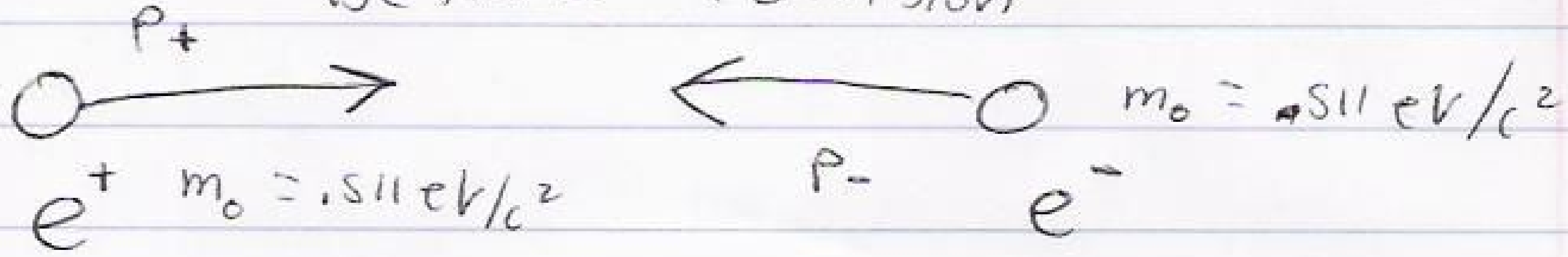
11/15/10

Before Collision



$E = 4.0\ GeV$ $E = 3.0\ GeV$

$1\ eV = 1.6022 \times 10^{-19}\ C \cdot \frac{1J}{C} = 1.6022 \times 10^{-19}\ J$

$1\ GeV = 10^{9}\ eV = 1.6022 \cdot 10^{-10}\ J$

After collision

$E_1 = hf_1$ $E_2 = hf_2$

h = Planck's constant.

f = frequency

$$E^2 = (m_0 c^2)^2 + (pc)^2$$

E = kinetic energy + rest energy m_0 = rest mass $m_0 c^2$ = rest energy

Mass not conserved, but energy & momentum are

$m_e = 9.11 \cdot 10^{-31} kg = .511 \underbrace{eV/c^2}_{\text{unit of mass}}$

$p = mv = \gamma m_0 v$

$m = \gamma m_0$

$\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \dfrac{v^2}{2c^2}$ $\Big\}$ for v no too close to c

$\gamma = \dfrac{1}{\sqrt{1-\frac{v}{c}}+\sqrt{1+\frac{v}{c}}} \approx \dfrac{1}{2\sqrt{1-\frac{v}{c}}}$ for $v \approx c$

Before collision:

$P_+ = \frac{1}{c}\sqrt{E^2 + \cancel{-}(m_e c^2)^2}$

$P_- = \frac{1}{c}\sqrt{E^2 - (m_e c^2)^2}$

$P_+ = \frac{1}{c}\sqrt{(4.0 \cdot 10^9 eV)^2 - (.511 eV)^2}$

$P_+ \approx \frac{1}{c}\sqrt{(4.0 \times 10^9 eV)^2} \approx 4.0 \times 10^9 eV/c$

$P_- \approx 3.0 \times 10^9 eV/c$

$P = P_+ - P_- = 1.0 \times 10^9 eV/c$

$E = E_+ + E_- = 7.0 \times 10^9 eV$

After collision:

$1.0 \times 10^9 eV/c = p = p_2 - p_1 = \frac{E_2}{c} - \frac{E_1}{c} = \frac{h}{c}(f_2 - f_1)$

$7.0 \times 10^9 eV = E = E_1 + E_2 = h(f_1 + f_2)$

Photons: $m_0 = 0 \Rightarrow E^2 = (pc)^2 \Rightarrow E = pc \Rightarrow p = \frac{E}{c}$

$$\left.\begin{array}{l} 1.0\ GeV/h = f_2 - f_1 \\ 7.0\ GeV/h = f_1 + f_2 \end{array}\right\} \therefore \left\{\begin{array}{l} 8.0\ GeV/h = 2f_2 \\ 6.0\ GeV/h = 2f_1 \end{array}\right.$$

$$= \left\{\begin{array}{l} 4.0\ GeV/h = f_2 \\ 3.0\ GeV/h = f_1 \end{array}\right. \qquad = \left\{\begin{array}{l} 4.0\ GeV = E_2 \\ 3.0\ GeV = E_1 \end{array}\right.$$

