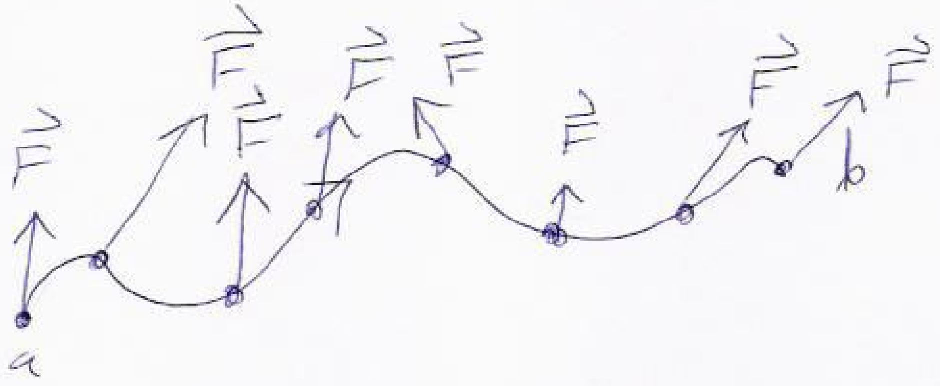
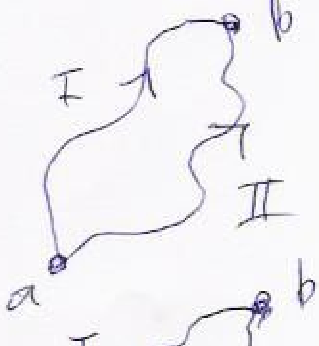
Work $W = W_{ba} = \int_a^b \vec{F} \cdot d\vec{l}$



If ~~loops~~ no work is done on loops, we call the force conservative,



and W_{ba} only depends $\vec{F}$, b, and a, not the path taken.

$0 = W_{loop} = W_I - W_{II} \Rightarrow W_I = W_{II}$

$\underbrace{\text{Electrostatic}}$ generated by stationary charges fields are conservative just like gravity.

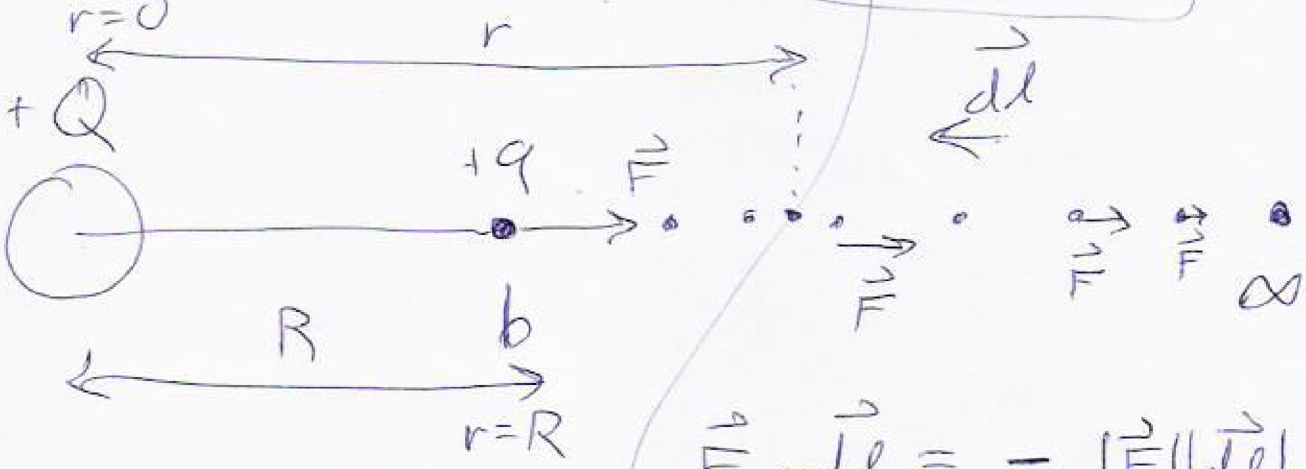
For a conservative force,

$$U_{ba} = -W_{ba} = -\int_a^b \vec{F} \cdot \vec{d\ell}$$

To have an absolute potential energy, pick a point/place to have potential energy 0.

