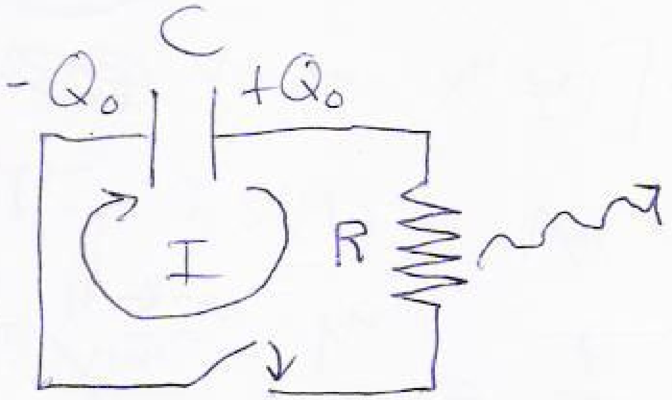


Close the switch.

Then, After time $\tau = RC$,

$$Q = e^{-1} Q_0 \text{ on capacitor}$$

$$e^{-1} = 0.367794\ldots$$

Here's why: $q(t)$ = charge on positive plate @ time t after switch closed. $dq/dt = -I$

$$C = \frac{q}{V} \qquad R = \frac{V}{I} \qquad C, R \text{ constant}$$

$$I = \frac{V}{R} = \frac{q/C}{R} = \frac{q}{RC} \Rightarrow \frac{dq}{dt} = -\frac{q}{RC}$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{RC}\,dt \Rightarrow \int_{Q_0}^{Q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\boxed{\int_{3m}^{5m} \frac{dx}{x} = \underbrace{\ln(5m) - \ln(3m)}_{\text{nonsense}} = \ln\frac{5m}{3m} = \ln\frac{5}{3}}$$

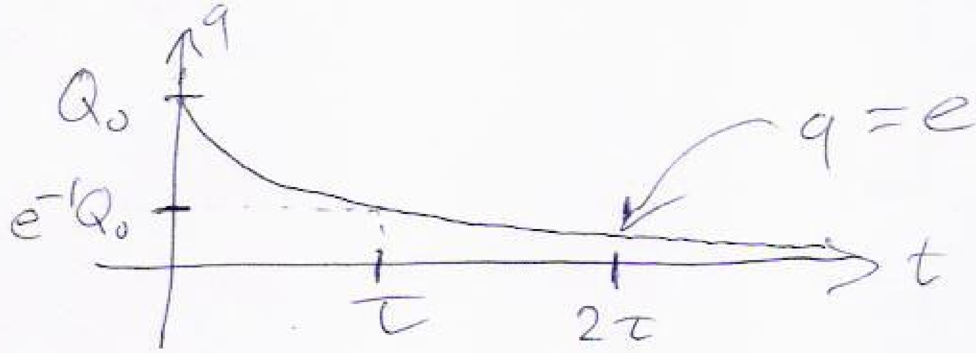
$$\ln\frac{Q}{Q_0} = \frac{-t}{\tau}$$

$$\frac{Q}{Q_0} = e^{-t/\tau}$$

$$\tau = RC$$

$q = Q = Q_0 e^{-t/\tau}$

$t = \tau \Rightarrow Q = Q_0 e^{-\tau/\tau} = Q_0 e^{-1}$



$q = e^{-2} Q_0 \approx 0.14 Q_0$

$e^{-3} Q_0 \approx 0.05 Q_0$

$I = - \frac{dq}{dt} = -Q_0\left(-\frac{1}{\tau}\right)e^{-t/\tau}$

$I = \frac{Q_0}{\tau} e^{-t/\tau}$ $\quad t = 0 \Rightarrow I_0 = \frac{Q_0}{\tau} e^0 = \frac{Q_0}{\tau}$

$V = \frac{Q}{C} = \frac{Q_0}{C} e^{-t/\tau}$ $\quad t = 0 \Rightarrow V_0 = \frac{Q_0}{C} e^0 = \frac{Q_0}{C}$

