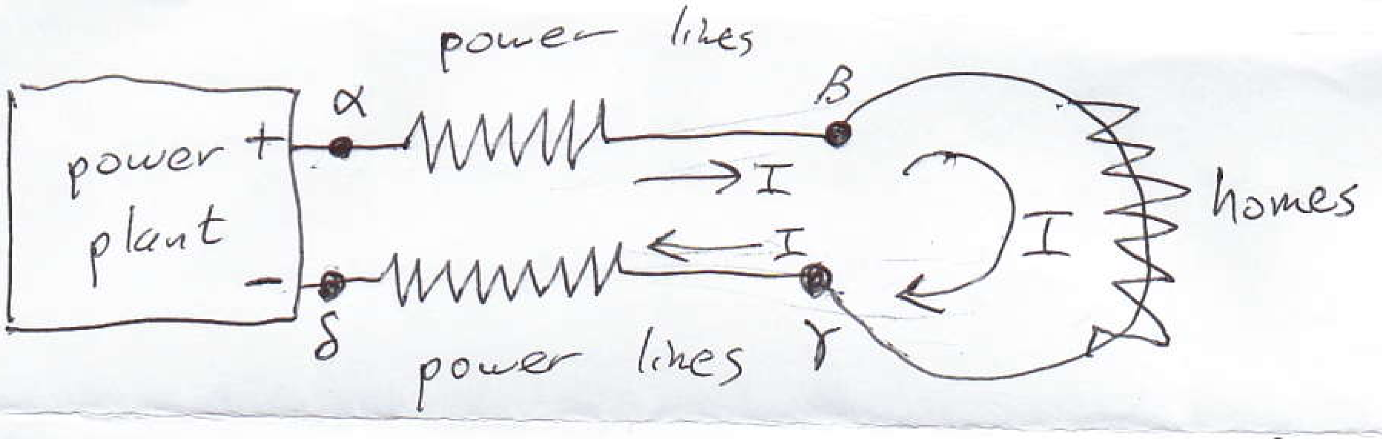
TODO (Milovich): suggest exercises 22–26

Power transmission:

1.0×10^5 homes, 2.0×10^3 W per home



transmission loss: $P_{\alpha\beta} + P_{\gamma\delta}$

useful electrical power: $P_{\beta\gamma} = \overbrace{2.0 \times 10^8 \text{ W}}^{2.0 \times 10^2 \text{ MW}}$

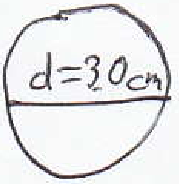
total power generated: $P_{\alpha\beta} + P_{\gamma\delta} + P_{\beta\gamma}$

$$e = \text{efficiency} = \frac{P_{\beta\gamma} \leftarrow 200\text{ MW}}{P_{\alpha\beta} + P_{\beta\gamma} + P_{\gamma\delta}}$$

What should $V_{\alpha\delta}$ be to get $e = \underbrace{0.99}_{99\%}$?

power lines: aluminum: $\rho = 2.8 \times 10^{-8}\ \Omega \cdot \text{m}$

$\ell = 1.0 \times 10^2 \text{ km} = 1.0 \times 10^5 \text{ m}$ $\quad R = \frac{\rho \ell}{A}$



$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 \qquad d = 3.0\times10^{-2}\,m$$

$$A = 7.069\times10^{-4}\,m^2$$

$$V = IR \qquad V_{\alpha\delta} = V_{\alpha\beta\gamma\delta} = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\delta}$$

$$P = IV \qquad P_{\alpha\beta} = I V_{\alpha\beta} = IIR_{line}$$

$$200MW = P_{homes} = P_{\beta\gamma} = I V_{\beta\gamma} = IIR_{homes}$$

$$P_{\gamma\delta} = I V_{\gamma\delta} = IIR_{line}$$

$$0.99 = e = \frac{P_{homes}}{P_{homes} + 2I^2R_{line}}$$

$$R_{line} = \frac{\rho l}{A} = \frac{(2.8\cdot10^{-8}\,\Omega\cdot m)(1.0\times10^{5}\,m)}{7.069\cdot10^{-4}\,m^2}$$