Common choice: $U = 0$ "at ∞":

$$U_b = U_{b,\infty} = -\int_\infty^b \vec{F} \cdot \vec{d\ell} = \boxed{\int_b^\infty \vec{F} \cdot \vec{d\ell}}$$



$$\vec{F} \cdot \vec{d\ell} = -|\vec{F}||\vec{d\ell}|$$
$$= -F\,d\ell$$

$$U_b = \boxed{\int_{r=R}^{r=\infty} -\frac{kQq}{r^2}(-dr)}$$

$d\ell = -dr$

$-r \leftarrow \quad \rightarrow +r$

$$U_b = \int_R^\infty \frac{kQq}{r^2}\,dr = \int_R^\infty kQq\,r^{-2}\,dr$$

$$= kQq \int_R^\infty r^{-2}\,dr = kQq \left.\frac{r^{-2+1}}{-2+1}\right|_R^\infty$$

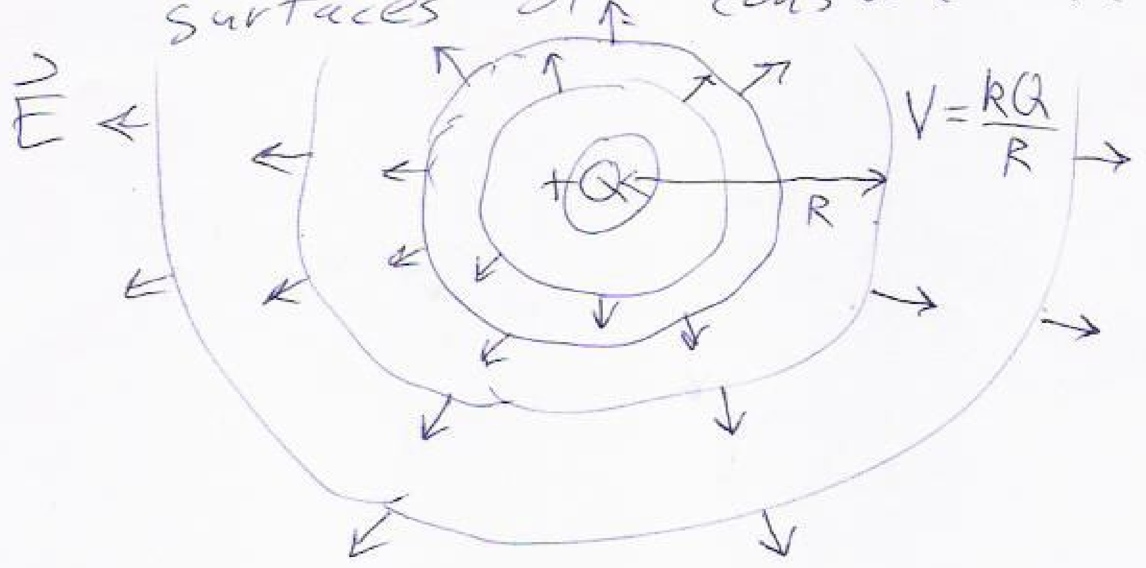
$$= -kQq\,r^{-1}\Big|_R^\infty = \left.\frac{-kQq}{r}\right|_R^\infty = \frac{-kQq}{\infty} - \left(\frac{-kQq}{R}\right)$$

