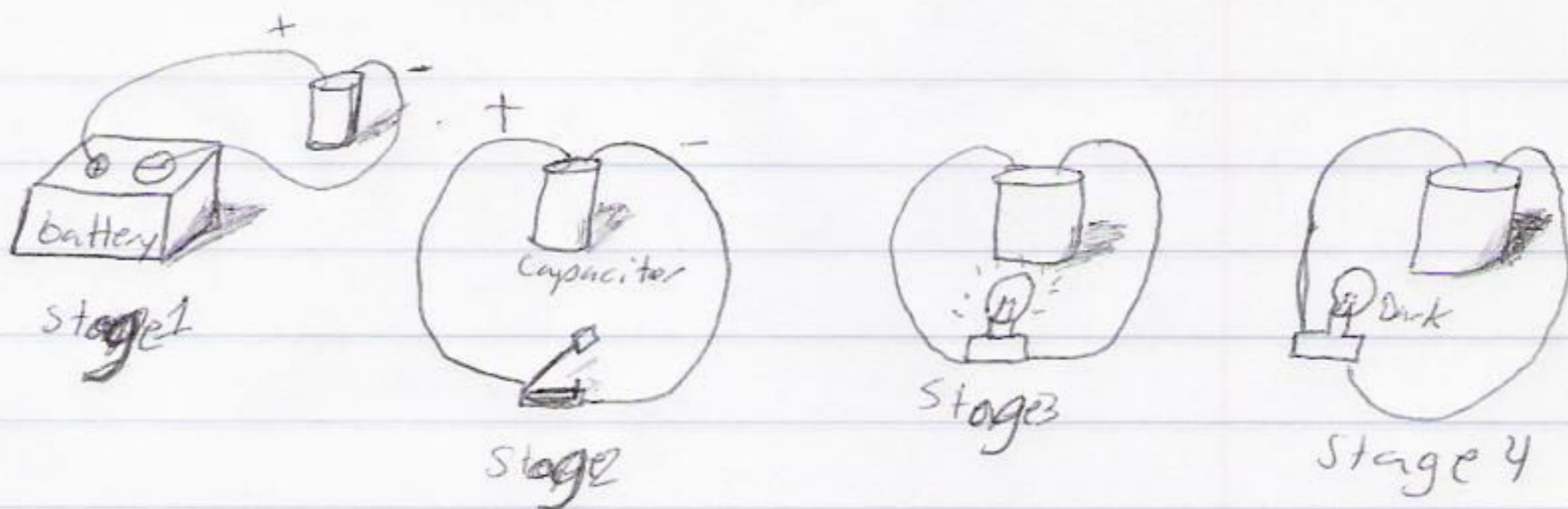

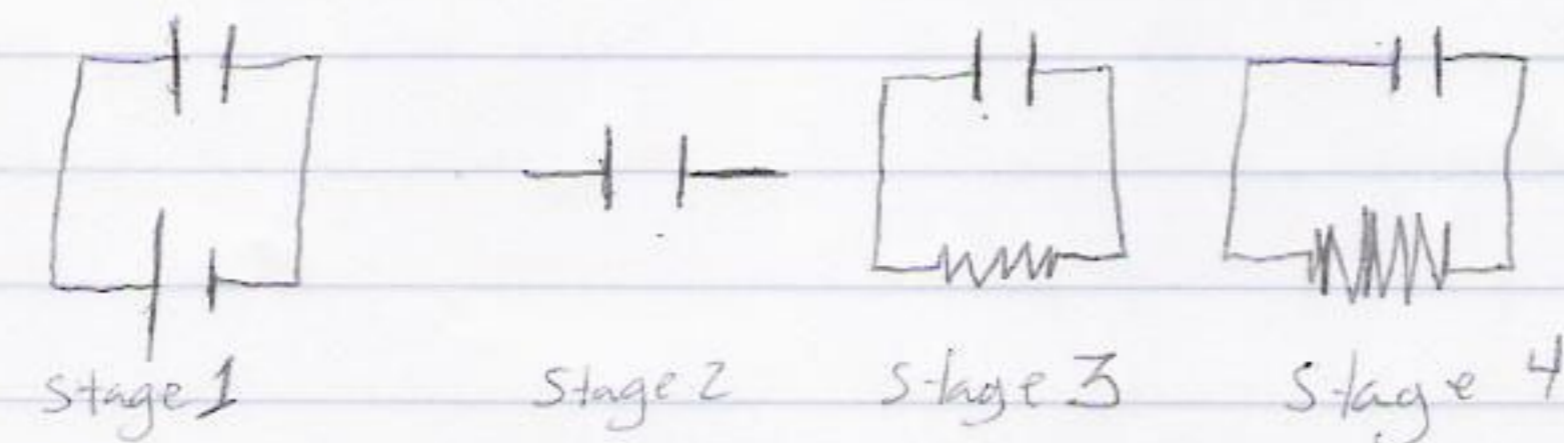


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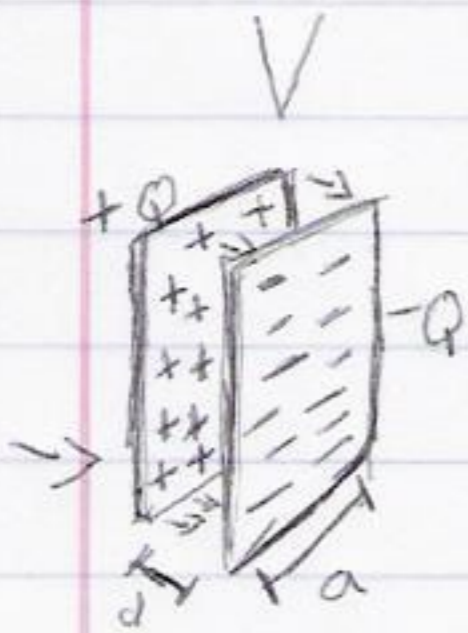
Notes



Stage 1

 Capacitors store energy
 Capacitors stores two opposite charges



Capacitance: $C = \frac{Q}{V}$




Two metal plates with nothing in between
 $d \ll a, b$ (Plates close together)

\vec{E} Points from + to - in between plates

$V = Ed \leftarrow \vec{E} = \frac{\sigma}{\epsilon_0}$ where $\sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A}$ where $A = ab$

Capacitance: $C = \frac{Q}{V}$

For capacitance  $V = V_{TS} = V_S - V_T = - \int_T^S \vec{E} \cdot d\vec{x} = - \int_T^S E dx \Rightarrow V = Ed \Rightarrow$

$$\frac{\sigma d}{\epsilon_0} = \left(\frac{Qd}{A\epsilon_0} \right) \Rightarrow C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\epsilon_0}} = \frac{A\epsilon_0}{d}$$

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

Actual capacitors are typically in the range of 1 mF to 1 pF (10^{-6} F to 10^{-12} F)

