Ch. 21 Electrostatics

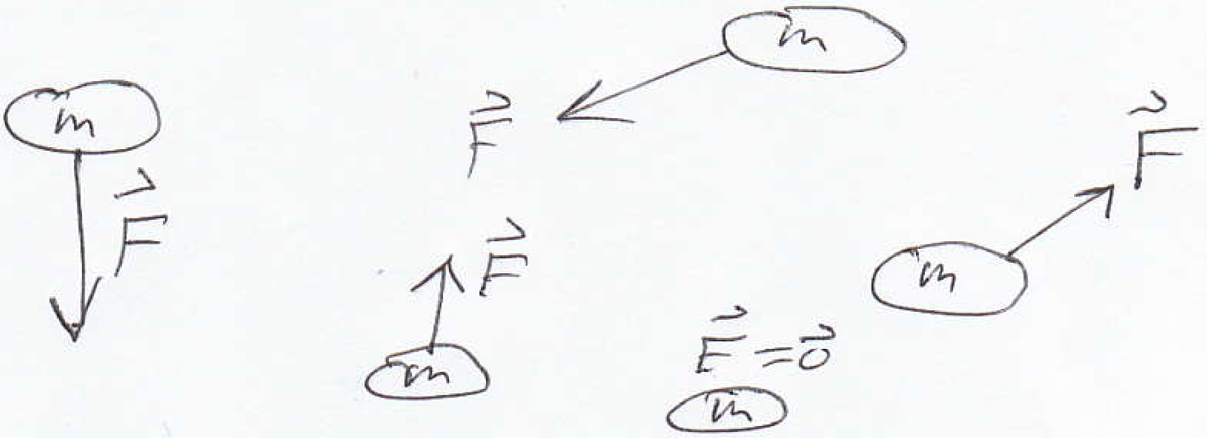
stationary charges
generating electric fields

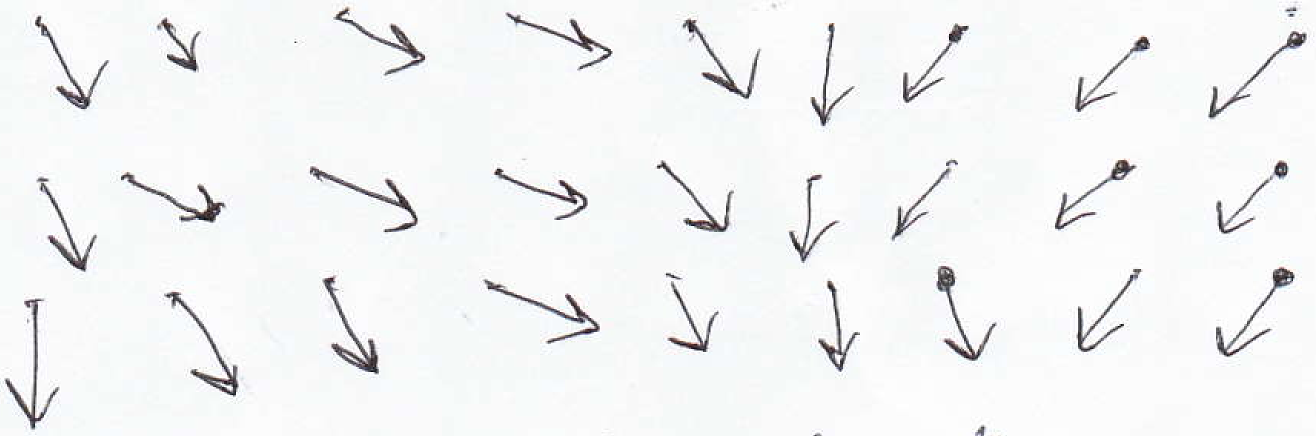
What is a Force Field?

__________ gravitational Field?

__________ electric Field?

Take a test object, see what the force acting on it is at different locations:





Force vectors depending on position (& time) & the test object.