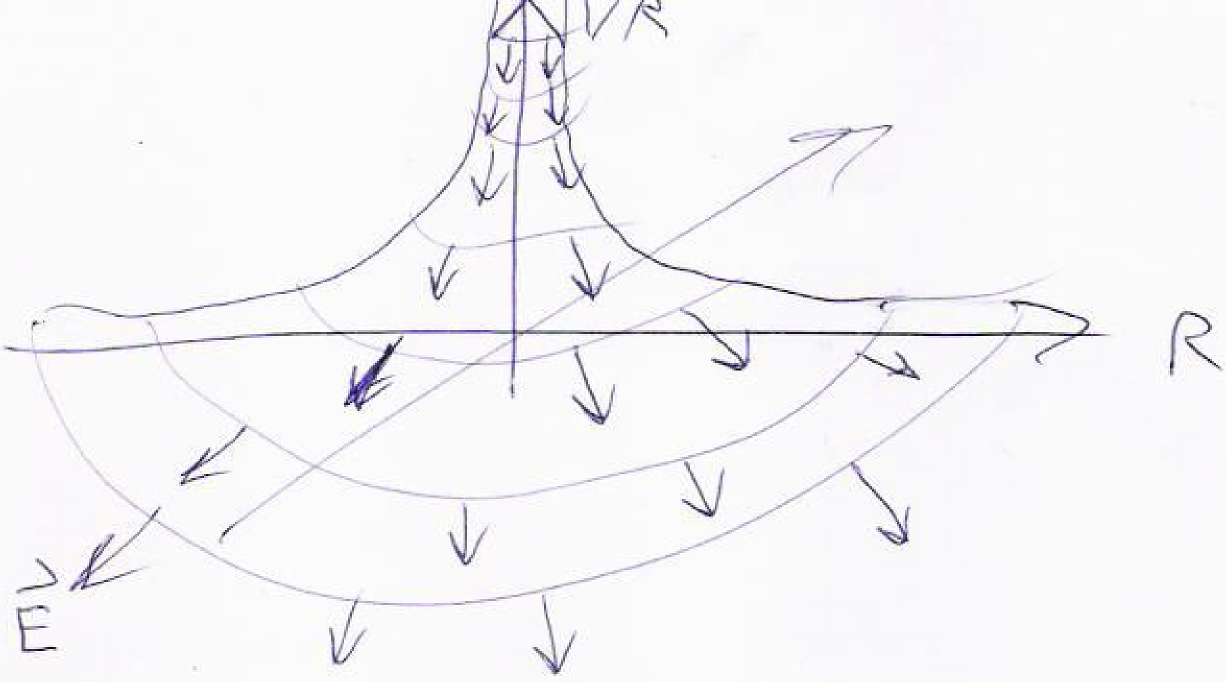
$$= 0 - \left(-\frac{kQq}{R}\right) = \boxed{\frac{kQq}{R} = U_b}$$

$$\lim_{r\to\infty} \frac{-kQq}{r} = 0$$

$$V_{ba} = \frac{U_{ba}}{q} \qquad V_b = \frac{U_b}{q} = \frac{kQ}{R}$$

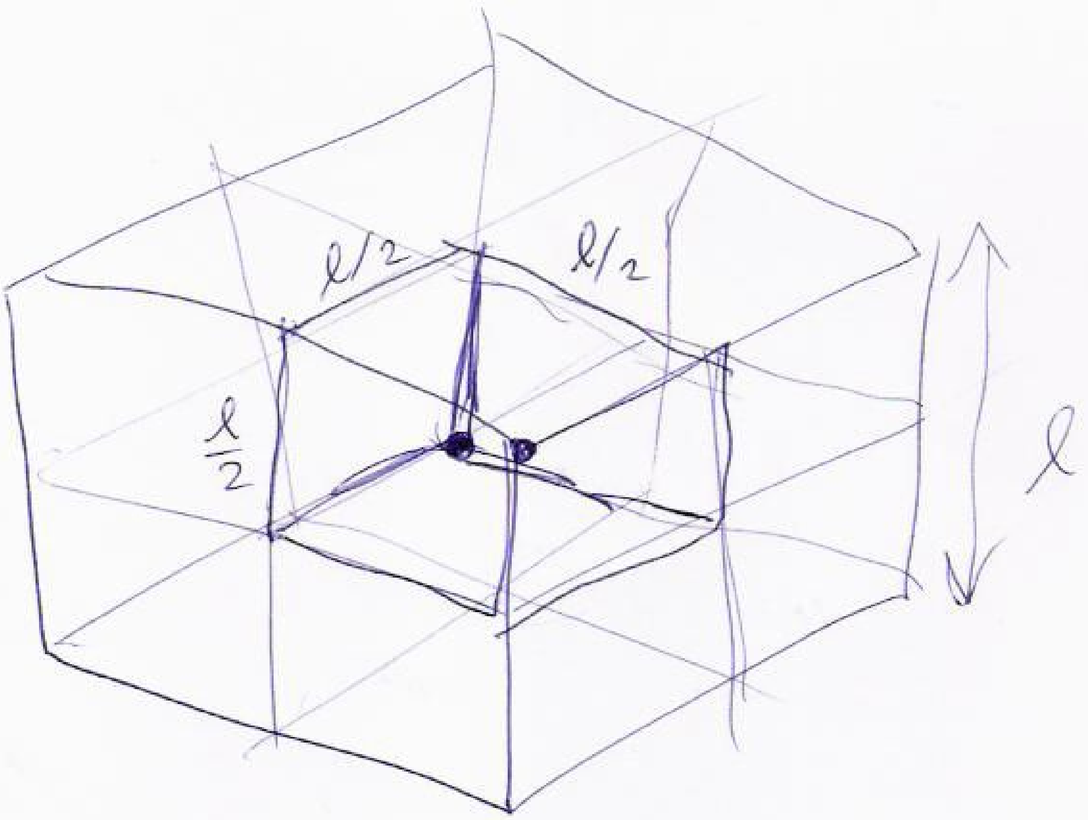
equipotential surfaces would be surfaces of constant R (spheres)