U = energy in capacitor $= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \boxed{\frac{Q_0^2}{2C} e^{-\frac{2t}{\tau}}}$

$P = IV$ = rate at which resistor converts current into heat

$P = \frac{Q_0^2}{\tau C} e^{-2t/\tau} = P_0 e^{-2t/\tau}$

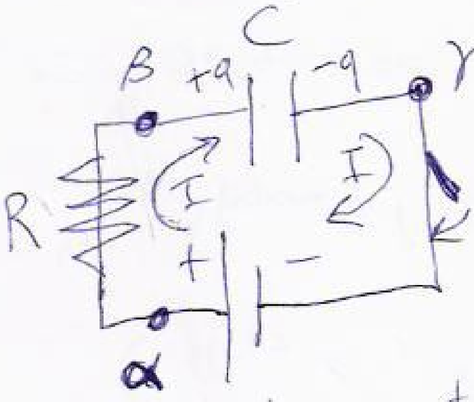
$t = \tau \Rightarrow P = P_0 e^{-2} \approx 0.14 P_0$

$\frac{dU}{dt} = -P \Rightarrow U = \boxed{\int_0^t -P\, dt} = \boxed{\frac{P_0 \tau}{2} e^{-\frac{2t}{\tau}}}$

$$e^{-10} = 4.5\ldots \times 10^{-5}$$

Waiting 10τ is as good as waiting forever.

Charging a capacitor



$t=0$: $q=0$ (switch closed)

Time t after switch closed: $q = ?$

V_0 = constant

$$V_{\alpha\gamma} = V_0 = \text{const}$$

$$V_0 = V_{\alpha\gamma} = V_{\alpha\beta\gamma} = \underbrace{V_{\alpha\beta}}_{IR} + \underbrace{V_{\beta\gamma}}_{q/C}$$

$$\Downarrow$$

$$\frac{dq}{dt} = I \Rightarrow V_0 = \frac{dq}{dt}R + \frac{q}{C}$$

$$V_0 - \frac{q}{C} = \frac{dq}{dt}R \Rightarrow dt\left(V_0 - q/C\right) = R\,dq$$

$$\Downarrow$$

$$dt = \frac{R\,dq}{V_0 - q/C} = \frac{RC\,dq}{CV_0 - q}$$

$\tau \to RC$

$Q_{max} \to CV_0$

$$\int_0^t \frac{dt}{\tau} = \int_0^q \frac{dq}{Q_{max} - q}$$

$u = Q_{max} - q$

$du = -dq$

$$\frac{t}{\tau} = \int_{Q_{max}-0}^{Q_{max}-q} \frac{-du}{u} = \int_{Q_{max}-q}^{Q_{max}-0} \frac{du}{u}$$

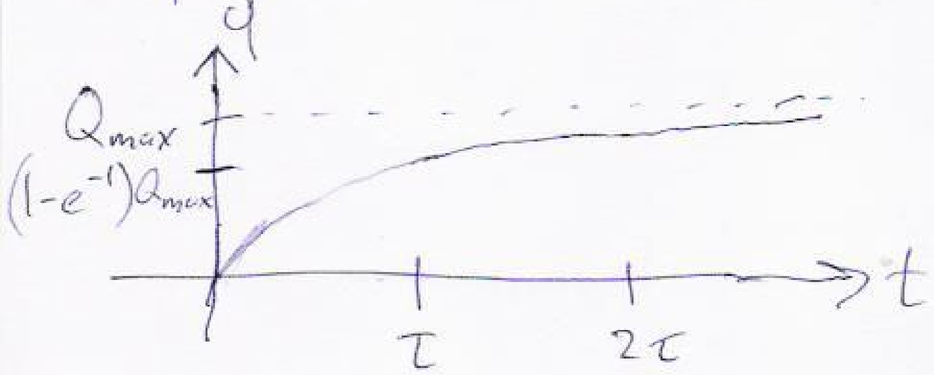
$$\frac{t}{\tau} = \ln \frac{Q_{max}}{Q_{max}-q} \Rightarrow e^{t/\tau} = \frac{Q_{max}}{Q_{max}-q}$$

