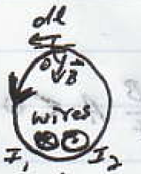
10-6-10

Ampere's Law

$$\int_{loop} \vec{B}\cdot d\vec{l} = \mu_0 \underbrace{I_{encl}}_{\text{Net enclosed current}}$$



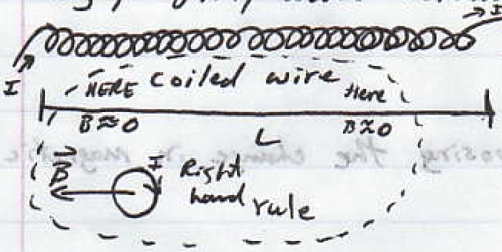
$\vec{B}\cdot d\vec{l} = B(dl)\cos\theta$

Right hand rule tells which current is (+).

Here $I_{encl} = -I_1 + I_2$

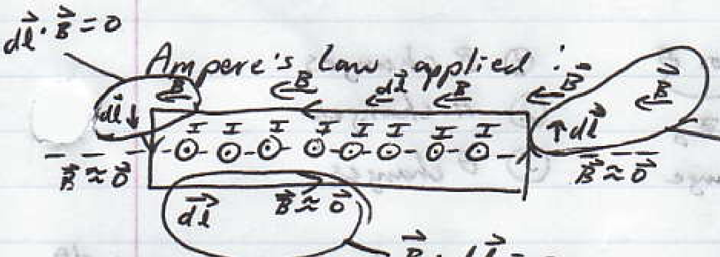
$L >> R$

Long, tightly wound solenoid



Inside $\vec{B}$ is approx. constant.

Ampere's Law applied:



$d\vec{l} \perp \vec{B} = d\vec{l}\cdot\vec{B} = 0$

$$\mu_0 I_{encl} = \int_{loop} \vec{B}\cdot d\vec{l} = \int_{\substack{top\\side}} \vec{B}\cdot d\vec{l} = Bl \underbrace{\cos 0°}_{\vec{B} \parallel d\vec{l}} = Bl \Rightarrow Bl = \mu_0 I_{encl} = \mu_0 N I$$

$$B = \mu_0\left(\frac{N}{l}\right) I$$

$$B = \mu_0 n I$$

N = # of coils that go through loop.

$n = \frac{N}{l}$ = # of coils per unit length

Faraday's Law of Induction:

$$\underbrace{\mathcal{E}}_{\text{Induced EMF}} = -\frac{d\Phi}{dt}$$

$\vec{B}\cdot d\vec{A} = B(dA)\cos\theta$

$$\underbrace{I}_{\text{Induced current}} = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt}$$

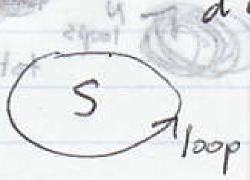
Electric flux: $\Phi_E \int_S \vec{E}\cdot d\vec{A}$

Magnetic flux: $\Phi_B \int_S \vec{B}\cdot d\vec{A}$

Added by Milovich:

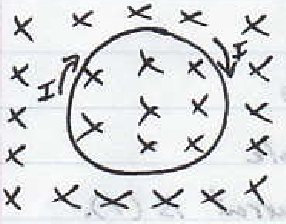
General form of Faraday's Law:

$$\int_{loop} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$



Remember that current

Creates a magnetic field.



$\Phi_B = BA$

$\frac{d\Phi_B}{dt} = \frac{dB}{dt} A < 0$

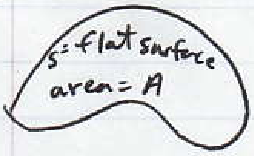
$\underbrace{I}_{\text{Induced current}} = -\frac{1}{R}\frac{d\Phi_B}{dt}$ is positive

$\left.\begin{matrix} d\vec{A} \\ \vec{B} \end{matrix}\right\}$ direction constant : $d\vec{A}$ in, $\vec{B}$ in, $\vec{B}$ uniform

$\frac{dB}{dt} < 0$

Lenz's Law:

Induced current creates a magnetic field opposing the change in magnetic flux.

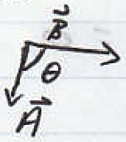


If $\vec{B}$ is uniform,

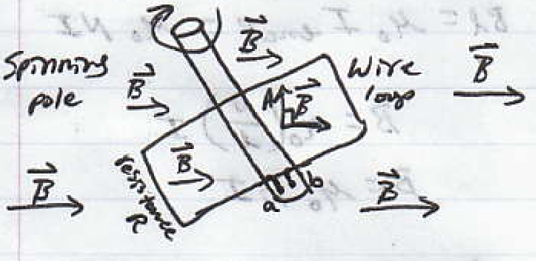
then $\Phi_B = \int_S \vec{B}\cdot d\vec{l} = \underbrace{BA\cos\theta}_{\text{3 ways } \Phi_B \text{ can change}}$

① B changes
② A changes
③ θ changes

$\vec{A} \perp S$



angular speed: ω



choose coordinate system such that $\frac{d\theta}{dt} = \omega$

$\left|\frac{d\theta}{dt}\right| = \omega \qquad \frac{d\theta}{dt} = \pm\omega$

$|V_{ab}| = |\mathcal{E}| = -\frac{d\Phi_B}{dt} = \frac{-d}{dt}(BA\cos\theta) = -BA(-\sin\theta)\frac{d\theta}{dt}$

↑ Induced

$= BA(\sin\theta)\,\omega$

[redacted]

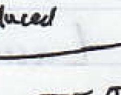
loop enclosed area A

Assuming ω is constant,

$\theta = \theta_0 + \omega t$

$BA\omega\sin(\theta_0 + \omega t)$

$\underset{\text{induced}}{I} = \frac{V_{ab}}{R} = \frac{BA\omega}{R}\sin(\theta_0 + \omega t)$

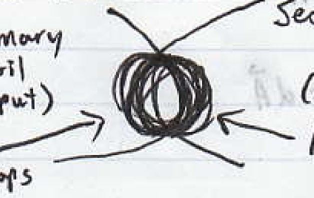


$\mathcal{E} = -\frac{d\Phi_B}{dt}$

[redacted]

total: $\mathcal{E} = -N\frac{d\Phi_B}{dt}$

Transformer



induced $V_p = \mathcal{E}_p = -N_p\frac{d\Phi_B}{dt}$

$V_s = \mathcal{E}_s = -N_s\frac{d\Phi_s}{dt}$

$\frac{V_s}{V_p} = \frac{N_s}{N_p}$