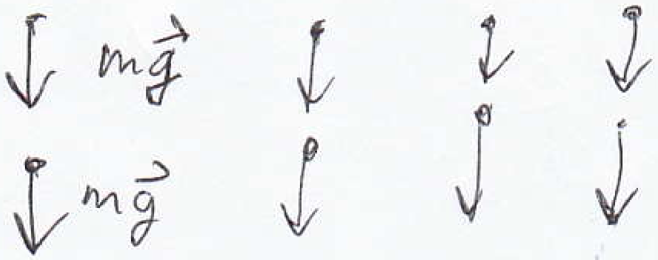
$$\vec{F} = m\vec{a} \qquad \frac{\vec{F}}{m} = \vec{a}$$

force

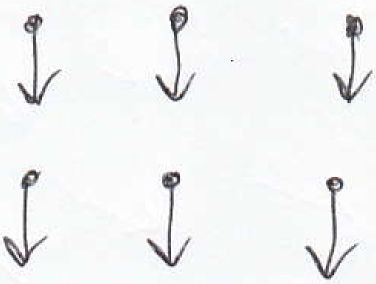
acceleration.

mass = gravitational charge

Force field of the earth's gravity: $\vec{g} = 9.80\,m/s^2$



$$\vec{F} = m\vec{g}$$



$\vec{g} = 9.80\ m/s^2$ gravitational field near earth

$$\vec{g} = \frac{\text{gravitational force}}{\text{gravitational } \cancel{\text{mass}} \text{ charge}}$$

Electric fields

$$\vec{E} = \frac{\text{electric force}}{\text{electric charge}}$$

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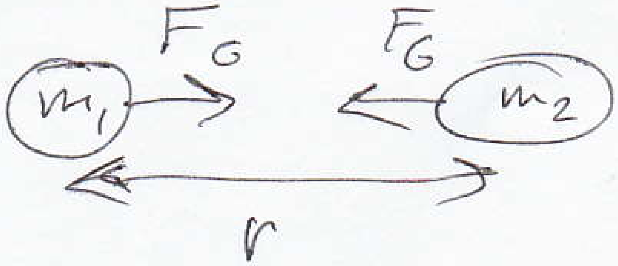
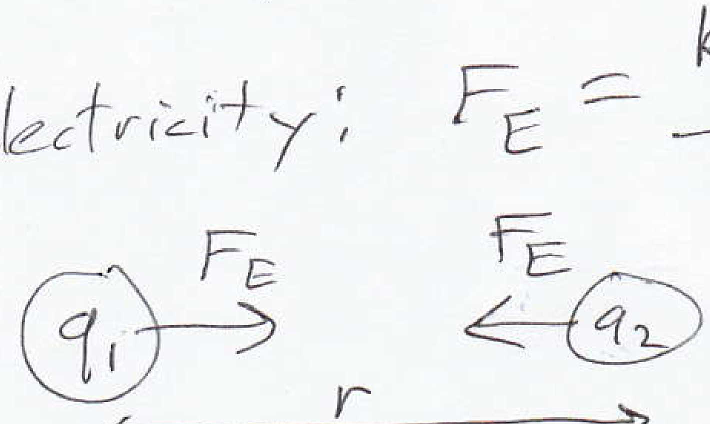
gravity: $F_G = \dfrac{G\, m_1 m_2}{r^2}$



electricity: $F_E = \dfrac{k\, q_1 q_2}{r^2}$



Generally, the test object has small charge compared to the charge of the object(s) causing the field you're measuring

