Section 37
Bohr model of
the hydrogen atom

All particles are also waves;

$$\text{momentum} = p = \frac{h}{\lambda}$$

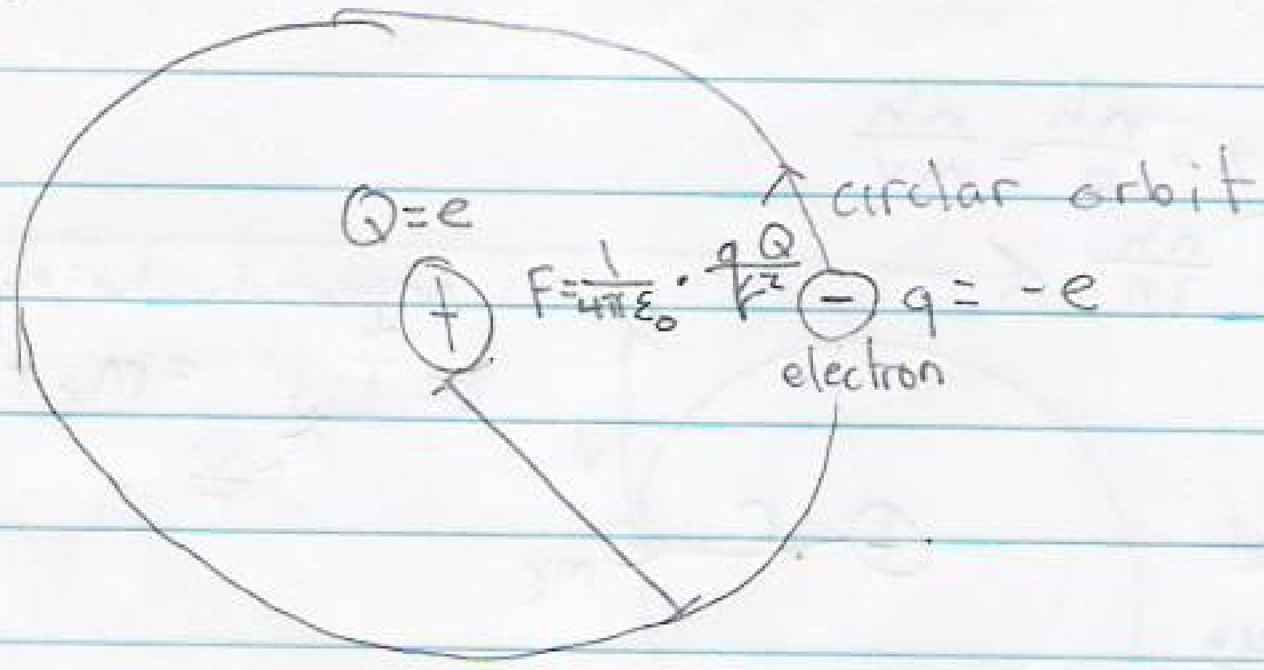
h = planck's constant $= 6.63 \cdot 10^{-34}$ $\underbrace{J \cdot s}$

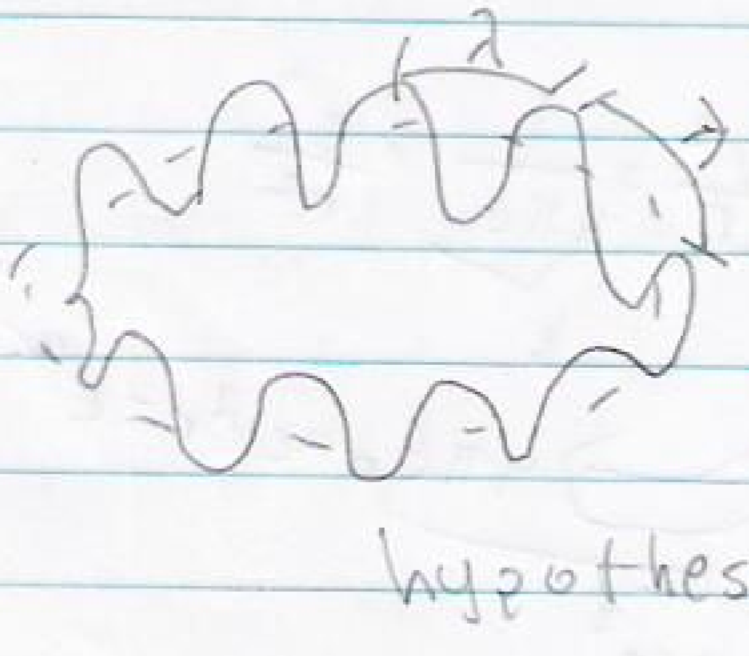
$J \cdot s = N \cdot m \cdot s = kg \cdot m \cdot s = \frac{kg \cdot m^2}{s}$

