Ch 31

Last time: energy density $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$

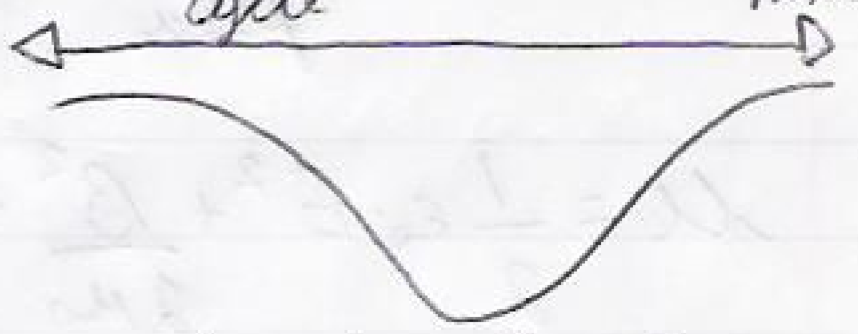
For sinusoidal EM waves, the energy at a fixed position oscillates sinusoidally.

$$u_{rms} = \sqrt{\frac{1}{T}\int_0^T u^2 \, dt} = \frac{1}{2} u_{peak}$$

EM waves: $E = BC$

$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ = speed of light

$\lambda = \frac{\text{lenght}}{\text{cycle}}$; $f = \frac{\text{cycle}}{\text{time}}$



$$C = \lambda f = \text{speed} = \frac{\text{lenght}}{\text{time}} = \frac{\text{lenght}}{\cancel{\text{cycle}}} \cdot \frac{\cancel{\text{cycle}}}{\text{time}}$$

$$k = \frac{\text{radians}}{\text{lenght}} \quad \frac{1}{f} = T = \frac{\text{time}}{\text{cycle}}$$

$$\omega = \frac{\text{radians}}{\text{time}} = 2\pi f$$

Function: $A\cos(\underbrace{kx - \omega t}_{\text{radians}})$

If an EM wave has frequency 500 MHz, what is its wave lenght?

$$c = \lambda f \qquad \lambda = \frac{c}{f} = \frac{3.00 \cdot 10^8 \text{ m/s}}{5.00 \cdot 10^8 /\text{s}} = \frac{3.00 \cdot 10^8}{5.00 \cdot 10^8} = \frac{3}{5} \text{ m} = .6 \text{ m} \; \boxed{= 60 \text{ cm}}$$

Direction of wave travel
= direction of energy flow
= direction of $\vec{E} \times \vec{B}$

Poynting vector = $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_0} = \underbrace{\dfrac{\text{energy}}{\text{Volume}}}_{u} \cdot \underbrace{\text{speed}}_{c}$

rate of energy flow

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \frac{1}{2}\varepsilon_0 \left(\frac{B}{\sqrt{\varepsilon_0 \mu_0}}\right)^2 + \frac{B^2}{2\mu_0} = \frac{1}{2}\cancel{\varepsilon_0}\frac{B^2}{\cancel{\varepsilon_0}\mu_0} + \frac{B^2}{2\mu_0} = \boxed{\frac{B^2}{\mu_0}}$$

$$E = cB = \frac{B}{\sqrt{\varepsilon_0 \mu_0}} \Leftrightarrow E\underbrace{\sqrt{\varepsilon_0 \mu_0}}_{1/c} = B$$

$$\frac{B^2}{\mu_0} = \frac{(E\sqrt{\varepsilon_0 \mu_0})^2}{\mu_0} = \frac{E^2 \varepsilon_0 \mu_0}{\mu_0} = \boxed{\varepsilon_0 E^2}$$

$$\varepsilon_0 E^2 = \varepsilon_0 E\left(\frac{B}{\sqrt{\varepsilon_0 \mu_0}}\right) = \frac{\varepsilon_0 E B}{\sqrt{\varepsilon_0 \mu_0}} = \boxed{\sqrt{\frac{\varepsilon_0}{\mu_0}}\, EB}$$

