$\mathcal{E}_2 = -N_2 \, d\Phi_{B2}/dt$

Magnetic flux inside coil caused by current I_1 from coil 1.

Because $\Phi_{B2} \propto I_1$

More about inductance (two coils)

mutual inductance $M = \frac{-\mathcal{E}_1}{dI_2/dt} = \frac{-\mathcal{E}_2}{dI_1/dt} = \frac{+N \, d\Phi_{B_2}/dt}{dI_1/dt} = \frac{+N\Phi_{B_2}}{I_1}$

N_2 = # loops in second coil

often easier to compute

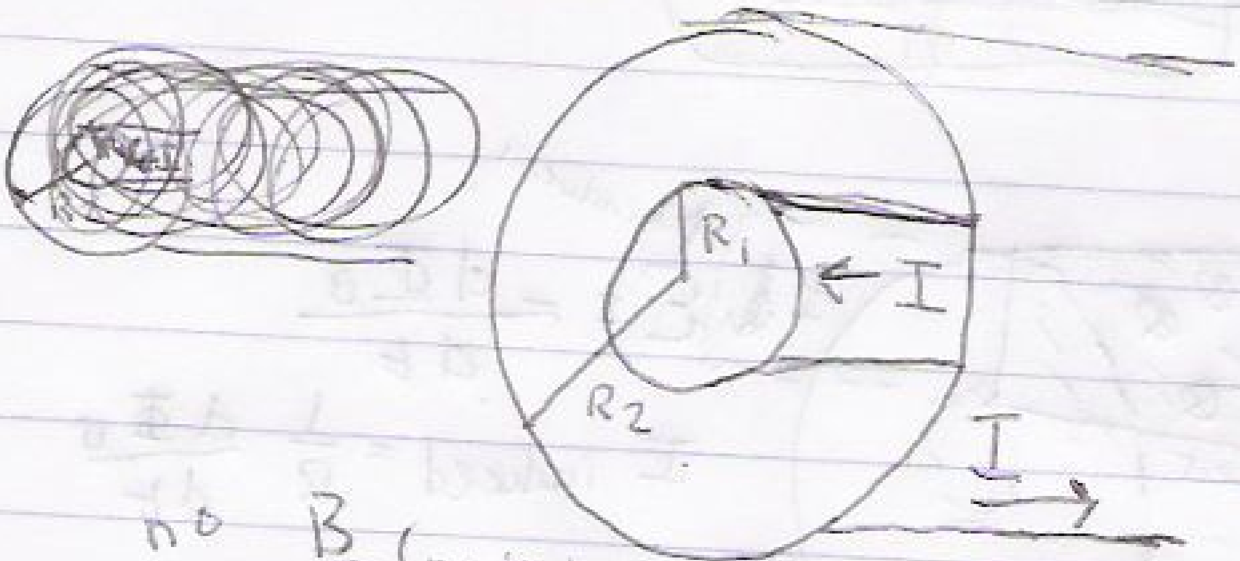
Last time:

self inductane — one coil

$L = \frac{-\mathcal{E}}{dI/dt} = \frac{+N \, d\Phi_B/dt}{dI/dt} = \frac{N\Phi_B}{I}$