λ = wavelength

units: $kg\left(\frac{m}{s}\right)$ $\quad kg \cdot \left(\frac{m}{s}\right) = \frac{kg \cdot m^2/s}{m}$ ✓

Apply to an orbital model the H atom:

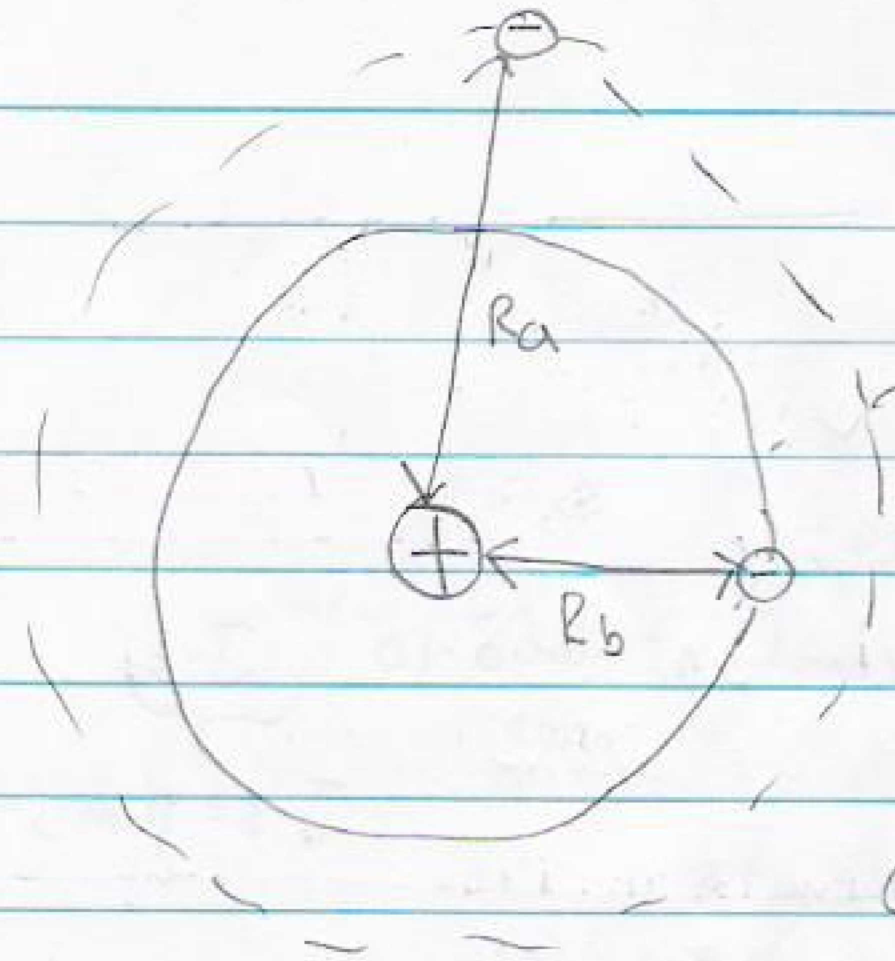




← picture

$2\pi r$ = circumference $= 11\lambda$

hypothesis: $\begin{cases} 2\pi r = n\lambda \\ p = h/\lambda \end{cases}$ for some $n = 1, 2, 3, \ldots$



