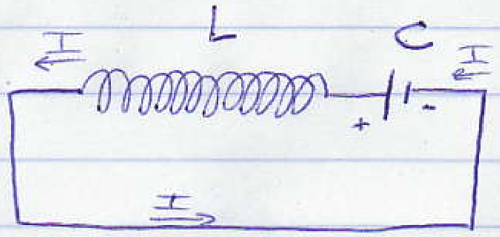
ALEX DLF
OCTOBER 25, 2010

LAST TIME

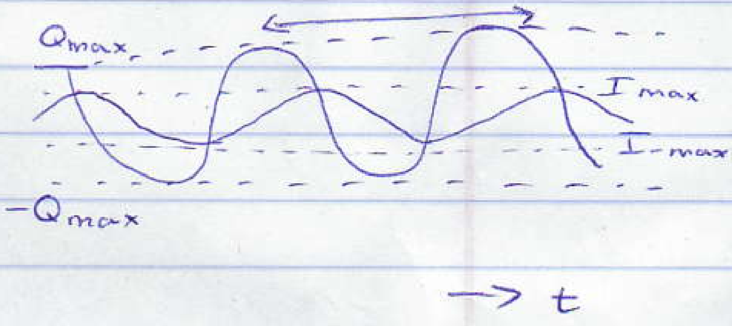


$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

$\Phi = A\cos(\omega t + \phi)$

$A \downarrow \Phi_{max}$

$\downarrow \omega = \frac{1}{\sqrt{LC}}$



A, ϕ depend on initial condition.

$$I = -\frac{dQ}{dt} = A\omega\sin(\omega t + \phi)$$

$$= A\omega\cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

$\uparrow$

$I = I_{max}$ at times after $\boxed{\frac{T}{4}}$

$Q = Q_{max}$

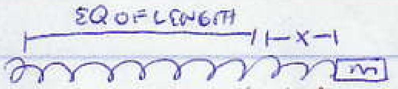
$\frac{\pi}{2} = \frac{2\pi}{4} = 90° = \frac{1}{4}$ of a revolution

* ENERGY in CAPACITOR

$$\frac{Q^2}{2C} = U_C$$

IN INDUCTOR $\frac{1}{2}LI^2 = U_L$

~ COMPARE TO SPRING:



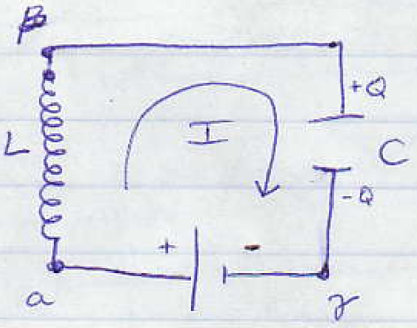
spring constant k

$\frac{1}{2}kx^2 = U_{spring}$

$\frac{1}{2}mv^2 = K$ = kinetic energy

$$v = \frac{dx}{dt}$$

$\underbrace{LC + dc\ \text{source}}_{\text{EQ battery}}$

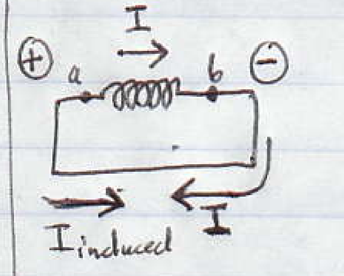


Battery emf = $V_{a\gamma} = V_a - V_\gamma$

$\boxed{\mathcal{E} \text{ opposes } \frac{dI}{dt}}$

$\boxed{\mathcal{E} = -L\,dI/dt}$

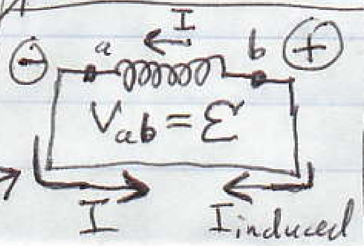
$dI/dt > 0$



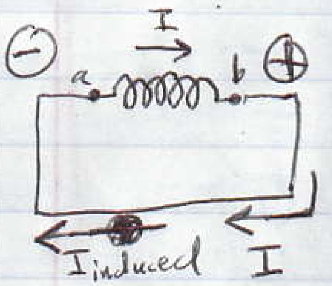
$V_{ab} = V_a - V_b$

$V_{ab} = -\mathcal{E}$

$V_{ab} = +L\frac{dI}{dt}$



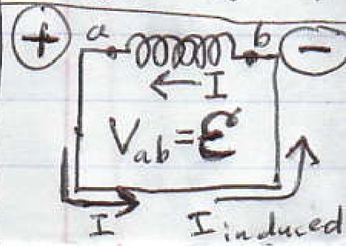
$dI/dt < 0$



$V_{ab} = -\mathcal{E}$

$V_{ab} = +L\frac{dI}{dt}$

$V_{ab} = V_a - V_b$



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Kirchoff's Law (Loop)