$$= 3.961\,\Omega$$

R_{line}

$$\frac{1}{e} = \frac{P_{homes} + 2I^2R_{line}}{P_{homes}}$$

$$\frac{P_{homes}}{e} = P_{homes} + 2I^2R_{line}$$

$$P_h\left(\frac{1}{e} - 1\right) = \frac{P_{homes}}{e} - P_{homes} = 2I^2R_{line}$$

$$\frac{P_h\left(\frac{1}{e}-1\right)}{2R} = I^2 \Rightarrow \sqrt{\frac{P_h\left(\frac{1}{e}-1\right)}{2R}} = I$$

$$I = \sqrt{\frac{W}{\Omega}}\ 5.0497\ldots \times 10^2 = 5.0 \times 10^2 A$$

$$\Omega = V/A = \frac{\frac{J}{C}}{\frac{C}{s}} = \frac{J}{C} \cdot \frac{s}{C} = \frac{J \cdot s}{C^2}$$

$$W = J/s$$

$$\frac{W}{\Omega} = \frac{\frac{J}{s}}{\frac{J \cdot s}{C^2}} = \frac{\cancel{J}}{s} \frac{C^2}{\cancel{J} \cdot s} = \frac{C^2}{s^2} = A^2$$

$5.0 \times 10^2 A$

4.0Ω

$$\underbrace{P_{\alpha\beta} + P_{\gamma\delta}}_{\text{transmission loss}} = I^2 R_{line} + I^2 R_{line} = \underbrace{2 I^2 R_{line}}_{2.02\ldots \times 10^6 W}$$

$$A^2 \Omega = W$$

$$2.0 \times 10^8 W = P_{homes} = P_{\beta\gamma}$$

$$\text{total power} = P_{\alpha\beta} + P_{\beta\gamma} + P_{\gamma\delta} = I(\underbrace{V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\delta}}_{V_{\alpha\delta}})$$

$$V_{\alpha\delta} = \frac{2.0\times10^{8}\,W + 2.02\ldots\times10^{6}\,W}{I}$$

$I \leftarrow 5.0\times10^{2}\,A$

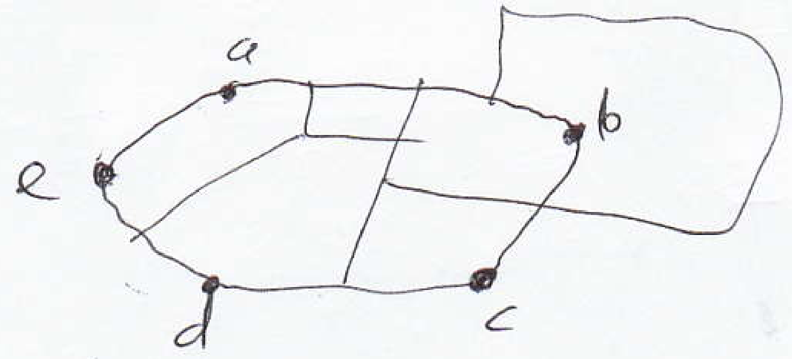
$$V_{\alpha\delta} = 4.0\times10^{5}\,V$$

high voltage for efficient transmission
lower voltage for safety

What should the voltage $V_{\alpha\delta}$ be for 99.99% efficiency?