$$\Downarrow$$

$$Q_{max} e^{-t/\tau} = Q_{max} - q \Leftarrow e^{-t/\tau} = \frac{Q_{max}-q}{Q_{max}}$$

$$\Downarrow$$

$$q = Q_{max} - Q_{max} e^{-t/\tau} = Q_{max}\left(1 - e^{-t/\tau}\right)$$



$$\lim_{t\to\infty} q = Q_{max}(1-0)$$

Graph

$V_{\alpha\beta}$, $V_{\beta\gamma}$

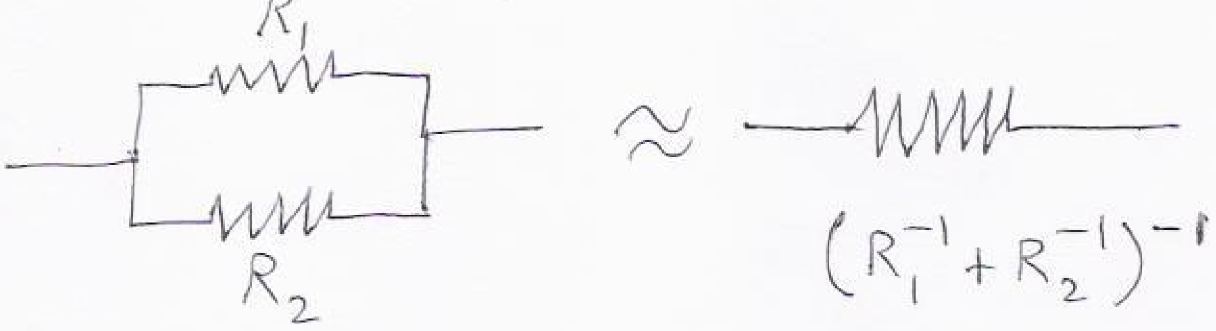
$P_{\alpha\beta}$, $U_{\beta\gamma}$,

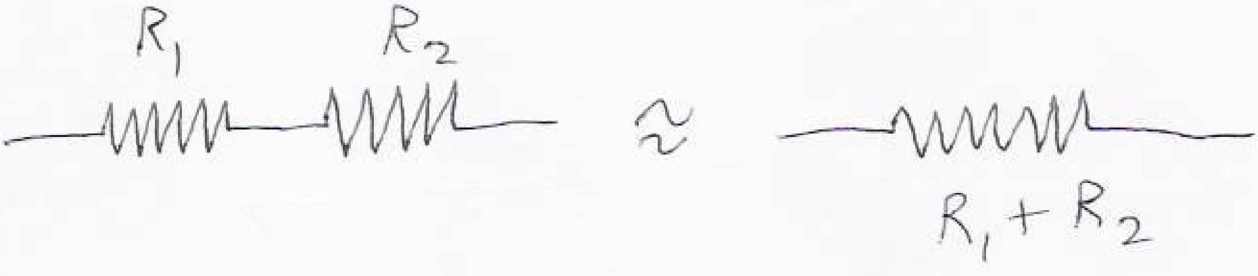
$P_{\beta\gamma}$ too ...