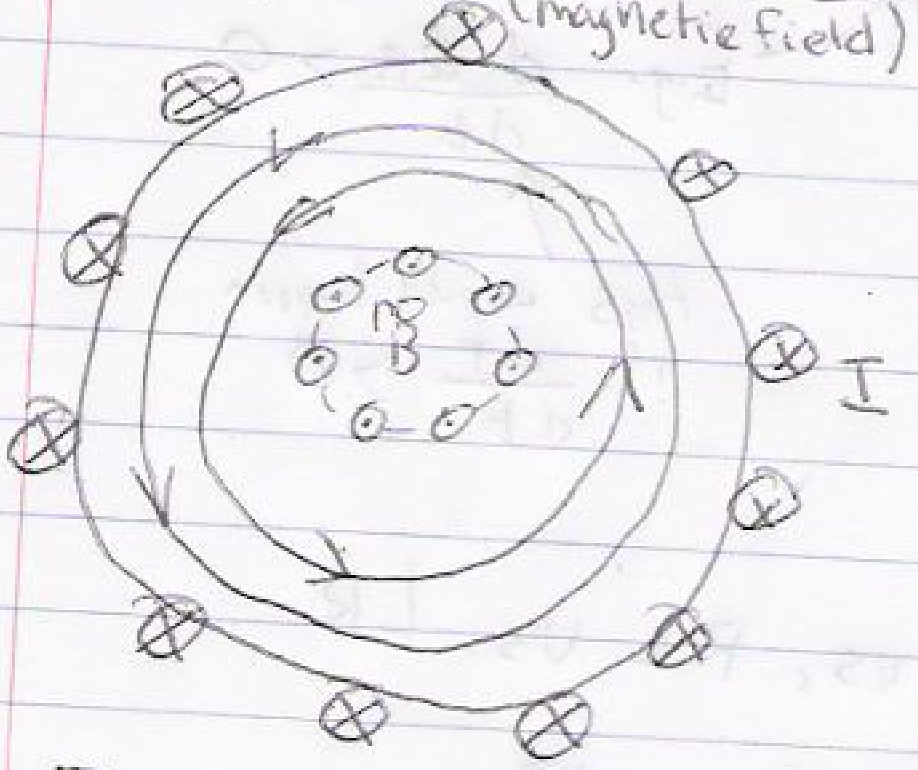
Because $\Phi_B \propto I$

coaxial cable



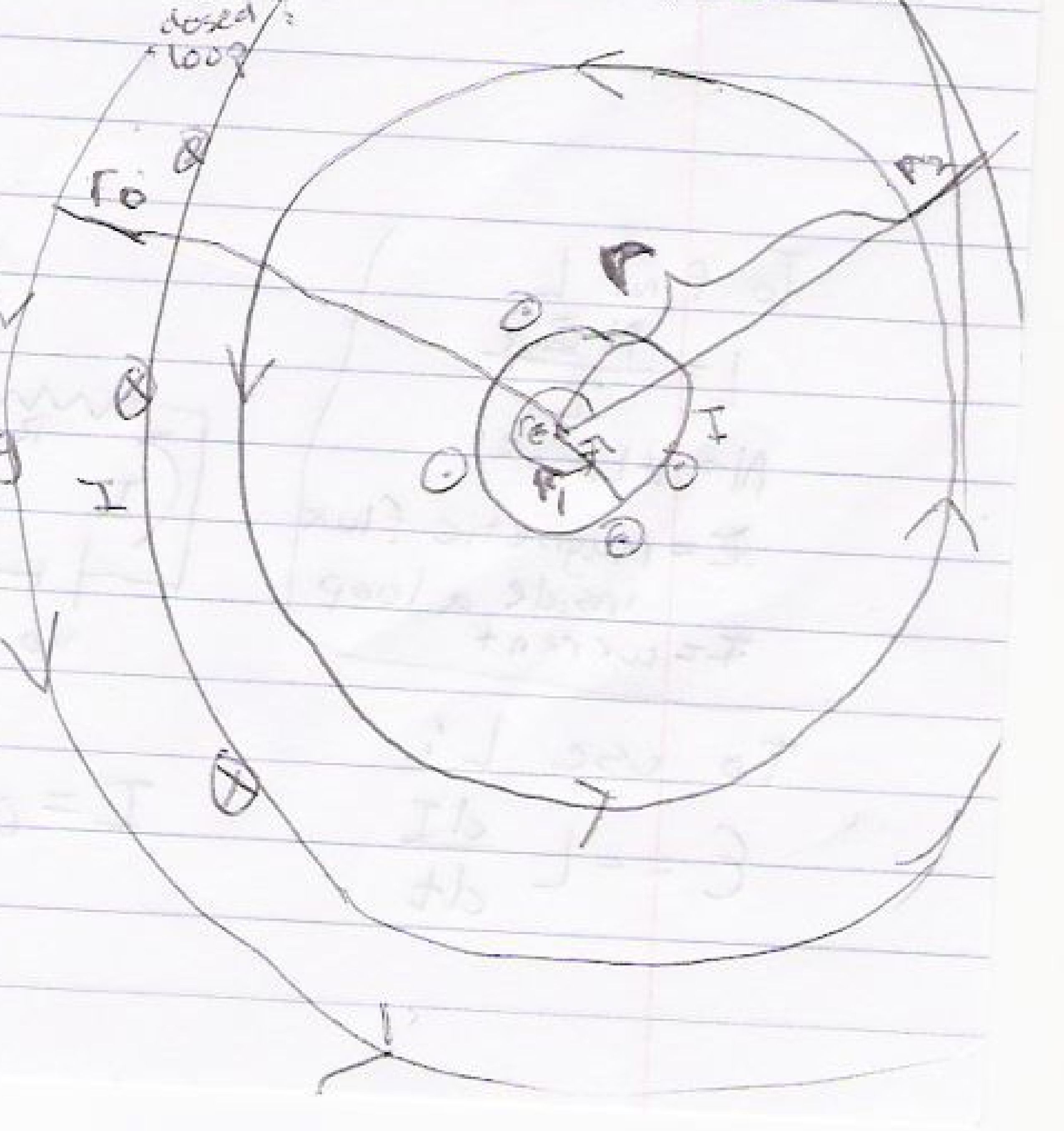
$L = \frac{1 \cdot \Phi_B}{I}$

no B (magnetic field)