Photon has energy $\Delta E = h f$ ← frequency of photon

Loss of energy $= -\Delta E$ ← depends on R_a & R_b

If R_a and R_b could be any distances, then $f = \frac{\Delta E}{h}$ could be any frequency

Only certain frequencies are observed

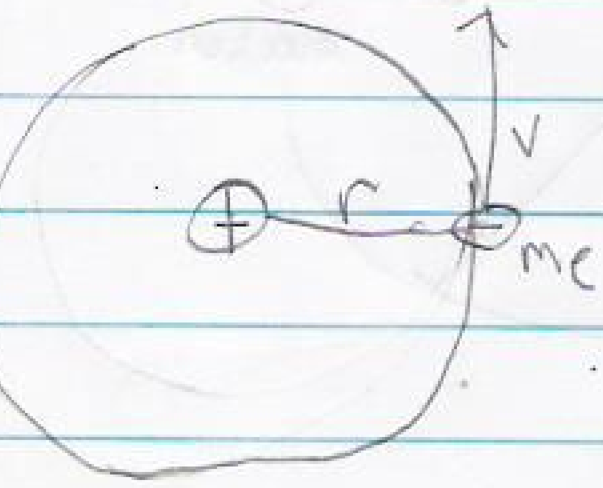
All equal (some constant) $\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$

where $\begin{cases} m = 1, 2, 3, 4, \text{ or } 5, \text{ or} \ldots \\ n = m+1, m+2, m+3, \text{ or } m+4 \text{ or} \ldots \end{cases}$

$$2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$$

$$\underbrace{mvr}_{\substack{\text{angular momentum} \\ L \text{ is quantized}}} = \frac{nh}{2\pi}$$

$$L = \overset{\overset{\text{momentum of inertia} = m_e r^2}{\downarrow}}{I}\underset{\frac{v}{r}}{\omega} = m_e r^2 \cdot \left(\frac{v}{r}\right) = m_e v r$$



$$L = \frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi}, \text{ or } \frac{4h}{2\pi}, \ldots$$

$$\left.\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \\ m_e a &= m_e \frac{v^2}{r} \end{aligned}\right\} \Rightarrow m_e \frac{v^2}{r} = \underbrace{\frac{1}{4\pi\epsilon_0}}_{k} \cdot \frac{e^2}{r^2}$$

↑ circular orbit

$$4\pi\epsilon_0 m_e v^2 r = e^2$$

$$v = \frac{nh}{2\pi m_e r}$$

$$4\pi\epsilon_0 m_e \left(\frac{nh}{2\pi m_e r}\right)^2 r = e^2$$

$$\boxed{r = \frac{n^2 h^2 \epsilon_0}{\pi m_e e^2} \quad \text{orbital radius is quantized}}$$

notation: $r_n = \frac{n^2 h^2 \epsilon_0}{\pi m_e e^2}$

kinetic potential

$E = K + U$

$K = \frac{1}{2} m_e \left(\frac{nh}{2\pi m_e r}\right)^2$

$U = \underbrace{-e}_{q}\,\underbrace{V}_{\text{voltage}} = -e\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{e}{r}\right) = \frac{-e^2}{4\pi\epsilon_0 r}$