E.g. $R = 41\,\Omega$ $C = 470\,\mu F = 4.70\times10^{-4} F$

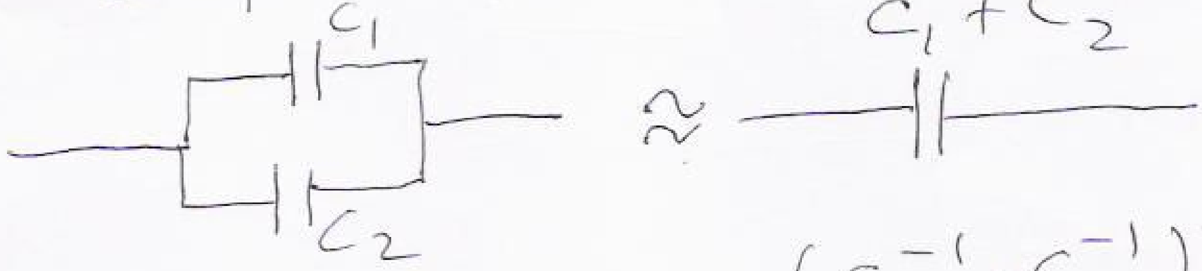
$$\tau = RC = (41 \cdot 4.7\times10^{-4})\,s = 1.9\times10^{-2}\,s$$

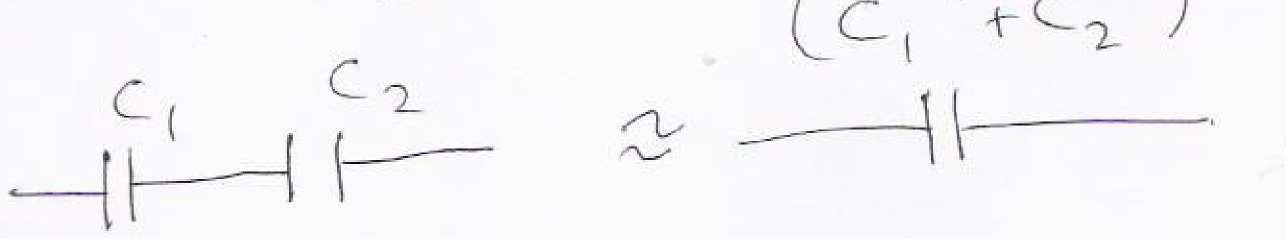
Resistors in parallel & series





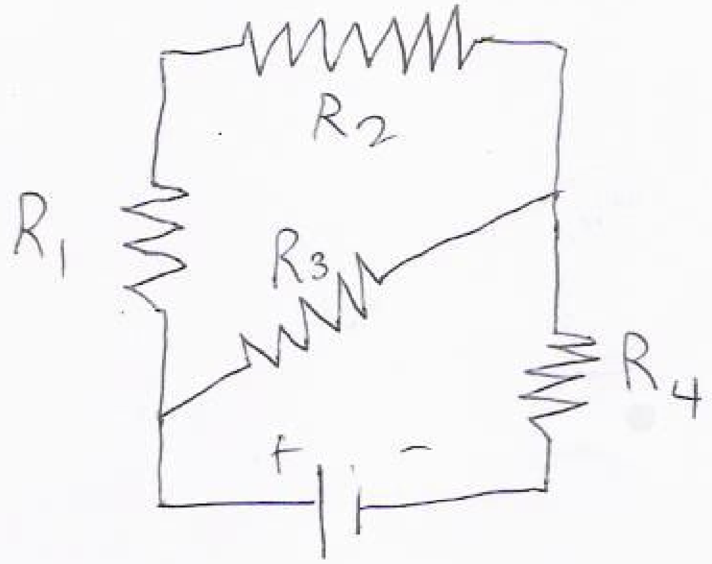
Compare to capacitors:

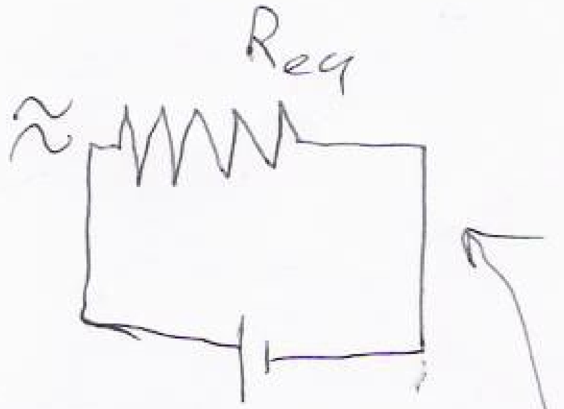


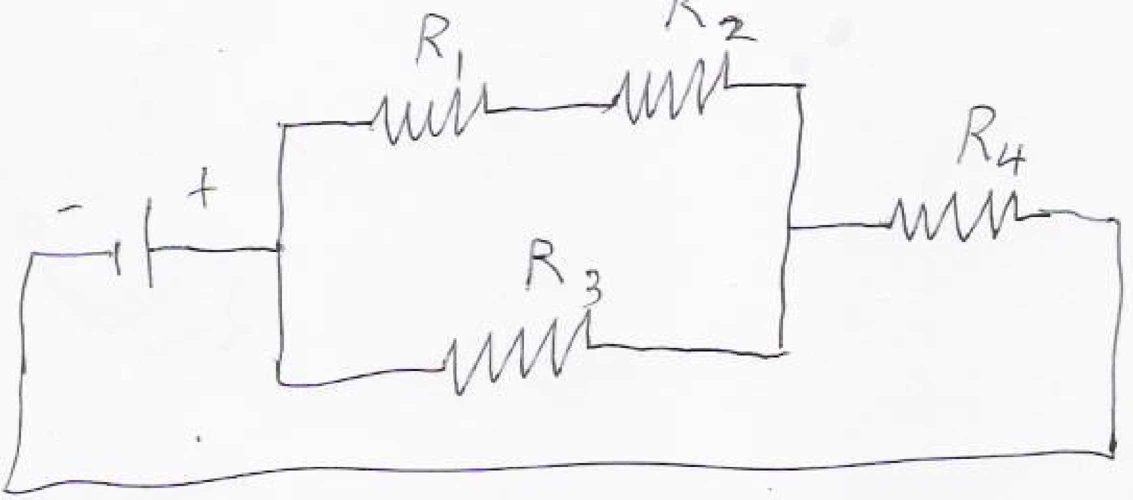


In more complicated circuits, you may need to use Kirchoff's rules (loop & junction) directly to find R_{eq} & C_{eq}.

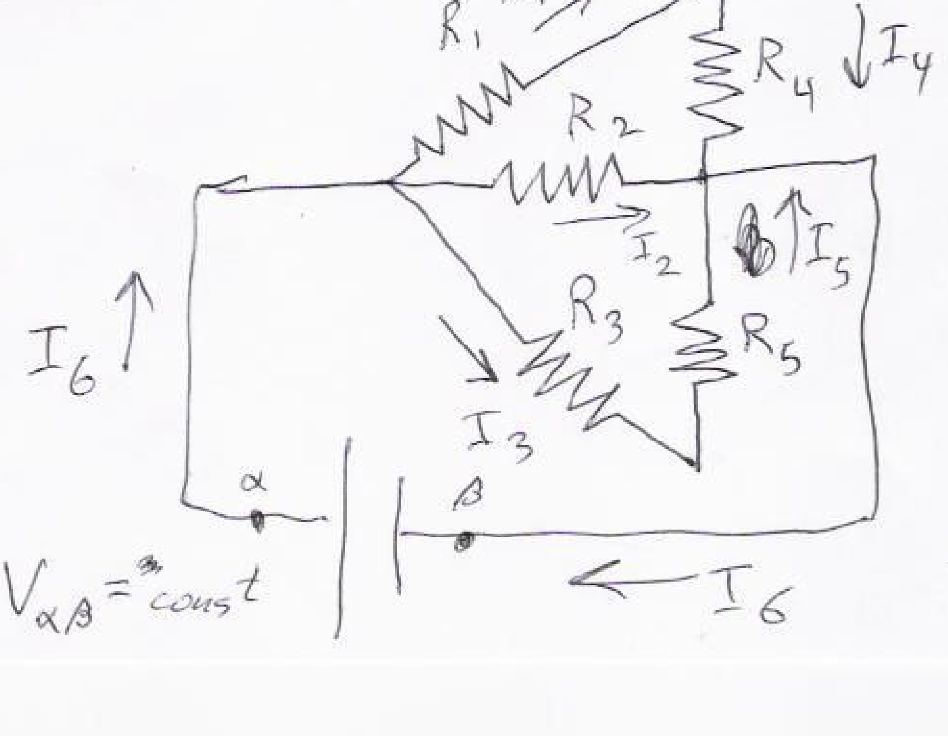
Sometimes you can repeatedly use series & parallel