Ampere Law

$\oint_{\text{closed loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$



I = total current on each ring

By symmetry

$B \cdot 2\pi r = \mu_0 I$

$\hookrightarrow B = \mu_0 I/(2\pi r)$

$\Rightarrow B = 0$

$B \cdot 2\pi r_i = \mu_0 \cdot 0$

$B \cdot 2\pi r_0 = \mu_0 \cdot 0$

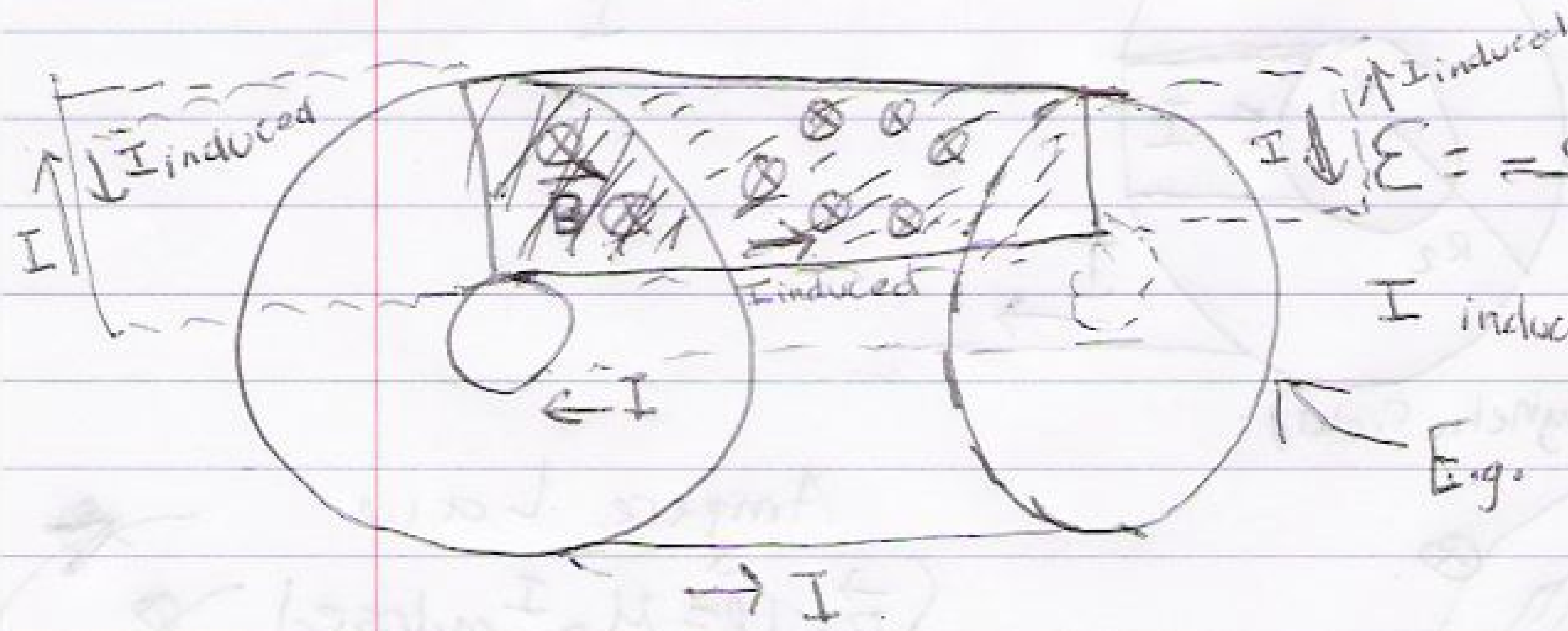
$\hookrightarrow B = 0$

$$L = \frac{1 \cdot \Phi_B}{I} \qquad \Phi_B = \int_{area} \vec{B} \cdot d\vec{A}$$

$$\int_{length} \int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r} \, dr \, dl = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right) l$$

$$L = \frac{1 \cdot \Phi_B}{I} = \frac{\mu_0 I \ln\left(\frac{r_2}{r_1}\right) l}{2\pi I}$$

$$= \boxed{\frac{\mu_0 \ln\left(\frac{r_2}{r_1}\right) l}{2\pi}}$$



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$I \text{ induced} = \frac{1}{R} \frac{d\Phi_B}{dt}$$

E.g. $\frac{d\Phi_B}{dt} > 0$

this would happen if $\frac{dI}{dt} > 0$

To find L

$L = \frac{N \Phi_B}{I}$

N = # Loops

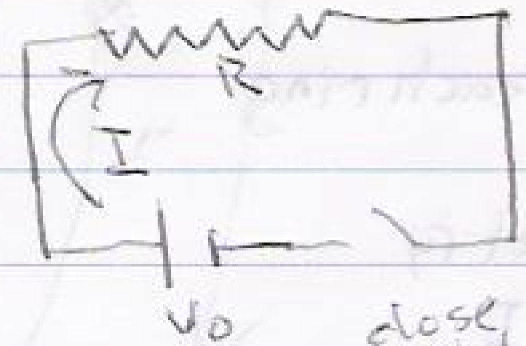
Φ = magnetic flux inside a loop

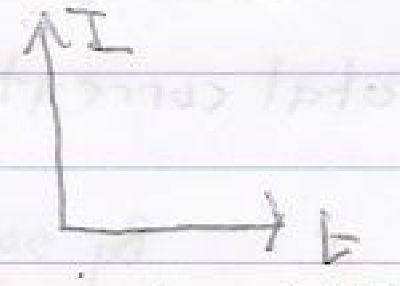
I = current

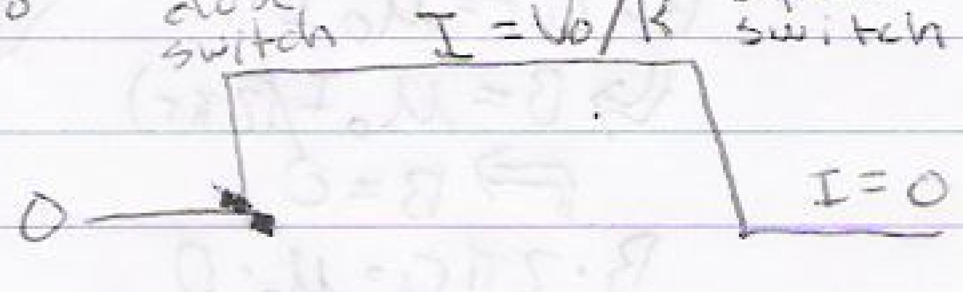
To use L:

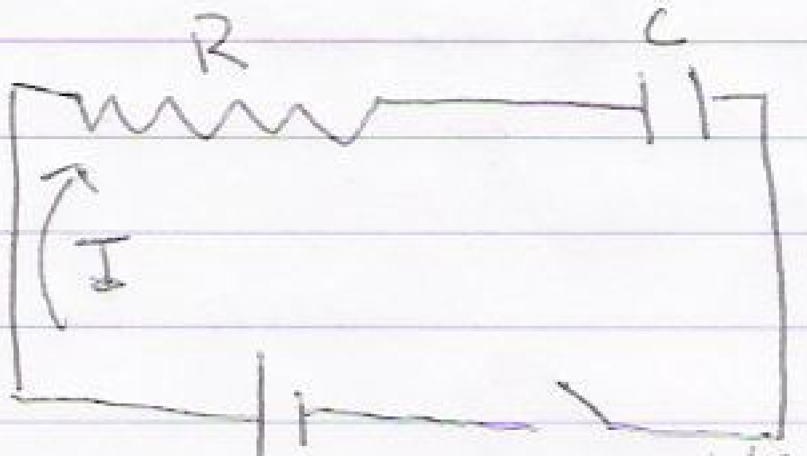
$$\mathcal{E} = -L \frac{dI}{dt}$$

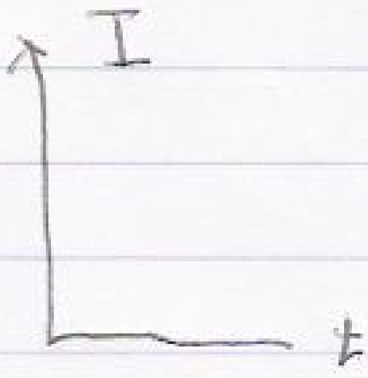
R vs. RC vs. LR

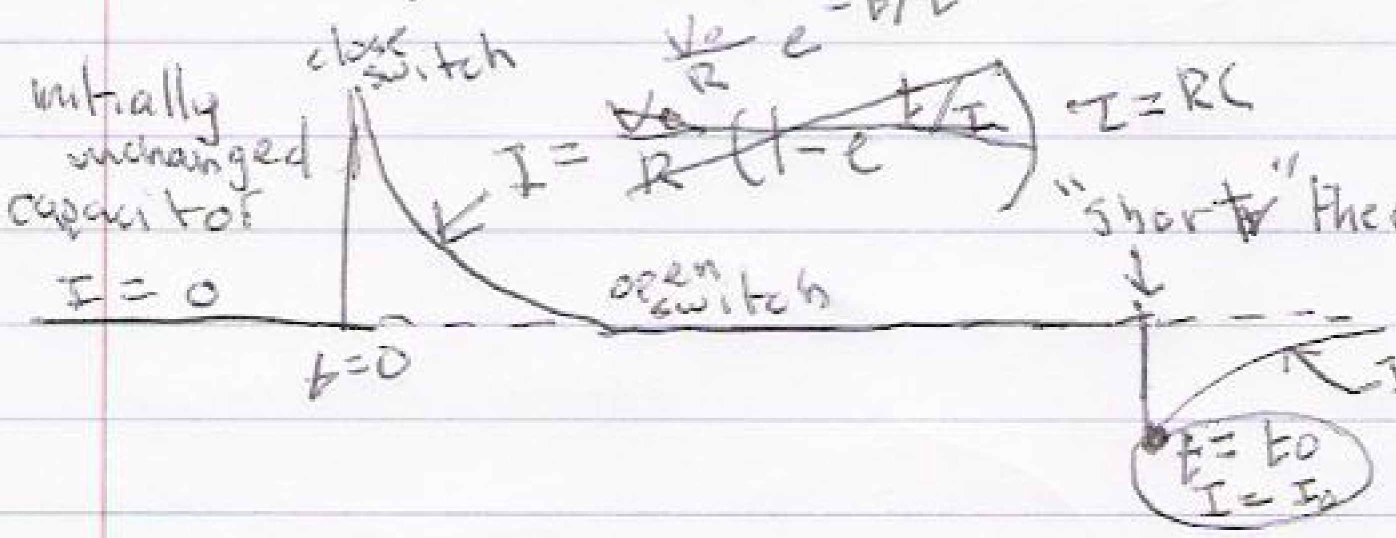










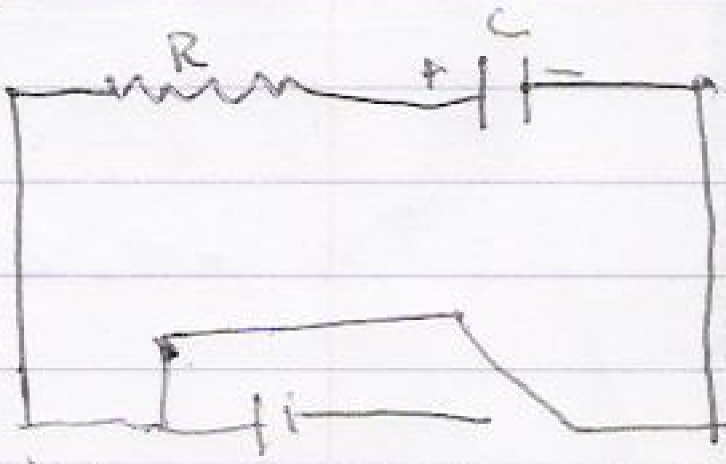


$\frac{V_0}{R}e^{-t/\tau}$

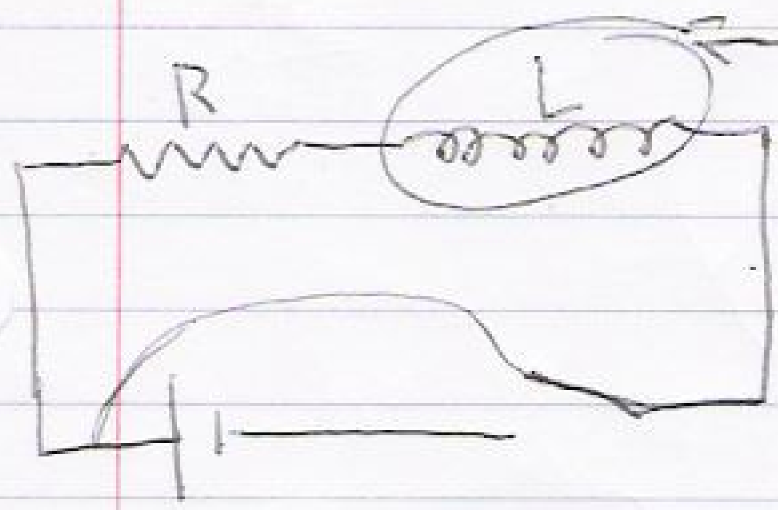
$I = \cancel{\frac{V_0}{R}(1-e^{t/\tau})}$ $\tau = RC$