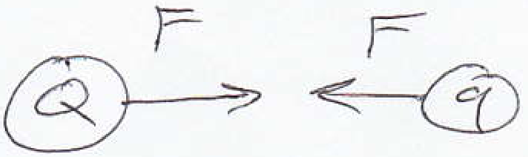
test mass

Gravity near earth $$F = \frac{G M_E m}{R_E^2}$$

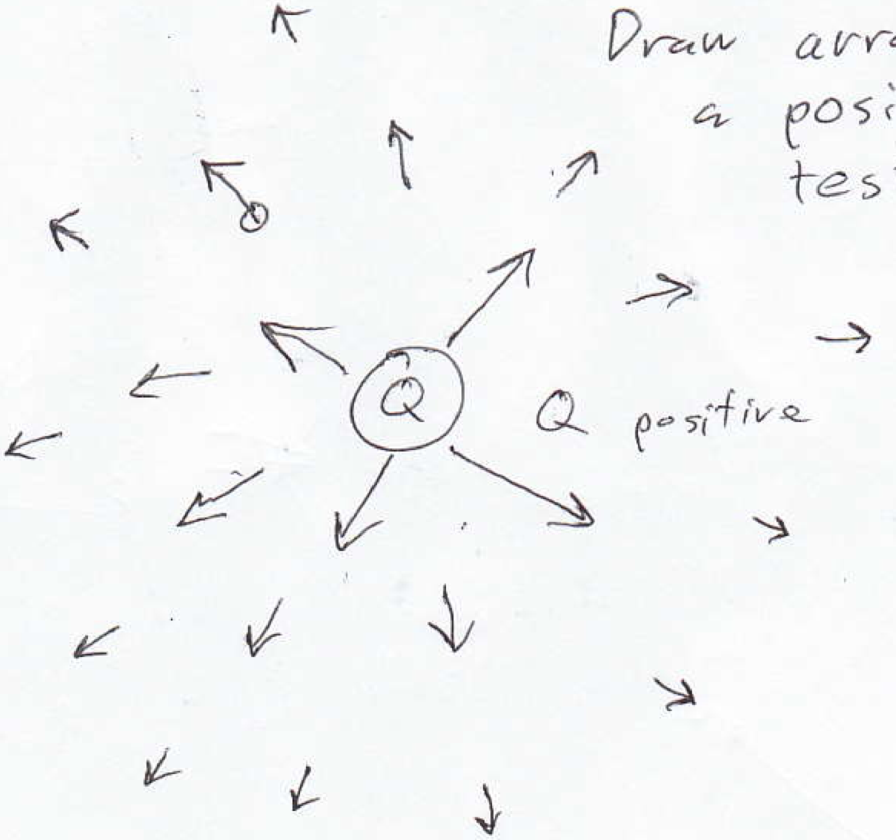
$$g = \frac{G M_E}{R_E^2} = 9.80 \, m/s^2$$

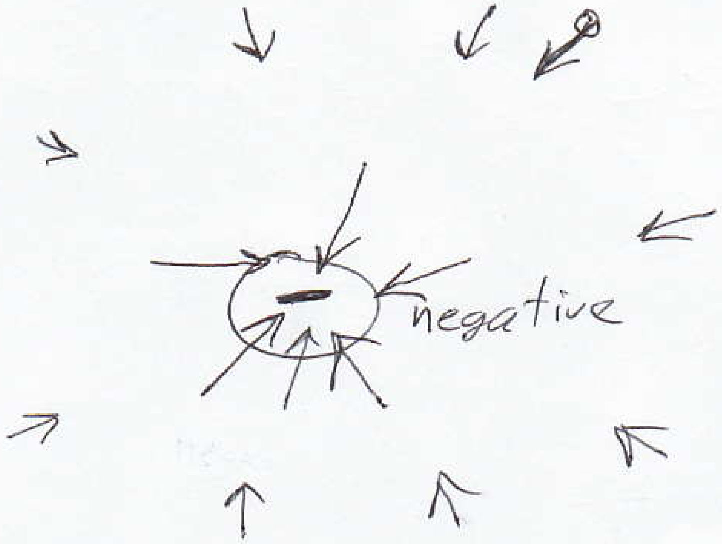
Similarly, given a big charge Q & a little test charge q