Kirchoff's laws (Ch. 26)

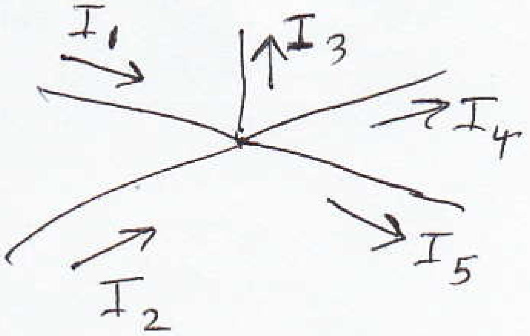
Loop law: In a loop of a circuit, the voltage drop along the loop is 0.



$$0 = V_{abcdea} = V_{ab} + V_{bc} + V_{cd} + V_{de} + V_{ea}$$

Equiv: $-V_{ea} = V_{ae} = V_{ab} + V_{bc} + V_{cd} + V_{de}$

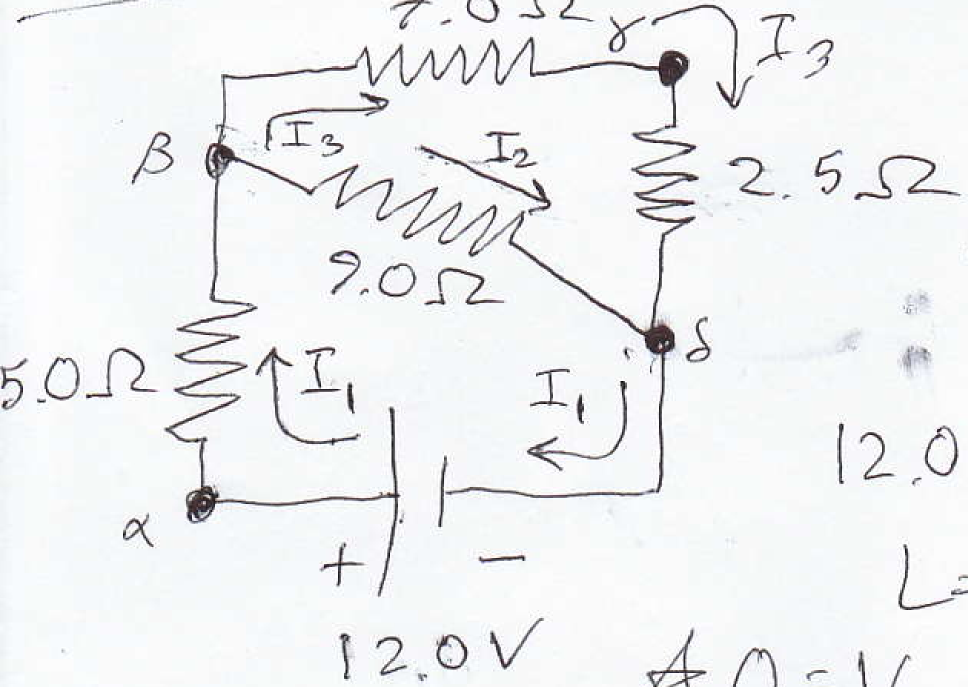
Junction law:



At junctions, current in = current out

$$I_1 + I_2 = I_3 + I_4 + I_5$$

(I's can be + or −.)



Junction law

At β: $I_1 = I_2 + I_3$

δ: $I_2 + I_3 = I_1$

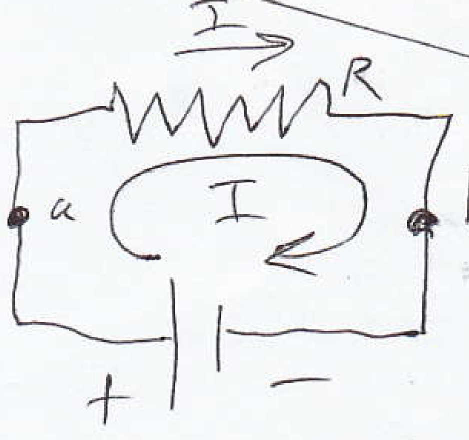
$$12.0V = V_\alpha - V_\delta = V_{\alpha\delta}$$