$$u = \frac{1}{c} \cdot S \Rightarrow S = u \cdot c$$

$$\boxed{\frac{1}{c} \cdot S} = \frac{\sqrt{\varepsilon_0 \mu_0}\, EB}{\mu_0}$$

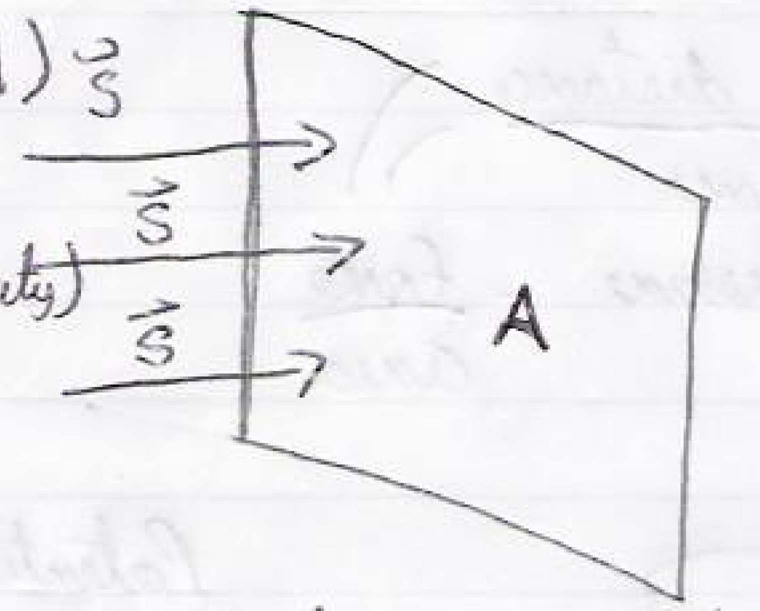
E, B oscillate, so $S = \dfrac{EB}{\mu_0}$ oscillates too.

$$\langle S \rangle = \bar{S} = \frac{1}{T}\int_0^T S \, dt = \frac{E_{rms} B_{rms}}{\mu_0} = \frac{(E_{peak}/\sqrt{2})(B_{peak}/\sqrt{2})}{\mu_0} = \boxed{\frac{E_{peak} B_{peak}}{2\mu_0}}$$

(compare to average power $\bar{P} = I_{rms} V_{rms}$.)

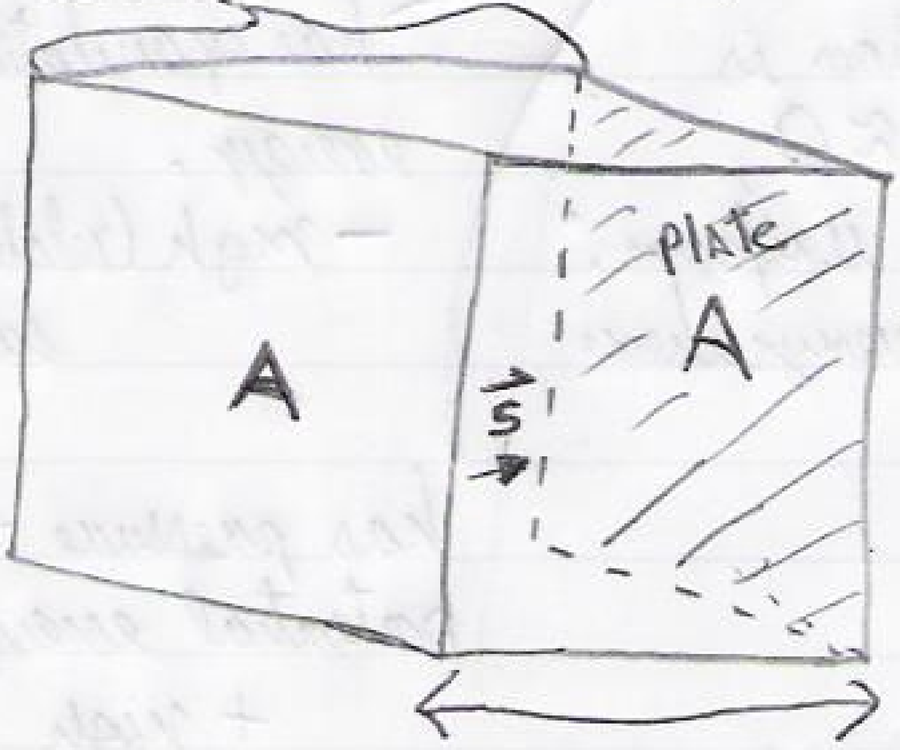
A Flat plate with area A

absorbs radiation (EM) $\vec{S}$
that are planar waves
(for simplicity)



How much energy is absorbed per unit time?

all energy will be absorbed in time dt — Volume $= dV = A\,dx$



$dx = c \cdot dt$

$S = \frac{\text{Energy}}{\text{Volume}} \cdot \text{speed} = u \cdot c = \frac{du}{dV} \cdot c$