$$F = \frac{kQq}{r^2} \qquad E = \frac{F}{q} = \frac{kQ}{r^2}$$

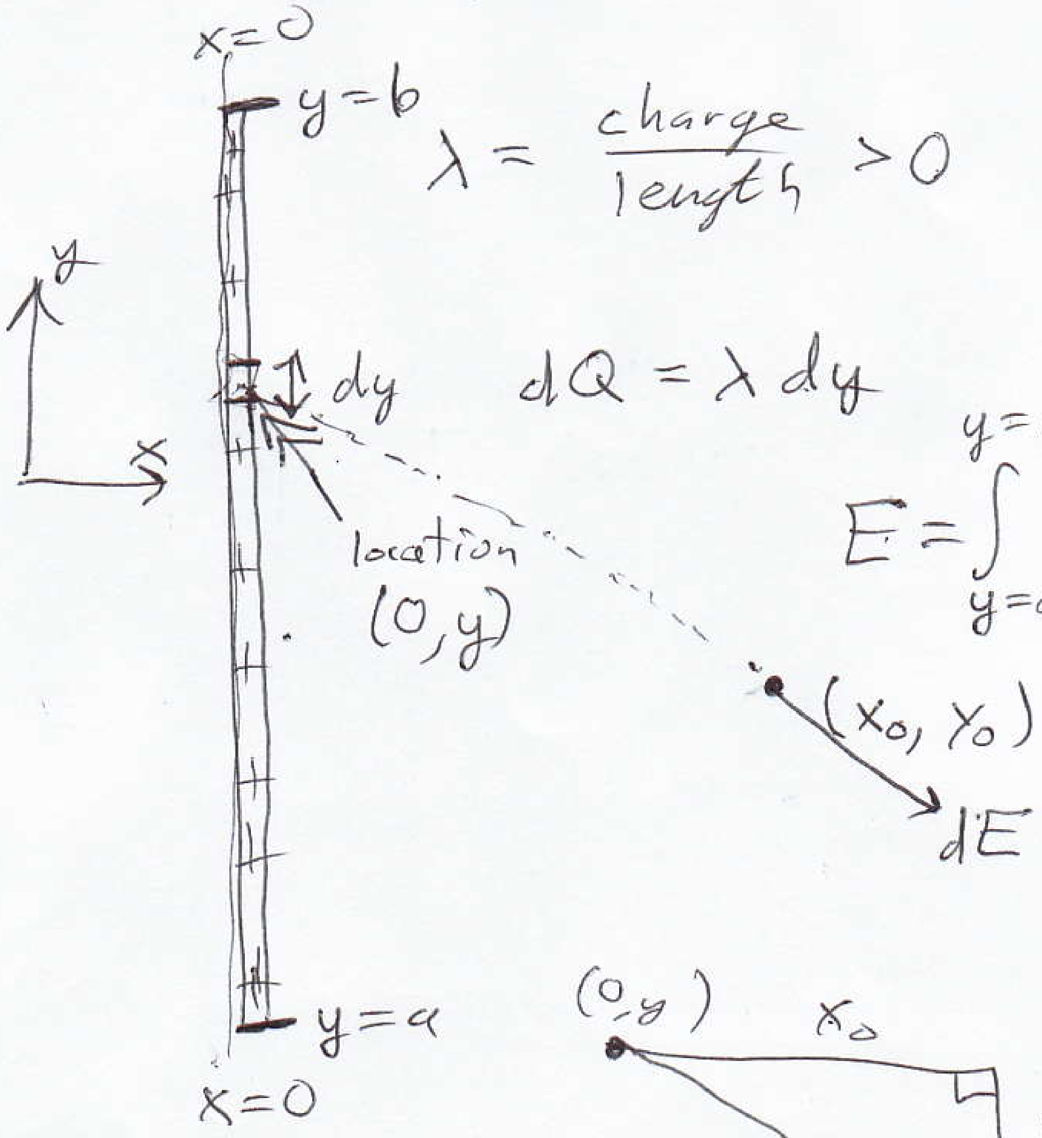


Convention:
Draw arrows for a positive test charge





Field generated by a long line of charge:



$$\lambda = \frac{\text{charge}}{\text{length}} > 0$$

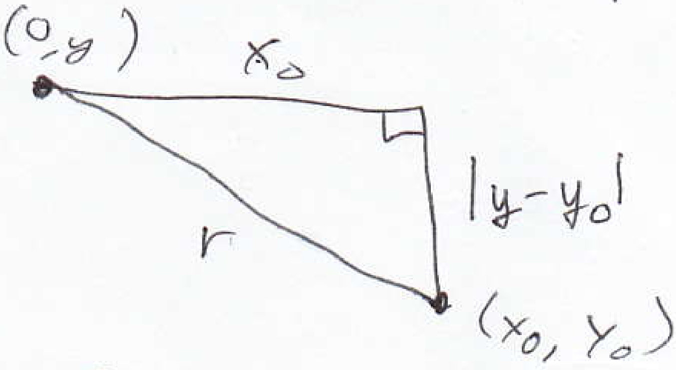
$$dQ = \lambda\, dy$$

constant

$$E = \int_{y=a}^{y=b} \frac{k\lambda\, dy}{x_0^2 + (y-y_0)^2}$$

constant

$$dE = \frac{dQ \cdot k}{r^2}$$

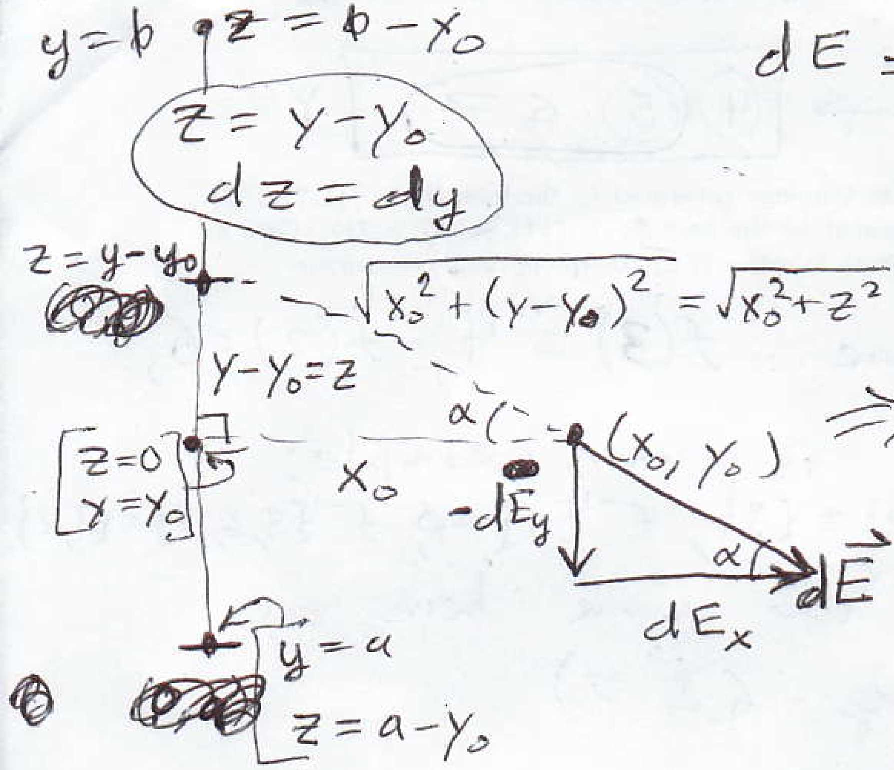


$$r^2 = (y-y_0)^2 + x_0^2$$

constant

$$dE = \frac{k\, dQ}{x_0^2 + (y-y_0)^2} = \frac{k\lambda\, dy}{x_0^2 + (y-y_0)^2}$$

constant



$$dE = k\lambda\, dz/(x_0^2+z^2)$$

$$\Rightarrow \begin{cases} \dfrac{dE_x}{dE} = \dfrac{x_0}{\sqrt{x_0^2+z^2}} \\[2ex] \dfrac{-dE_y}{dE} = \dfrac{z}{\sqrt{x_0^2+z^2}} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} dE_x = \dfrac{k\lambda x_0\, dz}{(x_0^2+z^2)^{3/2}} \\[2ex] dE_y = \dfrac{-k\lambda z\, dz}{(x_0^2+z^2)^{3/2}} \end{cases}$$