$$0 = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\alpha}$$

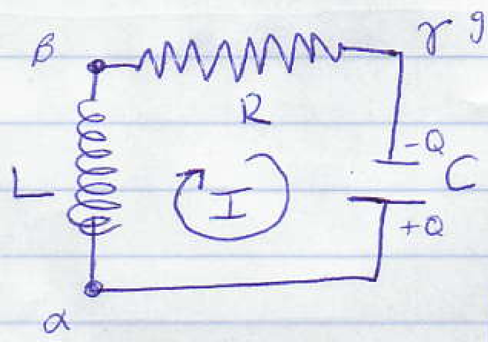
$$0 = +L\frac{dI}{dt} + \frac{Q}{C} - V_{\alpha\gamma}$$

w/out battery $= 0 = -L\frac{dI}{dt} + \frac{Q}{C}$

SOLUTION : $Q = A\cos(\omega t + \phi) + C\,V_{\alpha\gamma}$

$$I = \frac{dQ}{dt} = -\underbrace{A\omega}_{I_{max}}\sin(\omega t + \phi) = A\omega\cos\left(\omega t + \phi + \frac{\pi}{2}\right)$$

LRC circuit



$$0 = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\alpha}$$

$$\begin{cases} 0 = +L\frac{dI}{dt} + IR - \frac{Q}{C} \\ I = -\frac{dQ}{dt} \end{cases}$$

$$0 = \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC}$$

Compare to spring: $0 = \frac{d^2x}{dt^2} + \left(\frac{b}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x$

k = spring constant
b = "damping term"

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$$\Phi = Ae^{-t/\tau} \cos(\omega_d t + \phi)$$

$$\tau = \frac{L}{2R} \qquad \omega_d = \sqrt{\omega^2 - \left(\frac{1}{\tau}\right)^2} < \omega$$

$$\uparrow \ \omega = \frac{1}{\sqrt{LC}}$$

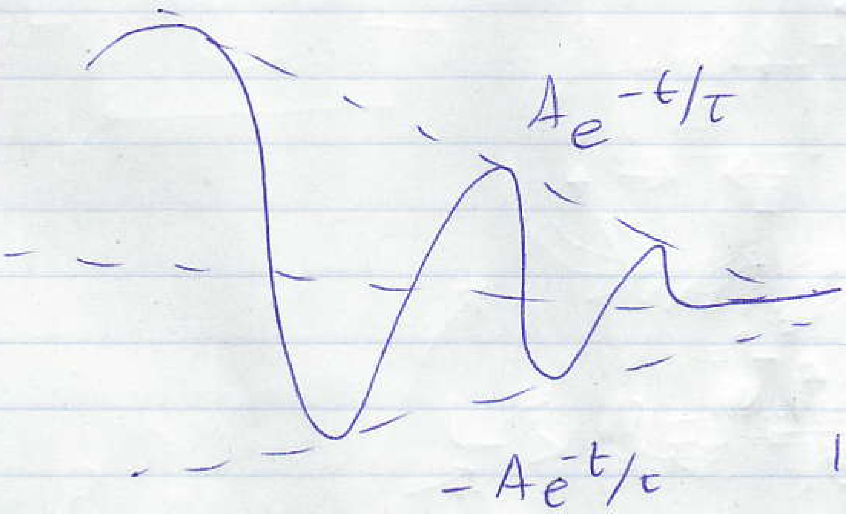
UNDERAMPED $\omega > \frac{1}{\tau}$

$$\Phi = (A + Bt)e^{-t/\tau}$$

CRITICALLY DAMPED $\omega = \frac{1}{\tau}$

$$Q = e^{-t/\tau}\left(A e^{t\sqrt{1/\tau^2 - \omega^2}} + B e^{-t\sqrt{(1/\tau)^2 - \omega^2}}\right)$$