$I = I_0 e^{-(t-t_0)/\tau}$

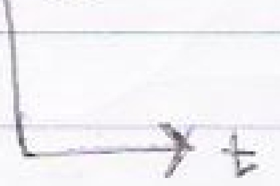
$(t-t_0)/\tau$



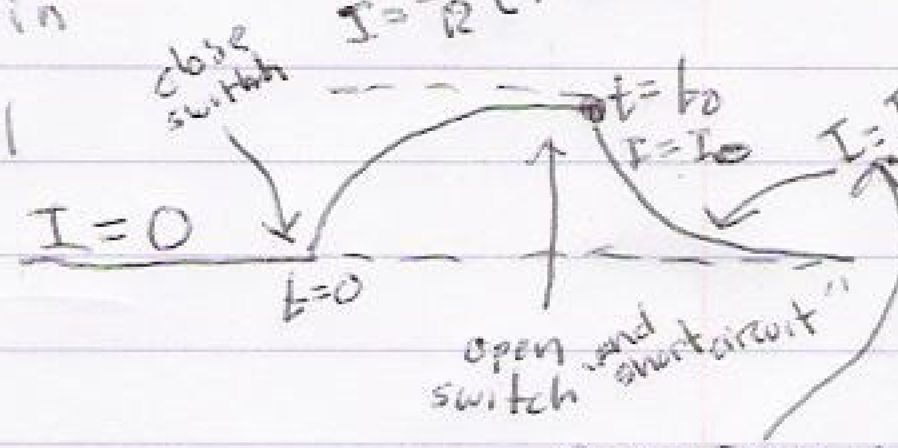
$(t-t_0)/\tau$



symbol for significant self-inductance in coil & negligible resistance in coil



$I = \frac{V_0}{R}(1-e^{t/\tau})$

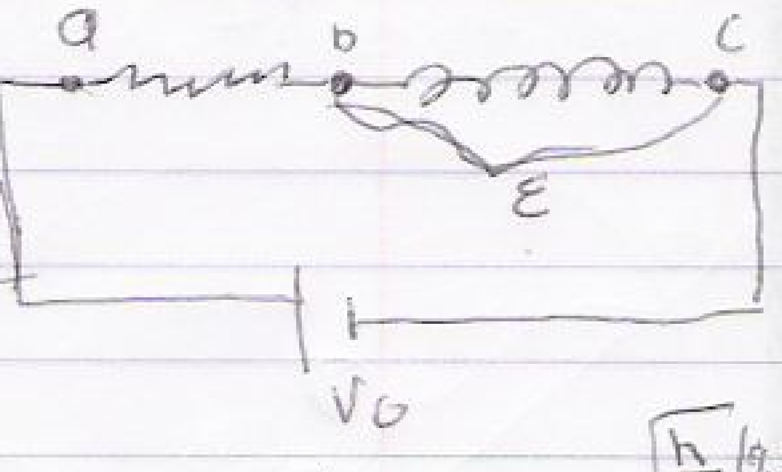


$I = I_0 e^{-(t-t_0)/\tau}$ $\tau = L/R$

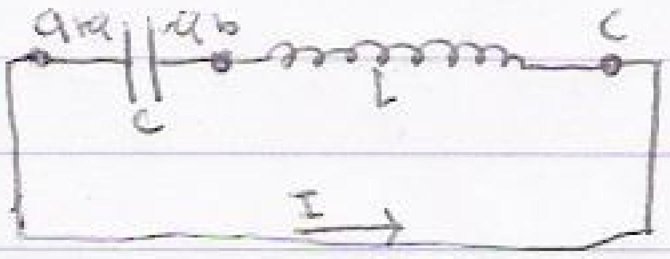
Comes from a differential equation

$0 = V_{ab} + V_{bc} + V_{ca}$

$0 = V_0 + IR - L\frac{dI}{dt}$



LC circuit



$I = -\frac{dQ}{dt}$

$I = -\frac{dQ}{dt}$

$\frac{dI}{dt} = -\frac{d^2Q}{dt^2}$

$0 = V_{ab} + V_{bc} + V_{ca}$

$0 = \underbrace{\frac{Q}{C}}_{} - \underbrace{L\frac{dI}{dt}}_{\varepsilon \text{ induced}} + 0 \longrightarrow 0 = \frac{Q}{C} + L\frac{d^2Q}{dt^2}$

because $C = \frac{Q}{V_{ab}}$

$0 = \frac{d^2Q}{dt^2} + \frac{1}{LC}Q \longrightarrow Q(t) = A\sin(\omega t + \phi)$

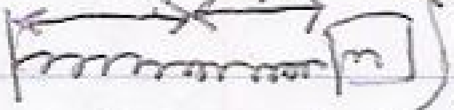
Q max

$\omega = \frac{1}{\sqrt{LC}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

Undamped spring:

$0 = \frac{d^2x}{dt^2} + \frac{k}{m}x$

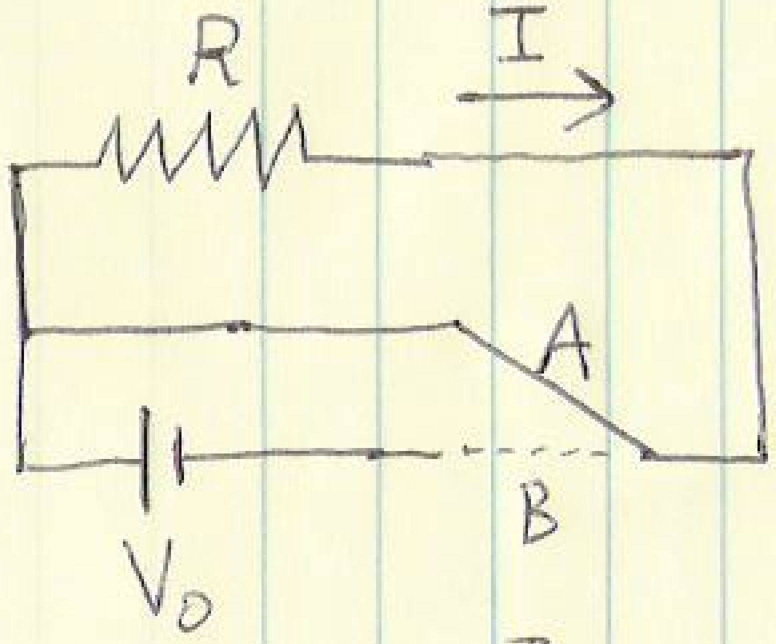


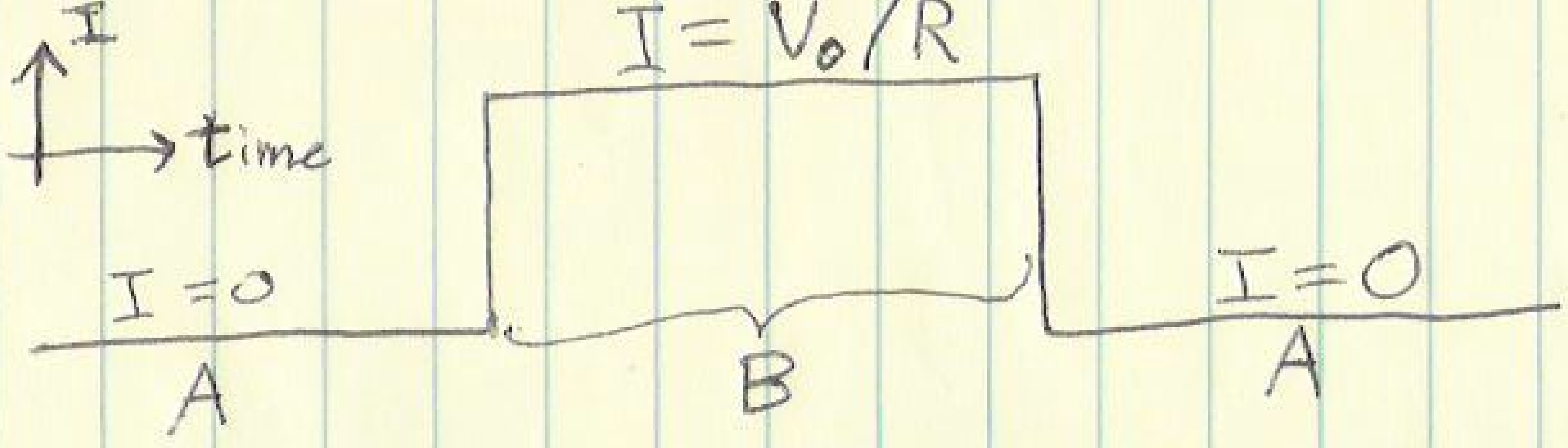
$\omega = \sqrt{\frac{k}{m}}$

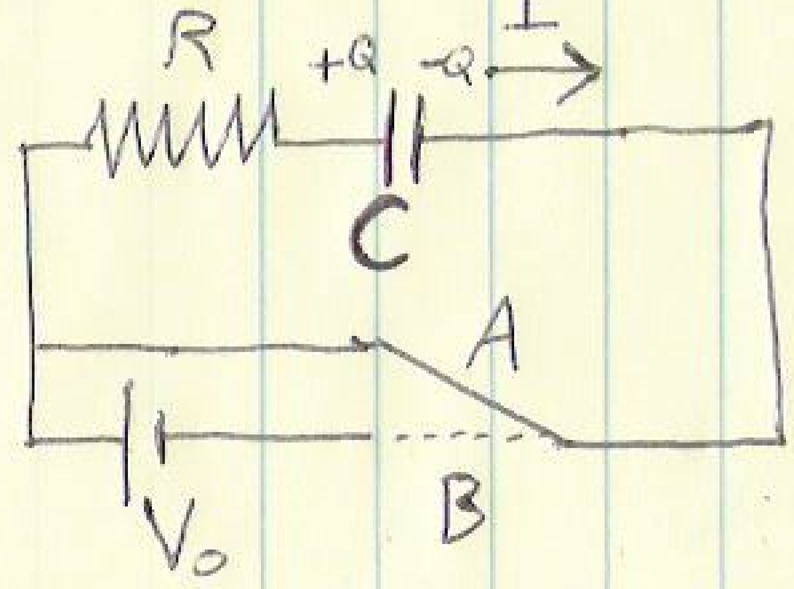
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