$$\Leftarrow \begin{cases} E_x = \displaystyle\int_{a-y_0}^{b-y_0} \frac{k\lambda x_0\, dz}{(x_0^2+z^2)^{3/2}} \\[2ex] E_y = \displaystyle\int_{a-y_0}^{b-y_0} \frac{-k\lambda z\, dz}{(x_0^2+z^2)^{3/2}} \end{cases}$$

For E_y, use $u = x_0^2+z^2$

$$\Downarrow$$

$$-\tfrac{1}{2}k\lambda\, du = -k\lambda z\, dz \Leftarrow du = 2z\, dz$$

$$\Downarrow$$

$$E_y = \int_{x_0^2+(a-y_0)^2}^{x_0^2+(b-y_0)^2} \frac{(-\frac{1}{2}k\lambda)\, du}{u^3} = -\frac{1}{2}k\lambda \frac{u^{-2}}{-2}\Bigg|_{x_0^2-(a-y_0)^2}^{x_0^2+(b-y_0)^2}$$

$$E_y = \frac{k\lambda}{4}\left[\frac{1}{(x_0^2+\underbrace{(b-y_0)^2}_{\to\infty})^2} - \frac{1}{(x_0^2+\underbrace{(a-y_0)^2}_{\to\infty})^2}\right]$$

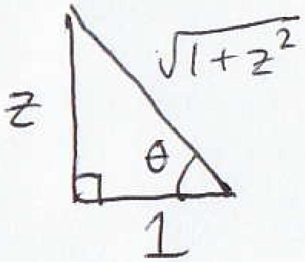
In the limit $a \to -\infty$, $b \to \infty$, we have $E_y \to 0$.

For E_x, use $z = x_0 \tan\theta \Rightarrow \begin{cases} dz = x_0 \sec^2\theta \, d\theta : \\ z^2 = x_0^2 \tan^2\theta \Rightarrow x_0^2 + z_0^2 = x_0^2 \sec^2\theta \end{cases}$

$$\Rightarrow E_x = \int_{\arctan(a-y_0)}^{\arctan(b-y_0)} \frac{(k\lambda x_0)(x_0 \sec^2\theta \, d\theta)}{x_0^3 \sec^3\theta} \qquad (x_0^2+z_0^2)^{3/2} = x_0^3 \sec^3\theta$$

$$\Rightarrow E_x = \frac{k\lambda}{x_0} \int_{\arctan(a-y_0)}^{\arctan(b-y_0)} \cos\theta = \frac{k\lambda \sin\theta}{x_0} \Bigg|_{\arctan(a-y_0)}^{\arctan(b-y_0)}$$

To find $\sin(\arctan(z))$ @ $z = b - y_0,\ a - y_0$: set up $\tan\theta = z$:



$$\Rightarrow \sin\theta = \frac{z}{\sqrt{1+z^2}} \Rightarrow E_x = \frac{k\lambda}{x_0}\left[\frac{b-y_0}{\sqrt{1+(b-y_0)^2}} \underbrace{- \frac{a-y_0}{\sqrt{1+(a-y_0)^2}}}_{= + \frac{(y_0-a)}{\sqrt{1+(y_0-a)^2}}}\right]$$

$$\Rightarrow E_x = \frac{k\lambda}{x_0}\left[\frac{1}{\sqrt{\underbrace{(b-y_0)^{-2}}_{\to 0}+1}} + \frac{1}{\sqrt{\underbrace{(y_0-a)^{-2}}_{\to 0}+1}}\right]$$

In the limit $a \to -\infty$, $b \to \infty$, we have $E_x \to \frac{2k\lambda}{x_0}$, which equals $\frac{\lambda}{2\pi\varepsilon_0 x_0}$.