OVERDAMPED $\omega < \frac{1}{\tau}$



If you add a battery with voltage V, then you add $\pm CV$ to Q

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SOURCE OF AC CURRENT

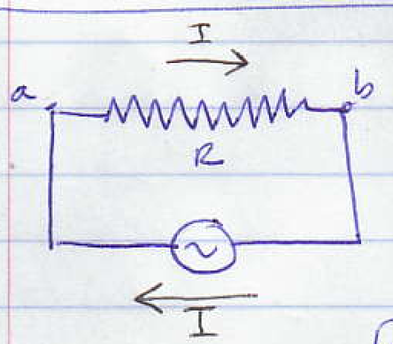
$$I = \underbrace{I_0}_{I_{max}} \cos(\omega t)$$

I_0 or I_{rmf} is given

120π Hz in wall outlet

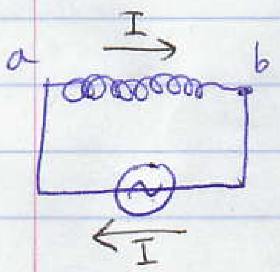
e.g.

$$I_{rmf} = \sqrt{\frac{1}{T}\int_0^T I^2\,dt} = \frac{I_{max}}{\sqrt{2}} = \frac{I_0}{\sqrt{2}}$$



$$V_{ab} = IR = I_0 R \cos\omega t$$

$$R = \frac{V_{ab}}{I_0} = \frac{V_{max}}{I_{max}} = \frac{V_{max}/\sqrt{2}}{I_{max}/\sqrt{2}} = \frac{V_{rms}}{I_{max}}$$



$$V_{ab} = L\frac{dI}{dt} = \underbrace{-\omega L I_0}_{V_{max}} \sin(\omega t) = V_{max}\cos\left(\omega t + \frac{\pi}{2}\right)$$

Peak positive V across inductors happens $\frac{T}{4}$ before peak positive current

voltages leads current by 90°

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_{max}/\sqrt{2}}{I_{max}/\sqrt{2}} = \frac{\omega L I_0}{I_0} = \omega L$$

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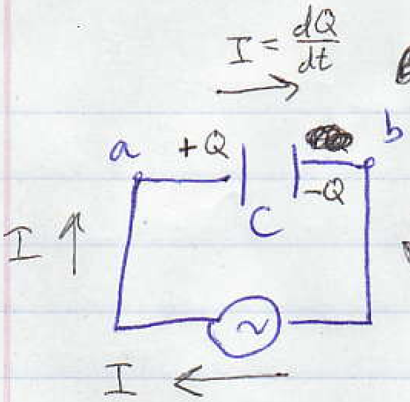
$I = \frac{dQ}{dt}$ Assuming $Q=0$ when $t=0$



$$V_{ab} = \frac{Q}{C} = \frac{1}{C}\int_0^t \underbrace{I_o \cos(\omega t)}_{I}\,dt$$

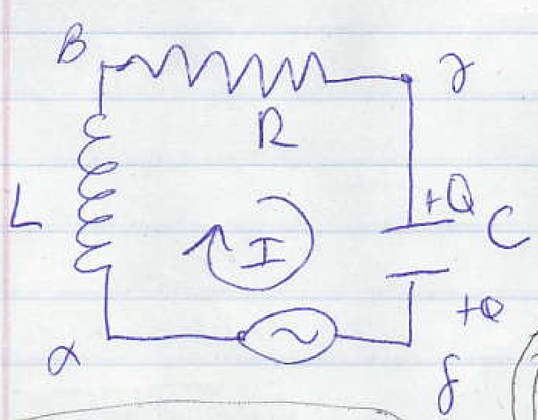
$$V_{ab} = \frac{I_o}{\omega C}\sin(\omega t)$$

$$X_C = \frac{V_{rms}}{I_{rms}} = \frac{1}{\omega C}$$

$$V_{ab} = \frac{I_o}{\omega C}\sin(\omega t) = \underbrace{\frac{I_o}{\omega C}}_{V_{max}} \cos\left(\omega t - \frac{\pi}{2}\right)$$

V LAGS current to 90°

LRC circuit w/ ac source



$$\left\{V_{\alpha\delta} = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\delta}\right.$$

$$V_{\alpha\delta} = V_{max}\cos(\omega t + \phi)$$

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{V_{max}}{I_{max}} = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + (X_L - X_C)^2}$$

impedence

Assuming $Q=0$ when $t=0$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$I = \frac{dQ}{dt}$$

$$V_{\alpha\beta} = X_L I_o \cos\left(\omega t + \frac{\pi}{2}\right)$$
$$V_{\beta\gamma} = R I_o \cos(\omega t)$$
$$V_{\gamma\delta} = X_C I_o \cos\left(\omega t - \frac{\pi}{2}\right)$$