$$V = \frac{U}{q} \quad \text{Voltage} = \frac{\text{Potential energy}}{\text{charge}} = \frac{\text{work done on charge}}{\text{charge}}$$

||
electric
Potential



Charging a capacitor from $q=0$ to $q=Q$ stores energy in the capacitor.

$$C = \frac{Q}{V} \quad CV = Q \quad V = \frac{Q}{C}$$

$$\Delta U = q\Delta V \quad \text{when } q \approx \text{constant}$$

$$\Delta U = V\Delta q \quad \text{when } V \approx \text{constant}$$

$$C = \text{capacitance}$$

$$V = \text{voltage}$$

$$q = Q = \text{charge}$$

$$U = \int_{q=0}^{q=Q} dU = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \cdot \frac{Q^2}{2} = \frac{Q^2}{2C} = \frac{Q^2}{2 \cdot \frac{Q}{V}} = \frac{QV}{2} = \frac{1}{2} CV^2$$

$$U = \int_{q=0}^{q=Q} dU = \int_{V=0}^{V=Q/C} q dV = \int_0^{Q/C} C V dV = C \cdot \frac{(Q/C)^2}{2} = \frac{Q^2}{2C}$$

Example 638



$k = \text{dielectric constant of the material}$

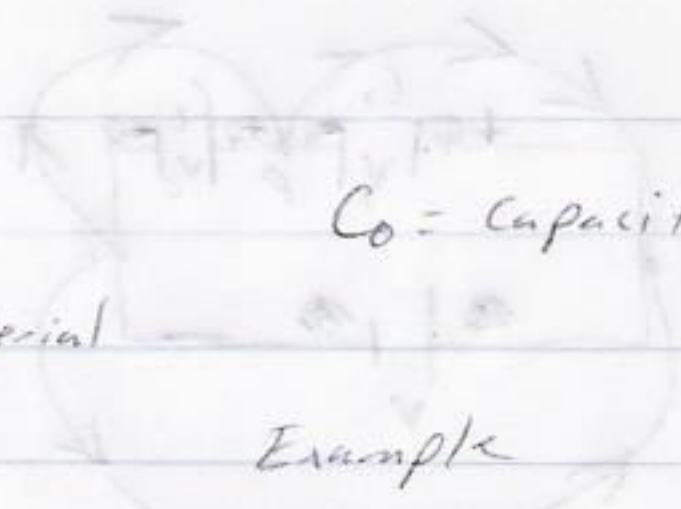
ϵ_0 = permittivity of Free space

C_0 = capacitance

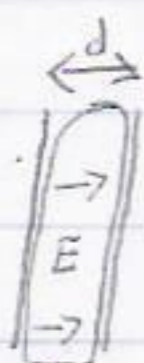
$\epsilon = k\epsilon_0$ = permittivity of the material