$$h = 6.63 \times 10^{-34}\ J \cdot s$$

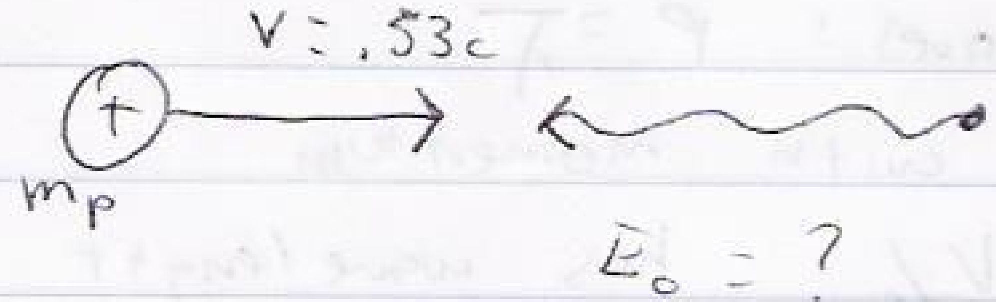
$$f_1 = \frac{3.0 \times 10^9\ eV}{6.63 \times 10^{-34}\ J.S} = \frac{4.8 \times 10^{-10}\ J}{6.63 \times 10^{-34}\ J \cdot S}$$

$$f_1 = 7.2 \times 10^{23}\ /s = 7.2 \times 10^{23}\ Hz$$

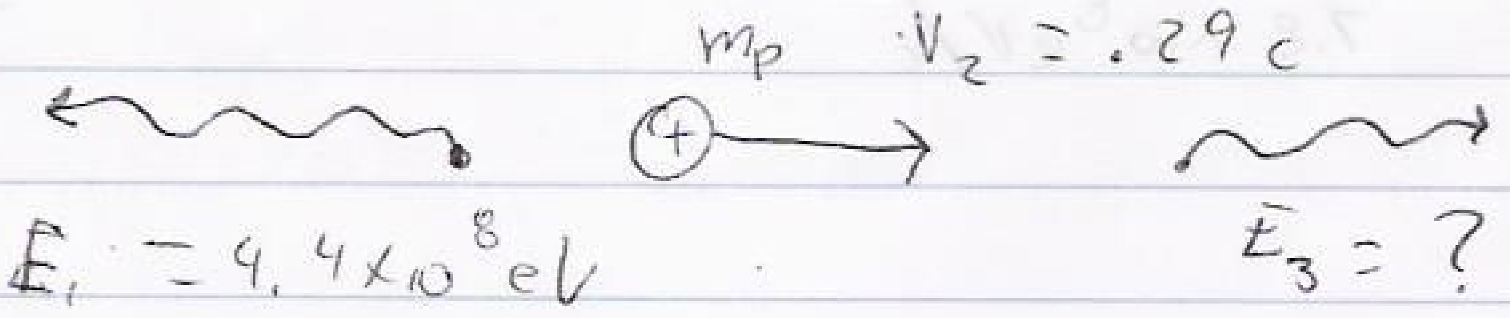
$$\lambda_1 = \frac{\ell}{cycle} = \frac{c}{f} = \frac{3.00 \times 10^8\ m/s}{7.2 \times 10^{23}/s}$$

$$\lambda_1 = 4.2 \times 10^{-16}\ m$$

Before:



After:



$$E_2 = \sqrt{(m_p c^2)^2 + (pc)^2}$$

$$p_2 = \gamma_2 m_p v_2$$

$$\gamma_2 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Before:

$$E_0 + E_+ = E = E_1 + E_2 + E_3$$

$$p_+ - p_0 = p = -p_1 + p_2 + p_3$$

solve for E_0 & E_3

Photons

$$\lambda = \frac{c}{f}$$

$$E = hf$$

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

all particles are waves: $p = \frac{h}{\lambda}$

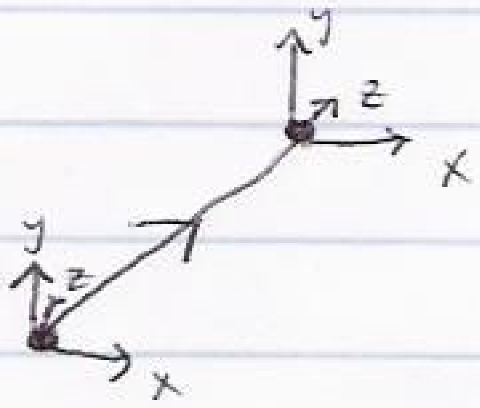
Eg, a proton with momentum 7.3×10^8 eV/c has wavelenght

$$\lambda = \frac{6.63 \times 10^{-34}\ J/s}{7.8 \times 10^8\ eV/c} = 1.6 \times 10^{-15}\ m$$

Lorentz invariants

(things all observers agree on)

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta\tau)^2$$



$\Delta\vec{x} = \vec{0}$

$\Delta t = \Delta\tau =$ "Propper time"

$=$ "wristwatch time"

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta\tau)^2 - 0 = (c\Delta\tau)^2$$

$$\left.\begin{array}{l}\Delta x = v\Delta t = v\gamma\Delta\tau \\ \Delta t = \gamma\Delta\tau\end{array}\right\} \stackrel{?}{=} \begin{array}{l}(c\Delta t)^2 - (\Delta x)^2 \\ (c\gamma\Delta\tau)^2 - (v\gamma\Delta\tau)^2 \\ (c\Delta\tau)^2\gamma^2\left(1 - \frac{v^2}{c^2}\right) = (c\Delta\tau)^2\end{array}$$