solution: $x(t)\ A\sin(\omega t + \phi)$

constant from initial conditions







$$Q = CV_0\left(1-e^{-t_1/\tau}\right)$$

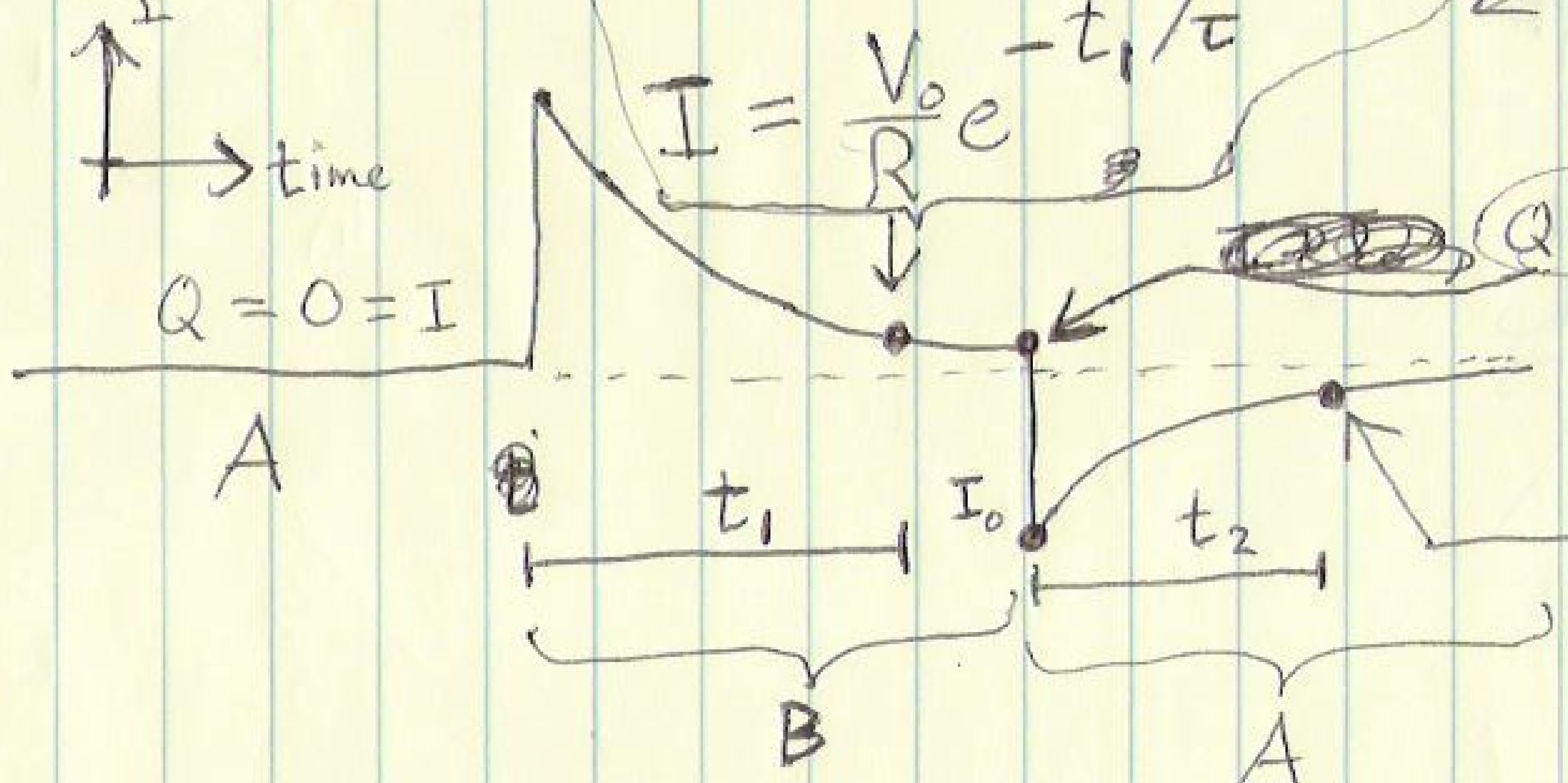
$$I = \frac{V_0}{R}e^{-t_1/\tau}$$

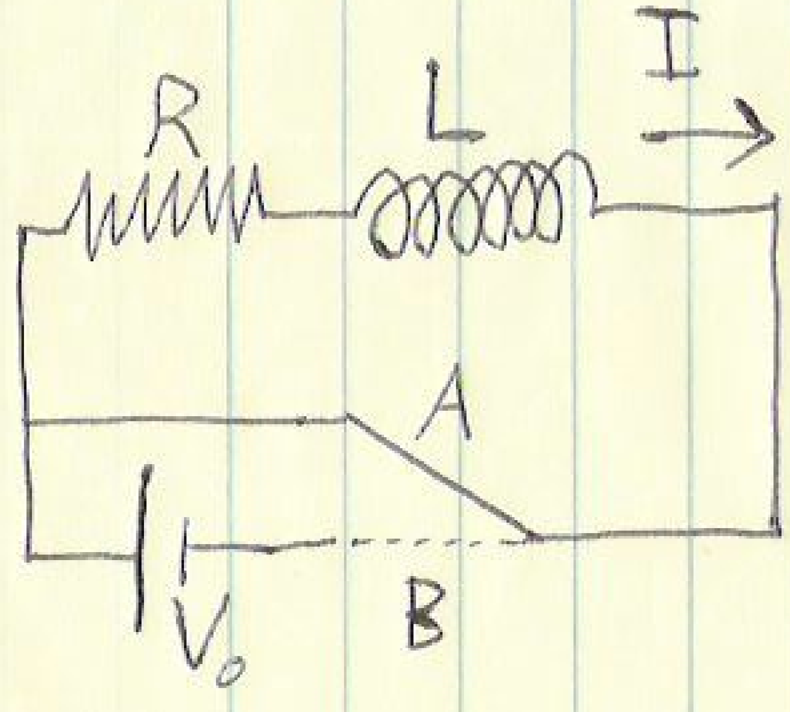
$\tau = RC$

