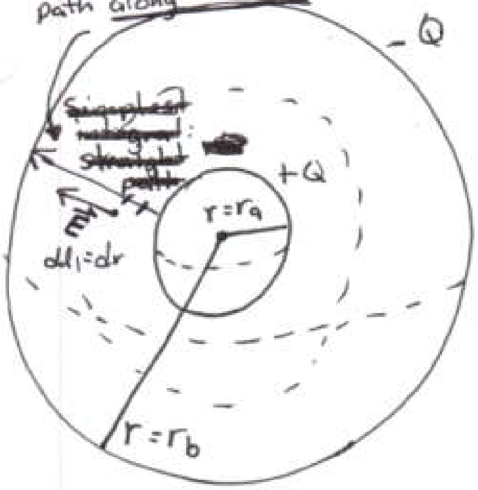
9/13/10

Spherical Capasitor

Simplest integral: straight path along a radius



Inner spherical shell: charge = +Q ; radius = r_a
Outer spherical shell: charge = -Q ; radius = r_b
In between: some insulator dielectric constan k (p. 638)

$C = \frac{Q}{V}$ ← want $V > 0$

Capasitance $C = ?$

$$V = V_a - V_b = \int_{r=r_b}^{r=r_a} -\vec{E} \cdot d\vec{l}$$

like $\frac{a^3 - b^3}{3} = \int_{x=b}^{x=a} x^2\,dx$

$$V = \int_{r=r_b}^{r=r_a} -E\,dr$$

Gauss' Law for a sphere of radius r where $r_a < r < r_b$

$$Q/\epsilon = Q/k\epsilon_0 = \Phi = \int_{\text{shell radius}} \vec{E} \cdot d\vec{A} = \int_{\text{shell}} E\,dA = EA = 4\pi r^2 E$$

constant on shell by symmetry

$$Q/k\epsilon_0 = 4\pi r^2 E \Rightarrow E = \frac{Q}{4\pi k\epsilon_0 r^2}$$

$$V = \int_{r=r_b}^{r=r_a} -E\,dr = \frac{Q}{4\pi k\epsilon_0} \int_{r=r_b}^{r=r_a} -\frac{dr}{r^2} = \frac{Q}{4\pi k\epsilon_0} \left(\frac{1}{r}\right)\Bigg|_{r_b}^{r_a} = \frac{Q}{4\pi k\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$\boxed{4\pi k\epsilon_0 \left(\frac{r_a r_b}{r_b - r_a}\right)} = \frac{Q}{\frac{Q}{4\pi k\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)} = C = Q/V$$

<u>When charges are not moving:</u>

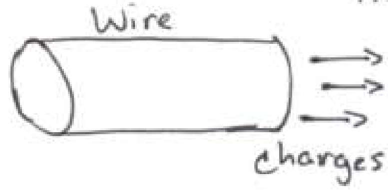
- Inside conductors: no charge
  $\vec{E} = \vec{0}$
  V constant
- On surface: all the charge is here
  $\vec{E} \perp$ surface (same as $\vec{E} \parallel d\vec{A}$)
  V constant

Same value

When charges are moving:

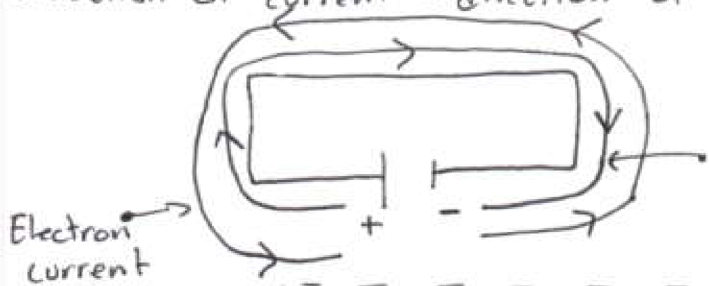
- Conductors don't have these properties
- Also, Coulomb's Law $F = \frac{kQ_1Q_2}{r^2}$ is not true in general (especially noticeable for fast moving charges)
- But Gauss' Law is still true

$\text{Current} = \frac{\text{charge}}{\text{time}}$; Specifically: rate at which charge passes through



(Conventional)