Add these to get E: plug in $r = \frac{n^2 h^2 \epsilon_0}{\pi m_e e^2}$

$\Downarrow$

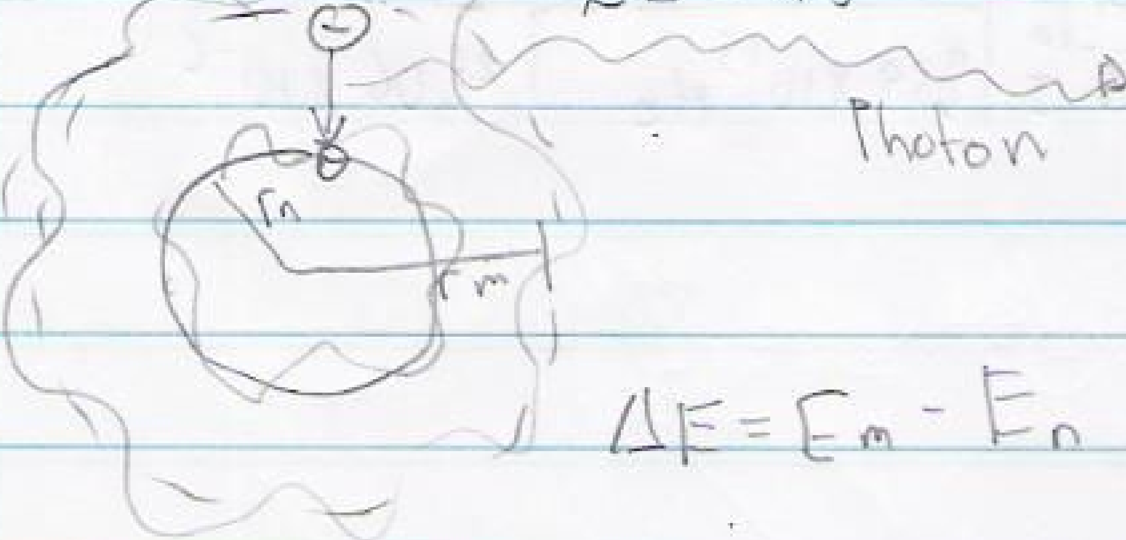
$E = \frac{-e^4 m_e}{8\epsilon_0^2 h^2 n^2} = \left(\frac{-13.6\,eV}{n^2}\right)$

+e ⊕ —— r —— here $V = \frac{ke}{r} = \frac{1}{4\pi\epsilon_0}\cdot\frac{e}{h}$

notation: $E_n = \underbrace{\frac{(-13.6\,eV)}{n^2}}_{\text{energy is quantized}}$

emitted photon } $\rightarrow \overbrace{f}^{\text{emitted frequencies quantized}} = \frac{\Delta E}{h} = \frac{(-13.6\,eV)}{h}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ matches experiments.

$\Delta E = hf$



$\Delta E = E_m - E_n$

compute the wavelengths of photons emitted from hydrogen when;

	(m, n)	ΔE	f photon	λ photon
Jesse	1, 2			
Allan	1, 3			
Jose	1, 4			
Teddy	1, 5			
Jorge V	1, 6			
Luis	1, 7			
	1, 8			
	1, 9			

	(m, n)	ΔE	f photon	λ photon
Jesse	(1, 2)	1.63×10^{-18} J	2.46×10^{15} Hz	1.22×10^{-7}
Alan	(1, 3)	1.93×10^{-18} J	2.91×10^{15} Hz	1.03×10^{7} m
Jose	(1, 4)	3.29×10^{-14} J	2.043×10^{14} Hz	9.74×10^{-7} m
Teddy	(1, 5)	2.09×10^{-12} J	3.16×10^{15} Hz	9.51×10^{-8} m
Jorge V	(1, 6)	2.18×10^{-12} J	3.19×10^{15} Hz	9.40×10^{-8} m
Luis	(1, 7)	4.45×10^{-20} J	6.7×10^{13} Hz	4.47×10^{-6} m
Alex	(2, 3)	3.02×10^{-19} J	1.56×10^{14} Hz	6.57×10^{-7} m
Jorge P	(2, 4)	4.08×10^{-20} J	6.16×10^{13} Hz	4.87×10^{-6} m
Freddy	(3, 4)	1.05×10^{20} J	1.59×10^{13} Hz	1.88×10^{-5} m
Milourd	(4, 5)	4.9×10^{-20} J	2.39×10^{13} Hz	4.06×10^{-6}