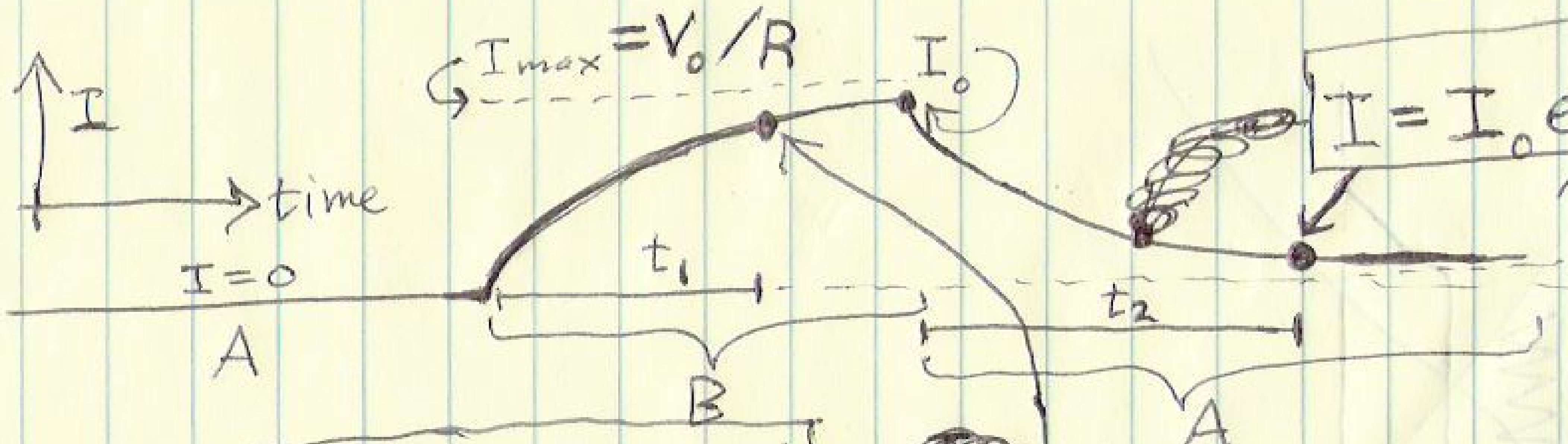
$Q=Q_0$

$$Q = Q_0 e^{-t_2/\tau}$$

$$I = -\underbrace{\frac{Q_0}{RC}}_{I_0} e^{-t_2/\tau}$$







$$I = I_0 e^{-t_2/\tau}$$

$$I = I_{max}\left(1-e^{-t_1/\tau}\right)$$

$\tau = L/R$

R vs. RC vs. LR circuits