* speed $= \frac{\text{lenght}}{\text{time}}$

* $c = \frac{dx}{dt}$

energy absorbed $= du$

$u = \frac{\text{Energy}}{\text{Volume}} = \frac{du}{dV}$ $\quad u_{rms} = \frac{u_{peak}}{2}$ ... Use S, not u here

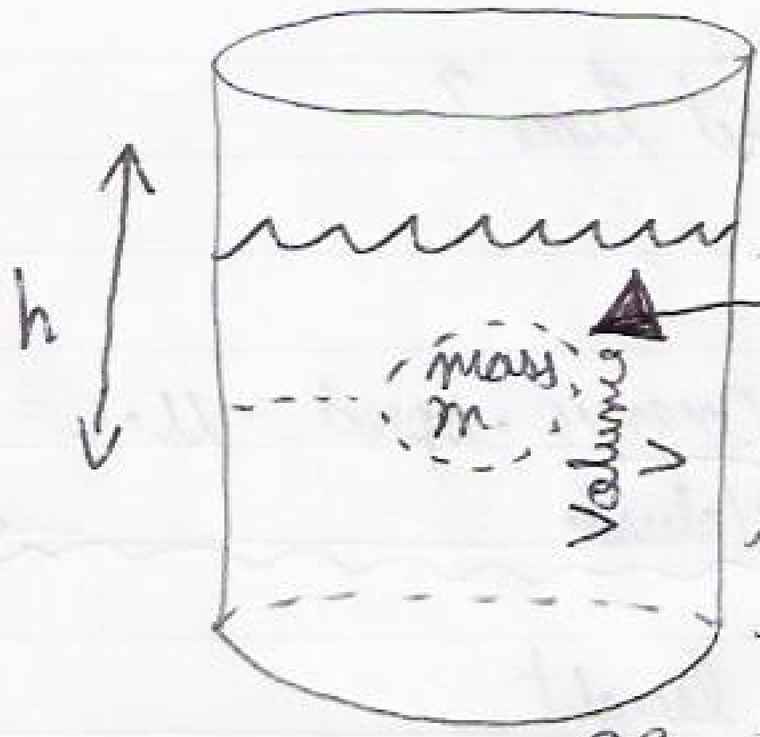
Average: $\bar{S} = \frac{E_{rms} B_{rms}}{\mu_0} = \frac{du}{dV} \cdot c = \frac{du}{A\,dx} \cdot \frac{dx}{dt} = \boxed{\frac{du}{A\,dt} = \bar{S}}$

$$\boxed{\underbrace{\bar{S}}_{\text{energy flow rate}} \cdot \underbrace{A}_{\text{area}} = \underbrace{\frac{du}{dt}}_{\text{power}}}$$

= Radiation Pressure =

$$\text{Energy density} = \frac{\text{energy}}{\text{Volume}} = \left(\frac{\text{Force} \cdot \text{distance}}{\text{Volume}}\right) \quad \text{pressure} = \frac{\text{force}}{\text{area}}$$

Potential energy
↓



At surface: $u = 0$

Net motion is stable & ≈ 0, so u is uniform: $u = 0$ everywhere

has gravitational potential energy. $-mgh$ (relative to surface)

has pressure-induced potential energy $+mgh$

$$\text{pressure} = P = \rho g h$$

$9.8\ m/s^2$ (g); depth (h); ρ: $\frac{\text{mass of fluid}}{\text{Volume}}$

$$= \frac{m}{V} g h = \frac{mgh}{V} = \frac{\text{energy from pressure}}{\text{Volume}} = \text{energy density from the pressure.}$$

$$\frac{\text{force}}{\text{area}} = \text{pressure} = \text{energy density} = u = \frac{\bar{S}}{c} = \boxed{\frac{E_{rms} B_{rms}}{c \mu_0}}$$

$$\text{force} = \text{area} \cdot \text{pressure} = \frac{A E_{rms} B_{rms}}{c \mu_0}$$

$$\| $$

$$A \cdot \frac{\bar{S}}{c} = \frac{A \cdot \bar{S}}{c} = \frac{\text{power}}{c}$$

$$\underbrace{\frac{dP}{dt} = \text{Force} = \frac{\text{Power}}{c} = \frac{dU/dt}{c}}$$

$$dp = \frac{dU}{c}$$

$$p = \frac{U}{c}$$ EM waves have momentum but no mass!

$$\text{EM momentum} = \frac{\text{EM energy}}{c}$$