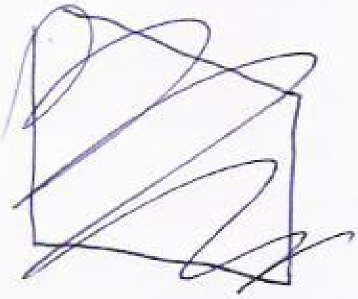


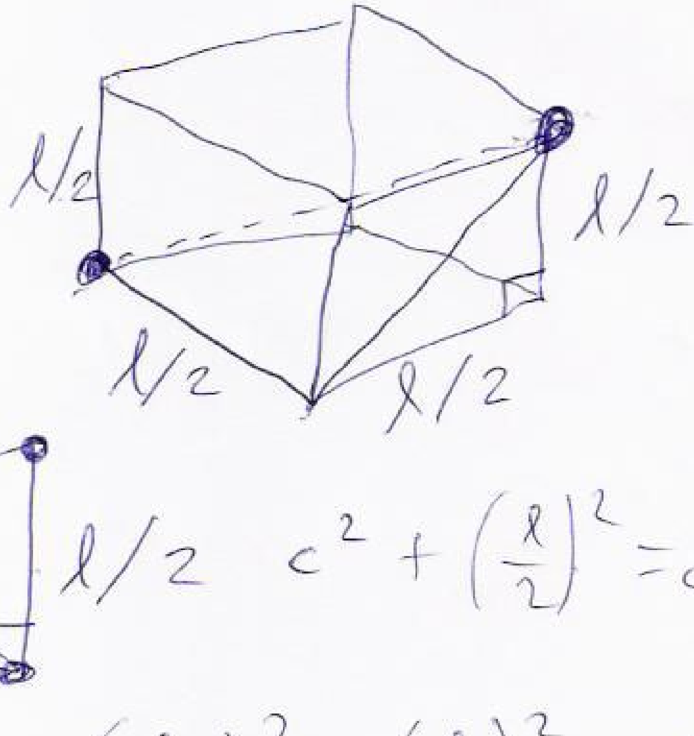


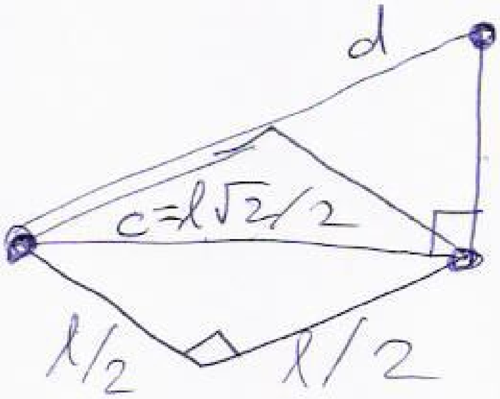
#72 (Ch. 23)