Example

$k > 1$
 k = dielectric constant of the material



$E = \frac{\sigma}{\epsilon_0}$



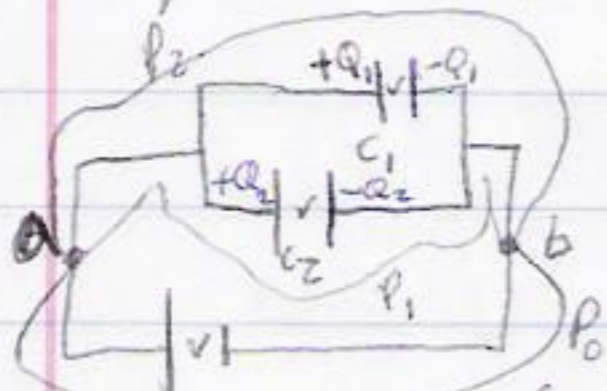
$E = \frac{\sigma}{\epsilon} = \frac{\sigma}{k\epsilon_0} < \frac{\sigma}{\epsilon_0}$ E gets smaller for a fixed Q

$\sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A}$

$\Rightarrow V = Ed$ gets smaller $\Rightarrow C = \frac{Q}{V}$ get bigger

$E = \frac{E_0}{k} \Rightarrow V = \frac{V_0}{k} \Rightarrow C = \frac{Q}{V} = \frac{Qk}{V_0} = kC_0$

Capacitors in parallel



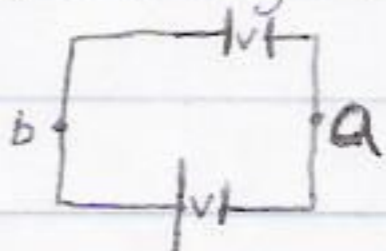
net charge of circuit = 0

$V = V_{ab} = V_a - V_b = - \int_b^a \vec{E} \cdot d\vec{l} =$

$- \int_{P_0} \vec{E} \cdot d\vec{l} = - \int_{P_1} \vec{E} \cdot d\vec{l} = - \int_{P_2} E_2 \Rightarrow$

$\Rightarrow d\vec{l} = - \int_{P_2} \vec{E} \cdot d\vec{l}$

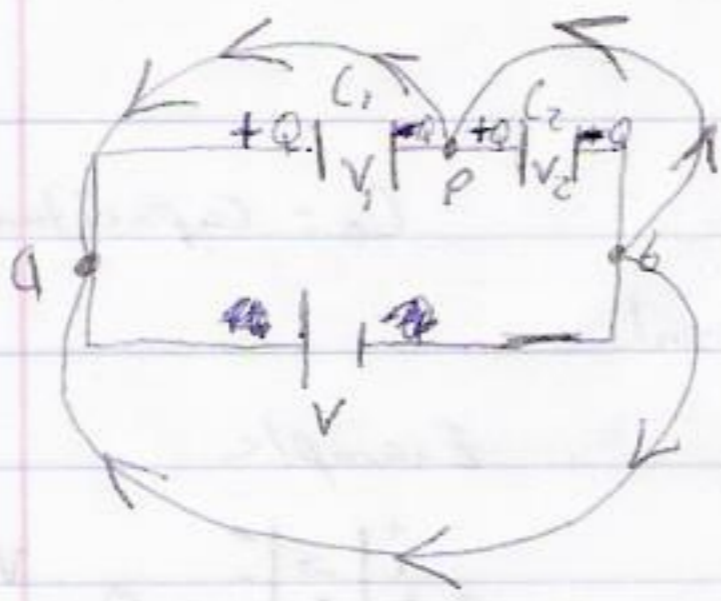
Equivalent Single Capacitor



Same amount of charge to leave each battery terminal: $Q = Q_1 + Q_2$

$C_{eq} = \frac{Q}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$

$C_{eq} = C_1 + C_2$



$$V_1 = V_a - V_p = - \int_p^a \vec{E} \cdot d\vec{l}$$

$$V_2 = V_p - V_b = - \int_b^p \vec{E} \cdot d\vec{l}$$

$$V = V_a - V_b = - \int_b^a \vec{E} \cdot d\vec{l}$$

$$V = V_1 + V_2$$

$$Q = Q = Q$$

$$C_{eq} = \frac{Q}{V} \Rightarrow \frac{1}{C_{eq}} = \frac{V}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} ; \text{ capacitors in series}}$$