Direction of current = direction of positive charge flow



$\text{Watt} = \frac{\text{Joule}}{\text{second}} = \frac{\text{newton}\cdot\text{meter}}{\text{second}} = \text{amp}\cdot\text{volt}$

units

$I = \frac{dQ}{dt}$ — variables

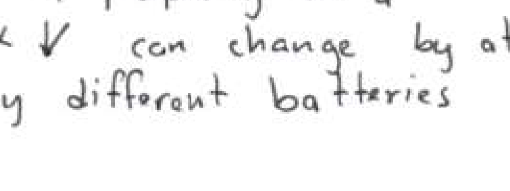
$\underset{\text{(ampere)}}{A} = \frac{C}{s} = \frac{\text{Coulomb}}{\text{Second}}$ — units

$R = \frac{V}{I}$ — variables

$\underset{\text{(ohm)}}{\Omega} = \frac{V}{A} = \frac{\text{volt}}{\text{amp}}$ — units

$P = IV$ — Variables

Current across wire $\propto$ voltage difference between ends



$I = \left(\frac{1}{R}\right)V = \frac{V}{R} \Leftrightarrow R = \frac{V}{I}$

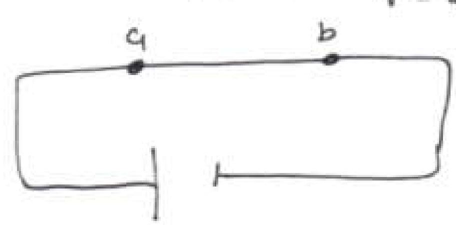
def. of resistance

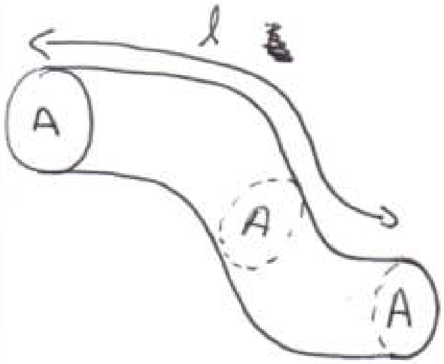
$\Updownarrow$

$V = IR$

R is a property of 

I & V can change by attaching, say different batteries





ρ = resistivity = constant depending only on material (p. 658) & temp.

$$R = \frac{\rho l}{A}$$

A = area of a cross-section
- assume all cross-sections have same area

constant depends on material

$$\rho = \underbrace{\rho_0}_{} \left[1 + \overset{}{\underset{\alpha}{\propto}} (T - T_0)\right]$$

ρ at temperature T_0 (constant)

usually
$20^\circ C$
$0^\circ C$
$300^\circ K$

Careful:
'α' is the greek 'a'!
'$\propto$' means 'is proportional to.'

$\propto > 0$

always, i.e., resistivity increases as temperature increases

$$\Delta R = \frac{(\Delta \rho) l}{A} = \frac{\overset{\alpha}{\propto} l \Delta T}{A}$$

power $P = \frac{du}{dt} = \frac{d}{dt}(qV) = \frac{dq}{dt} V + q\frac{dV}{dt} = IV + 0 \Rightarrow \boxed{P = IV}$

$\frac{du}{dt}$: $\frac{\text{energy}}{\text{time}}$

if V constant

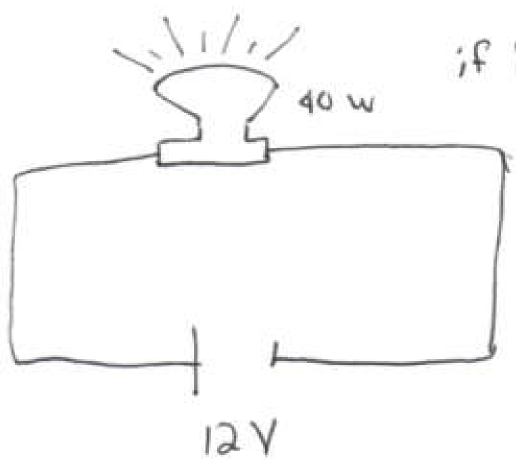
$V = IR \rightarrow \Updownarrow$

$I(IR) = \boxed{I^2 R}$

$I = \frac{V}{R} \quad \Updownarrow$

$\left(\frac{V}{R}\right)^2 R = \boxed{\frac{V^2}{R}}$

$$P = -\frac{dW}{dt} = \frac{du}{dt}$$



$P = 40$ watt
$V = 12$ volt

$I = P/V = (40/12)$ amps ≈ 3.3 amps

$R = V/I$ (and $= V^2/P$) $= 144/40\ \Omega = 3.6\ \Omega$

$\parallel$

$\frac{12 \text{ volt}}{3.3 \text{ amp}} \approx 3.6\ \Omega$