(Following pages)









$$c^2 + \left(\frac{l}{2}\right)^2 = d^2$$

$$\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2 = c^2$$

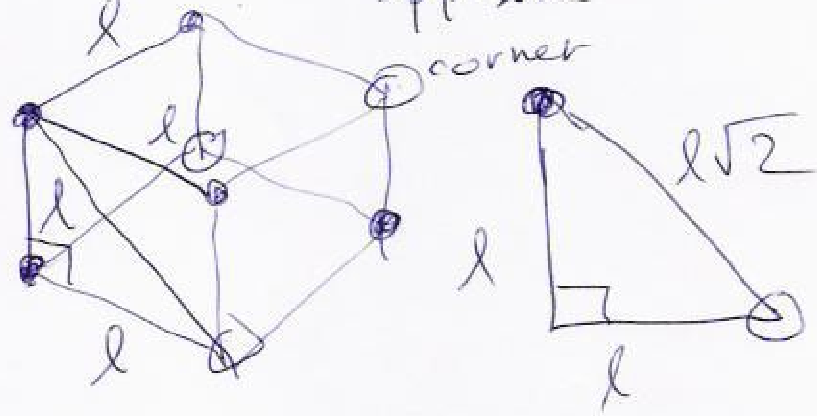
$$\frac{\sqrt{2}\,l}{2} = c \Leftarrow 2\left(\frac{l^2}{4}\right) = c^2$$

$$\frac{3\ell^2}{4} = \frac{2\ell^2}{4} + \frac{\ell^2}{4} = d^2 \Rightarrow \frac{\ell\sqrt{3}}{2} = d$$

$$a)\ V_{center} = 8\frac{kQ}{d} = 8kQ\frac{2}{\ell\sqrt{3}} = \frac{16kQ}{\ell\sqrt{3}} = \frac{4Q}{\ell\pi\varepsilon_0\sqrt{3}}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$b)\ V_{corner} = 3\frac{kQ}{\ell} + \underbrace{\frac{kQ}{\ell\sqrt{3}}}_{\text{opposite corner}} + 3\frac{kQ}{\ell\sqrt{2}}$$



$$c)\ \sum_{pairs}\frac{kQQ}{distance} = U_{system}$$

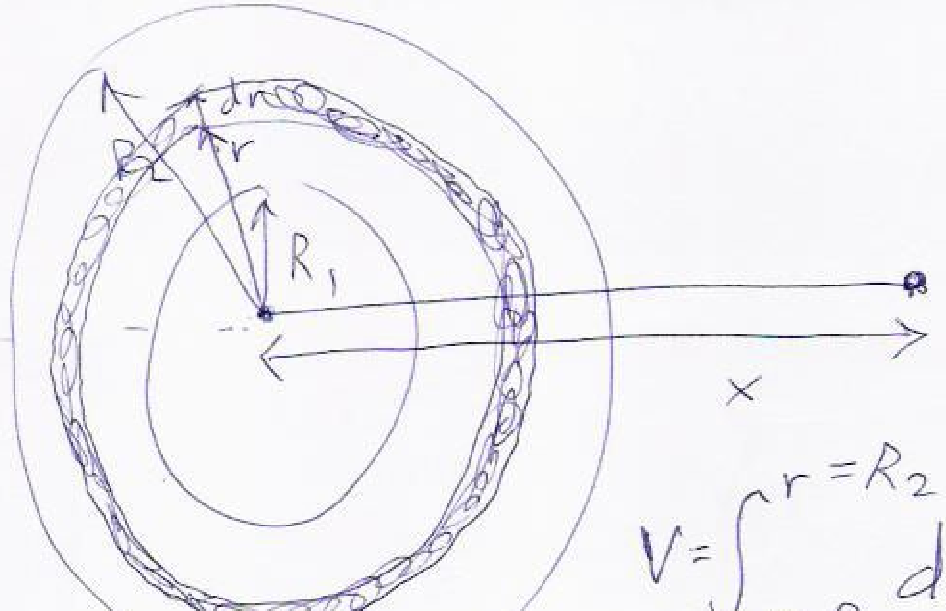
Each charge is paired with 7 other charges.