Loop law:

$$0 = V_{\alpha\beta\delta\alpha} = \underbrace{V_{\alpha\beta}}_{I_1(5.0\Omega)} + \underbrace{V_{\beta\delta}}_{I_2(9.0\Omega)} + \underbrace{V_{\delta\alpha}}_{-12.0V}$$

$$0 = V_{\beta\gamma\delta\beta} = V_{\beta\gamma} + V_{\gamma\delta} + V_{\delta\beta}$$

$$0 = I_3(7.0\Omega) + (2.5\Omega)I_3 - I_2(9.0\Omega)$$



$$-V_{ba} = V_{ab} = IR$$

Solve the 3 ☆'s for I_1, I_2, I_3.

To find voltages, use $V = (\pm) IR$:

$V_\alpha - V_\beta = V_{\alpha\beta} = (5.0\,\Omega) I_1$ $\quad V_{\beta\delta} = (9.0\,\Omega) I_2$

$V_\beta - V_\gamma = V_{\beta\gamma} = (7.0\,\Omega) I_3$ $\quad V_{\beta\delta} = V_\beta - V_\delta$

$$\begin{cases} I_1 = I_2 + I_3 \\ 0 = I_1(5.0\,\Omega) + I_2(9.0\,\Omega) - 12.0\,V \\ 0 = I_3(7.0\,\Omega) + I_3(2.5\,\Omega) - I_2(9.0\,\Omega) \end{cases}$$

$$\frac{12.0\,V}{\Omega} = \frac{I_1(5.0\,\Omega) + I_2(9.0\,\Omega)}{\Omega}$$

$$12.0\,A = 5.0\,I_1 + 9.0\,I_2$$

$$\boxed{0 = 9.5\,I_3 - 9.0\,I_2}$$

$$12.0\,A = 5.0(I_2 + I_3) + 9.0\,I_2$$

$$\boxed{12.0\,A = 14.0\,I_2 + 5.0\,I_3}$$

$$\begin{cases} 0 = -9.0 I_2 + 9.5 I_3 & (eq_1) \\ 12.0A = 14.0 I_2 + 5.0 I_3 & (eq_2) \end{cases}$$

$14 eq_1 + 9 eq_2$: $108A = 0 + 178 I_3$

$$\boxed{0.61A} = \frac{108A}{178} = I_3$$

$5 eq_1 - 9.5 eq_2$: $-114A = -178 I_2 + 0$

$$\boxed{0.64A} = \frac{-114A}{-178} = I_2$$

$$I_1 = I_2 + I_3 = \boxed{1.25A}$$

Then Find $V_{\alpha\beta}$, $V_{\beta\gamma}$, $V_{\gamma\delta}$ ...

$$V_{\alpha\beta} = I_1 (5.0\Omega) = \boxed{6.3V}$$

$$V_{\beta\gamma} = I_3 (7.0\Omega) = \boxed{4.2V}$$

$$V_{\gamma\delta} = I_3 (2.5\Omega) = \boxed{1.5V}$$

$$V_{\beta\delta} = I_2 (9.0\Omega) = \boxed{5.8V}$$

Bonus:

$$P_{\alpha\beta} = I_1^2 (5.0\Omega) = \boxed{7.8W}$$

$$P_{\beta\gamma} = I_3^2 (7.0\Omega) = \boxed{2.6W}$$

$$P_{\gamma\delta} = I_3^2 (2.5\Omega) = \boxed{0.92W}$$

$$P_{\beta\delta} = I_2^2 (9.0\Omega) = \boxed{3.7W}$$

total power: $P_{\alpha\beta} + P_{\beta\gamma} + P_{\gamma\delta} + P_{\beta\delta}$ equals 15.0W, which equals $I_1 \cdot V_{\alpha\delta} = (1.25A)(12.0V)$.

$P = IV = I^2 R$

(also, $P = IV = V^2/R$)