$$R_{eq} = \left((R_1 + R_2)^{-1} + R_3^{-1} \right)^{-1} + R_4$$



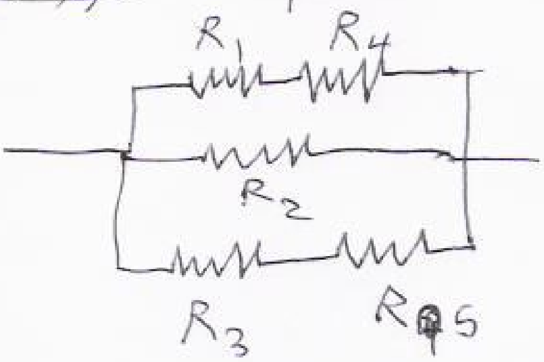
$V_{\alpha\beta} = \text{const}$

$R_{eq} = ? = \frac{V_{\alpha\beta}}{I_6}$

Takes more work...

~~Try Kirchhoff's rules~~

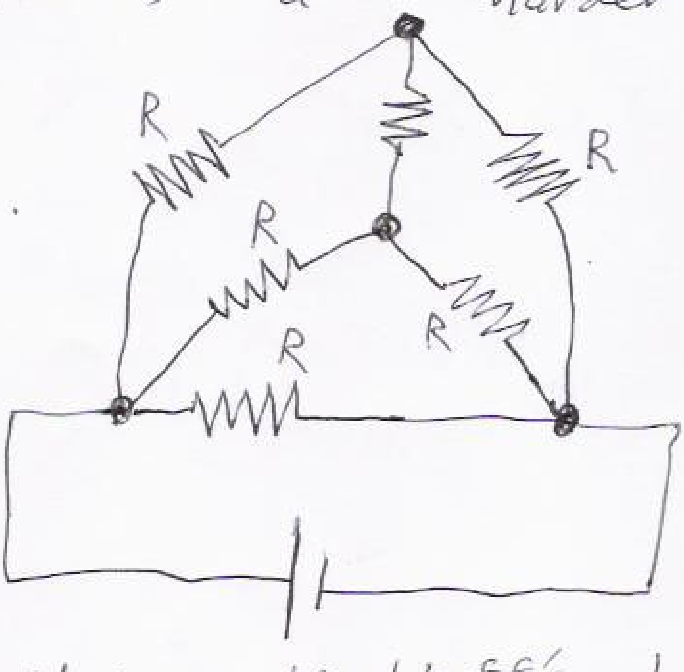
~~Actually, it's simpler:~~



$\approx$

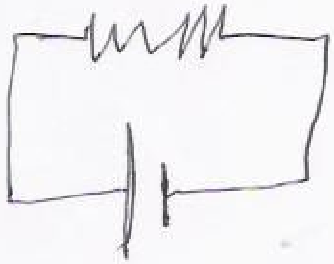
$$\frac{1}{\frac{1}{R_1+R_4}+\frac{1}{R_2}+\frac{1}{R_3+R_5}}$$

Here's a harder one:



$\approx$

$R_{eq} = ?$



Using Kirchkoff's Laws, I computed $R_{eq} = \frac{1}{2}R$

Challenge problem:

Prove that