$$\frac{8 \text{ charges} \times 7 \text{ other charges}}{2} = 28 \text{ pairs}$$

$$U_{system} = \frac{1}{2} \cdot 8\, V_{corner}\, Q$$

$$U_{system} = \frac{4kQ^2}{l}\left[3 + \frac{1}{\sqrt{3}} + \frac{3}{\sqrt{2}}\right]$$

Try #35 (p. 624)



Like Example 23-8 p. 615

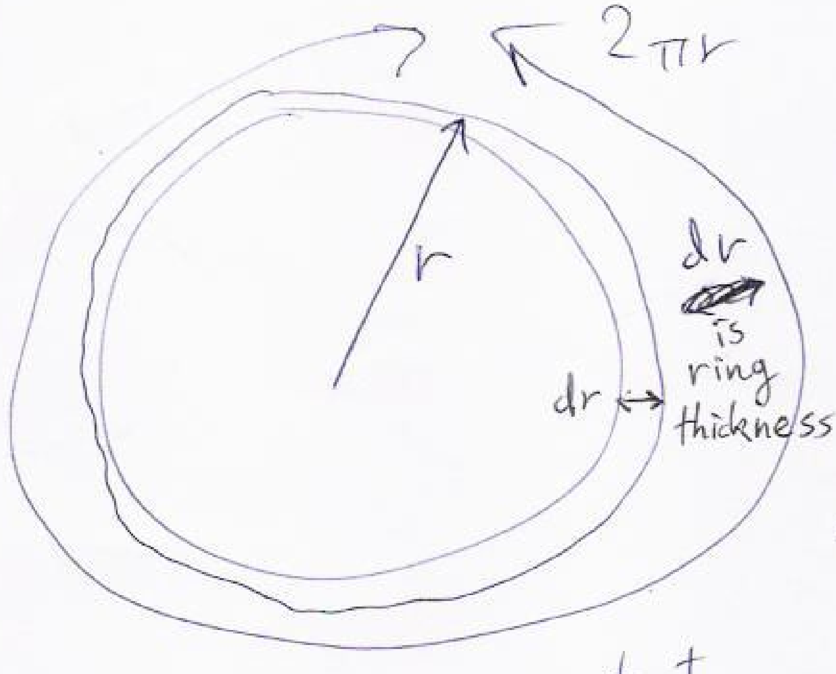
$$V = \int_{r=R_1}^{r=R_2} dV_r$$

dQ_r = charge of thin ring

$$dV_r = \frac{1}{4\pi\varepsilon_0}\frac{dQ_r}{(x^2+r^2)^{1/2}} = \frac{k\,dQ_r}{\sqrt{x^2+r^2}}$$

From Ex. 23-8

$$dQ_r = ?$$



$$dA_r = 2\pi r\,dr$$

$$\sigma = \frac{dQ_r}{dA_r}$$

$$dQ_r = 2\pi r \sigma\,dr$$

constant

$$V = \int_{R_1}^{R_2} \frac{2\pi r \sigma k\,dr}{\sqrt{x^2+r^2}}$$

const.

$$u = x^2 + r^2$$

$$du = 2r\,dr$$

$$u_1 = x^2 + R_1^2$$

$$u_2 = x^2 + R_2^2$$

$$V = \int_{x^2+R_1^2}^{x^2+R_2^2} \frac{\pi\sigma k\,du}{\sqrt{u}}$$

$$V = \int_{u_1}^{u_2} \pi\sigma k\,u^{-1/2}\,du = \pi\sigma k \left.\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right|_{u_1}^{u_2}$$

$$V = \pi \sigma k \, 2\sqrt{u} \, \Big|_{u_1}^{u_2}$$

$$V = 2\pi\sigma k \left(\sqrt{x^2 + R_2^2} - \sqrt{x^2 + R_1^2} \right)$$

$$V = \frac{\sigma}{2\varepsilon_0} \left(\sqrt{x^2 + R_2^2} - \sqrt{x^2 + R_1^2} \right)$$